RD Sharma Solutions Class 12 Maths Chapter 9 Ex 9.2

Chapter 9 Continuity Ex 9.2 Q2

When $x \neq 0$,

$$f(x) = \frac{x}{|x|} = \begin{cases} \frac{-x}{x} = -1 \ ; \ x < 0 \\ \frac{x}{|x|} = 1 \ ; x > 0 \end{cases}$$

So, $f(x)$ is a constant function when $x \neq 0$

hence, is continuous for all $x < 0$ and $x > 0$

Now,

Consider the point $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{-h}{|-h|} = -1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{h}{|h|} = 1$$

So, LHL $\neq$ RHL

Hence, function is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(i)

When $x \neq 1$

$f(x) = x^3 - x^2 + 2x - 2$ is a polynomial, so is continuous for $x < 1$ and $x > 1$

Now, consider the point $x = 1$

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} (1-h)^3 - (1-h)^2 + 2(1-h) - 2 = 1 - 1 + 2 - 2 = 0$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} (1+h)^3 - (1+h)^2 + 2(1+h) - 2 = 1 - 1 + 2 - 2 = 0$$

$f(1) = 4$

$\text{LHL} = \text{RHL} \neq f(1)$

Thus, function is not discontinuous at $x = 1$

Chapter 9 Continuity Ex 9.2 Q3(ii)

When $x \neq 2$, we have,

$$f(x) = \frac{x^4 - 16}{x - 2} = \frac{(x^2+4)(x^2-4)}{x-2} = \frac{(x^2+4)(x+2)(x-2)}{x-2} = f(x) = (x^2+4)(x+2)$$

which is a polynomial, so the function is continuous when $x < 2$ or $x > 2$

Now, consider the point $x = 2$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} \frac{(2-h)^4 - 16}{(2-h) - 2}$$

$$= \lim_{h \to 0} \frac{2^4 - 4.8h + 6.4h^2 - 4.2h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \to 0} \frac{16 - 32h + 24h^2 - 8h^3 + h^4 - 16}{-h}$$

$$= \lim_{h \to 0} 32 - 24h + 8h^2 - h^3 = 32$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} \frac{(2+h)^4 - 16}{(2+h) - 2} = \lim_{h \to 0} \frac{16 + 32h + 24h^2 + 8h^3 + h^4 - 16}{h}$$

$$= \lim_{h \to 0} 32 + 24h + 8h^2 + h^3$$

$$= 32$$

Also, $f(2) = 16$

Thus, $\text{LHL} = \text{RHL} \neq f(2)$

Hence, the function is discontinuous at $x = 2$

Chapter 9 Continuity Ex 9.2 Q3(iii)

When $x < 0$, we have, $f(x) = \frac{\sin x}{x}$

We know that $\sin x$ and the identity function continuous for $x < 0$, so the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$ $f(x) = 2x + 3$, which is a polynomial of degree 1 so $f(x) = 2x + 3$ is continuous for $x >$

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin h}{h} = 1$$

$f(0) = 2 \times 0 + 3 = 3$

Thus, L.H.L = R.H.L $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(iv)

When $x \neq 0$ $f(x) = \frac{\sin 3x}{x}$

We know that $\sin 3x$ and the identity function x are continuous for $x < 0$ and $x > 0$.

So, the quotient function $f(x) = \frac{\sin 3x}{x}$ is continuous for $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{\sin 3(-h)}{-h} = \lim_{h \to 0} \frac{-\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{\sin 3h}{h} = 3$$

$f(0) = 4$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(v)

When $x \neq 0$, we have, $f(x) = \frac{\sin x}{x} + \cos x$

We know that

$\sin x$ and $\cos x$ is continuous for $x < 0$ and $x > 0$.

The identity function x is also continuous for $x < 0$ and $x > 0$.

$\therefore$ The quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$ and $x > 0$.

And, the sum $\frac{\sin x}{x} + \cos x$ is also continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} + \cos(-h) = \lim_{h \to 0} \frac{-\sin h}{-h} + \cos h = 1 + 1 = 2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{\sin h}{h} + \cos h = 1 + 1 = 2$$

$f(0) = 5$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vi)

When $x \neq 0$, we have, $f(x) = \frac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$

We know that a polynomial is continuous for $x < 0$ and $x > 0$, Also the inverse trignometric functio
is continuous in its domain.

Here, $x^4 + x^3 + 2x^2$ is polynomial, so is continuous for $x < 0$ and $x > 0$ and $\tan^{-1} x$ is also continuo
for $x < 0$ and $x > 0$

So, the quotient function $f(x) = \frac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{(-h)^4 + (-h)^3 + 2(-h)^2}{\tan^{-1}(-h)} = \lim_{h \to 0} \frac{h^4 - h^3 + 2h^2}{\tan^{-1} h} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{h^4 + h^3 + 2h^2}{\tan^{-1} h} = 0$$

$f(0) = 10$

Thus, LHL = RHL $\neq f(0)$

Hence, the function is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(vii)

When $x \neq 0$, we have,

$$f(x) = \frac{e^x - 1}{\log_e(1+2x)}$$

We know that e^x and the constant function is continuous for $x < 0$ and $x > 0$

$\Rightarrow$ $e^x - 1$ is continuous for $x < 0$ and $x > 0$

Again, logarithmic function is continuous for $x < 0$ and $x > 0$

$\Rightarrow$ $\log_e(1+2x)$ is continuous for $x > 0$ and $x < 0$

So, the quotient function $f(x) = \dfrac{e^x - 1}{\log_e(1+2x)}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{e^{-h} - 1}{\log_e(1-2h)} = \lim_{h \to 0} \frac{\dfrac{e^{-h} - 1}{-h}}{\dfrac{\log_e(1-2h)}{-2h} \times -2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{e^{h} - 1}{\log_e(1+2h)} = \lim_{h \to 0} \frac{\dfrac{e^{h} - 1}{h}}{\dfrac{\log_e(1+2h)}{2h} \times 2} = \frac{1}{2}$$

$f(0) = 7$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is not continuous at $x = 0$

Chapter 9 Continuity Ex 9.2 Q3(viii)

We know that

(i) The absolute value function $g(x) = |x|$ is continuous on IR

(ii) Polynomial function are every where continuous.

So, the only possible point of discontinuity of $f(x)$ can be $x = 1$

Now

$$f(1) = |1 - 3| = |-2| = 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} |x - 3| = 2$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \left(\frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} \right)$$

$$= \frac{1}{4} - \frac{3}{2} + \frac{13}{4} = \frac{8}{4} = 2$$

Since

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1) = 2$$

$\therefore$ $f(x)$ is continuous at x

Hence $f(x)$ has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q3(ix)

When $x < -3$,

$f(x) = |x| + 3$

We know that $|x|$ is continuous for $x < -3$

$\therefore |x| + 3$ is continuous for $x < -3$

When $x > 3$

$f(x) = 6x + 2$ which is a polynomial of degree 1, so $f(x) = 6x + 2$ is continuous for $x > 3$

When $-3 < x < 3$

$f(x) = -2x$ which is again a polynomial so, it is continuous for $-3 < x < 3$

Now, consider the point $x = -3$

LHL $= \lim_{x \to -3^-} f(x) = \lim_{h \to 0} f(-3-h) = \lim_{h \to 0} |-3-h| + 3 = \lim_{h \to 0} |3+h| + 3 = 6$

RHL $= \lim_{x \to -3^+} f(x) = \lim_{h \to 0} f(-3+h) = \lim_{h \to 0} -2(-3+h) = 6$

$f(-3) = |-3| + 3 = 6$

Thus, LHL = RHL = $f(-3) = 6$

So, the function is continuous at $x = -3$

Now, consider the point $x = 3$

LHL $= \lim_{x \to 3^-} f(x) = \lim_{h \to 0} f(3-h) = \lim_{h \to 0} -2(3-h) = -6$

Chapter 9 Continuity Ex 9.2 Q3(xi)

The given function is $f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \le x \le 1 \\ 4x, & \text{if } x > 1 \end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

If $c < 0$, then $f(c) = 2c$

$\lim_{x \to c} f(x) = \lim_{x \to c} (2x) = 2c$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, f is continuous at all points x, such that $x < 0$

Chapter 9 Continuity Ex 9.2 Q3(xii)

The given function f is $f(x)=\begin{cases}\sin x-\cos x, & \text{if } x\neq 0\\ -1 & \text{if } x=0\end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

$$\text{If } c\neq 0,\ \text{then } f(c)=\sin c-\cos c$$
$$\lim_{x\to c} f(x)=\lim_{x\to c}(\sin x-\cos x)=\sin c-\cos c$$
$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points x, such that $x\neq 0$

The given function f is $f(x) = \begin{cases} -2, \text{ if } x \leq -1 \\ 2x, \text{ if } -1 < x \leq 1 \\ 2, \text{ if } x > 1 \end{cases}$

The given function is defined at all points of the real line.

Let c be a point on the real line.

Case I:

If $c < -1$, then $f(c) = -2$ and $\lim_{x \to c} f(x) = \lim_{x \to c} (-2) = -2$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, f is continuous at all points x, such that $x < -1$

Case II:

If $c = -1$, then $f(c) = f(-1) = -2$

The left hand limit of f at $x = -1$ is,

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} (-2) = -2$

The right hand limit of f at $x = -1$ is,

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} (2x) = 2 \times (-1) = -2$

$\therefore \lim_{x \to -1} f(x) = f(-1)$

Therefore, f is continuous at $x = -1$

Case III:

If $-1 < c < 1$, then $f(c) = 2c$

$\lim_{x \to c} f(x) = \lim_{x \to c} (2x) = 2c$

$\therefore \lim_{x \to c} f(x) = f(c)$

Therefore, f is continuous at all points of the interval $(-1, 1)$.

Case IV:

If $c=1$, then $f(c)=f(1)=2\times1=2$

The left hand limit of f at $x=1$ is,

$\lim\limits_{x\to1^-} f(x)=\lim\limits_{x\to1^-}(2x)=2\times1=2$

The right hand limit of f at $x=1$ is,

$\lim\limits_{x\to1^+} f(x)=\lim\limits_{x\to1^+}2=2$

$\therefore \lim\limits_{x\to1} f(x)=f(c)$

Therefore, f is continuous at $x=2$

Case V:

If $c>1$, then $f(c)=2$ and $\lim\limits_{x\to c} f(x)=\lim\limits_{x\to c}(2)=2$

$\lim\limits_{x\to c} f(x)=f(c)$

Therefore, f is continuous at all points x, such that $x>1$

Thus, from the above observations, it can be concluded that f is continuous at all points of the real line

Chapter 9 Continuity Ex 9.2 Q4(i)

We have given that the function is continuous at $x=0$

$\therefore$ LHL = RHL = $f(0)$ ....(1)

$$\text{LHL} = \lim_{x\to0^-} f(x) = \lim_{h\to0} f(0-h) = \lim_{h\to0}\frac{\sin(-2h)}{5(-h)} = \lim_{h\to0}\frac{-\sin 2h}{-5h} = \lim_{h\to0}\frac{\sin 2h}{2h}\times\frac{2h}{5h} = \frac{2}{5}$$

$f(0)=3k$

So, using (1) we get,

$\frac{2}{5}=3k$

$k=\frac{2}{15}$

Chapter 9 Continuity Ex 9.2 Q4(ii)

It is given that the function is continuous

$\therefore$ LHL = RHL = $f(2)$ ....(1)

$$\text{LHL} = \lim_{x\to2^-} f(x) = \lim_{h\to0} f(2-h) = \lim_{h\to0} k(2-h)+5 \quad = 2k+5$$

$$\text{RHL} = \lim_{x\to2^+} f(x) = \lim_{h\to0} f(2+h) = \lim_{h\to0}(2+h)-1=1$$

Thus, using (1), we get,

$2k+5=1$

$k=-2$

Chapter 9 Continuity Ex 9.2 Q4(iii)

It is given that the function is continuous

LHL = RHL = $f(0)$ .... (1)

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} k\left((-h)^2 + 3(-h)\right) = \lim\limits_{h \to 0} k\left(h^2 - 3h\right) = 0$

$f(0) = \cos 2 \times 0 = \cos 0^\circ = 1$

LHL $\neq f(0)$

Hence, no value of k can make f continuous

Chapter 9 Continuity Ex 9.2 Q4(iv)

First check the continuity of the function at $x = 3$

$f(3) = 2$ .....(A)

RHL = $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{h \to 0} f(3+h) = \lim\limits_{h \to 0} a(3+h) + b = 3a + b$.....(B)

$\therefore f(x)$ will be continuous at $x = 3$ if $3a + b = 2$.....(1)

Now, check the continuity at $x = 5$

$f(5) = 9$ ....(C)

LHL = $\lim\limits_{x \to 5^-} f(x) = \lim\limits_{h \to 0} f(5-h) = \lim\limits_{h \to 0} a(5-h) + b = 5a + b$

$f(x)$ will be continuous at $x = 5$ if $5a + b = 9$....(2)

Solving (1) & (2), we get

$a = \frac{7}{2}$ and $b = \frac{-17}{2}$

Chapter 9 Continuity Ex 9.2 Q4(v)

It is given that the function is continuous

At $x = -1$

$f(-1) = 4$

RHL = $\lim\limits_{x \to -1^+} f(x) = \lim\limits_{h \to 0} f(-1+h) = \lim\limits_{h \to 0} a(-1+h)^2 + b = a + b$

Since, $f(x)$ is continuous at $x = -1$

$\therefore a + b = 4$ ....(A)

Now, at $x = 0$,

$f(0) = \cos 0^\circ = 1$

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} a(-h)^2 + b = b$

Since, $f(x)$ is continuous at $x = 0$

$\therefore f(0) = \text{LHL}$

$\Rightarrow b = 1$

$\therefore$ from (A)

$a = 3$

Thus, $a = 3$, $b = 1$

Chapter 9 Continuity Ex 9.2 Q4(vi)

It is given that the function is continuous.

At $x = 0$

$$\text{LHL} = \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{\sqrt{1-Ph}-\sqrt{1+Ph}}{-h} = \lim_{h\to 0} \frac{\left(\sqrt{1-Ph}-\sqrt{1+Ph}\right)}{-h} \times \frac{\left(\sqrt{1-Ph}+\sqrt{1+Ph}\right)}{\left(\sqrt{1-Ph}+\sqrt{1+Ph}\right)}$$

$$= \lim_{h\to 0} \frac{(1-Ph)-(1+Ph)}{-h\left(\sqrt{1-Ph}+\sqrt{1+Ph}\right)} = \frac{2P}{2} = P$$

$$\text{RHL} = \lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{2h+1}{h-2} = \frac{-1}{2}$$

Since, $f(x)$ is continuous so,

$$P = \frac{-1}{2}$$

Chapter 9 Continuity Ex 9.2 Q4(vii)

The given function f is $f(x)=\begin{cases}5, & \text{if } x\le 2\\ ax+b, & \text{if } 2<x<10\\ 21, & \text{if } x\ge 10\end{cases}$

It is evident that the given function f is defined at all points of the real line.

If f is a continuous function, then f is continuous at all real numbers.

In particular, f is continuous at $x=2$ and $x=10$

Since f is continuous at $x=2$, we obtain

$$\lim_{x\to 2^-} f(x)=\lim_{x\to 2^+} f(x)=f(2)$$
$$\Rightarrow \lim_{x\to 2^-}(5)=\lim_{x\to 2^+}(ax+b)=5$$
$$\Rightarrow 5=2a+b=5$$
$$\Rightarrow 2a+b=5 \qquad \ldots(1)$$

Since f is continuous at $x=10$, we obtain

$$\lim_{x\to 10^-} f(x)=\lim_{x\to 10^+} f(x)=f(10)$$
$$\Rightarrow \lim_{x\to 10^-}(ax+b)=\lim_{x\to 10^+}(21)=21$$
$$\Rightarrow 10a+b=21=21$$
$$\Rightarrow 10a+b=21 \qquad \ldots(2)$$

On subtracting equation (1) from equation (2), we obtain

$8a=16$

$a=2$

By putting $a=2$ in equation (1), we obtain

$2\times 2+b=5$

$4+b=5$

$b=1$

Therefore, the values of a and b for which f is a continuous function are 2 and 1 respectively.

Chapter 9 Continuity Ex 9.2 Q4(viii)

Since the function is continuous at $x=\frac{\pi}{2}$ therefore

LHL of $f(x)$ at $x=\frac{\pi}{2}$ is

$$=\lim_{x\to\frac{\pi}{2}^-} f(x)$$

$$=\lim_{h\to 0} f\left(h-\frac{\pi}{2}\right)$$

$$=\lim_{h\to 0}\frac{k\cos\left(h-\frac{\pi}{2}\right)}{\pi-2\left(h-\frac{\pi}{2}\right)}$$

$$=\lim_{h\to 0}\frac{k\sinh}{2\pi-2h}$$

$$=\frac{k}{2}\lim_{h\to 0}\frac{\sin(\pi-h)}{(\pi-h)}$$

$$=\frac{k}{2}$$

Again

$$f\left(\frac{\pi}{2}\right)=3$$

Hence

$$LHL=f\left(\frac{\pi}{3}\right)$$

$$\frac{k}{2}=3$$

$$k=6$$

Chapter 9 Continuity Ex 9.2 Q5

We have given that $f(x)$ is continuous on $[0.\infty]$

$\therefore f(x)$ is continuous at $x = 1$ and $x = \sqrt{2}$

$\therefore$ At $x = 1$, LHL = RHL = $f(1)$ .....(A)

$f(1) = a$ .....(1)

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{(1-h)^2}{a} = \frac{1}{a}$$

Using (A) we get,

$$a = \frac{1}{a} \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$$

At $x = \sqrt{2}$ LHL = RHL = $f(\sqrt{2})$ .....(B)

$$f(\sqrt{2}) = \frac{2b^2 - 4b}{(\sqrt{2})^2} = \frac{2b^2 - 4b}{2} = b^2 - 2b \quad ......(2)$$

$$\text{LHL} = \lim_{x \to \sqrt{2}^-} f(x) = \lim_{h \to 0} f(\sqrt{2} - h) = \lim_{h \to 0} a = a.$$

So, using (B), we get,

$b^2 - 2b = a$

For $a = 1$, $b^2 - 2b - 1 = 0$

$$\Rightarrow b = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$$

For $a = -1$ $b^2 - 2b + 1 = 0$

$\Rightarrow (b-1)^2 = 0 \Rightarrow b = 1$

Thus, $a = -1$, $b = 1$ or $a = 1$, $b = 1 \pm \sqrt{2}$

Chapter 9 Continuity Ex 9.2 Q6

Since, $f(x)$ is continuous on $[0,\pi]$

$f(x)$ is continuous at $x=\frac{\pi}{4}$ and $x=\frac{\pi}{2}$

At $x=\frac{\pi}{4}$,

$$\text{LHL}=\text{RHL}=f\left(\frac{\pi}{4}\right).....(A)$$

$$\text{Now, } f\left(\frac{\pi}{4}\right)=2\frac{\pi}{4}.\cot\left(\frac{\pi}{4}\right)+b=\frac{\pi}{2}.1+b=\frac{\pi}{2}+b\ ....(1)$$

$$\text{LHL}=\lim_{x\to\frac{\pi}{4}^-}f(x)=\lim_{h\to 0}f\left(\frac{\pi}{4}-h\right)=\lim_{h\to 0}\left(\frac{\pi}{4}-h\right)+a\sqrt{2}\sin\left(\frac{\pi}{4}-h\right)=\frac{\pi}{4}+a\sqrt{2}.\frac{1}{\sqrt{2}}=\frac{\pi}{4}+a$$

Thus, using (A)

$$\frac{1}{2}+b=\frac{\pi}{4}+a$$

$$a-b=\frac{\pi}{4}.....(B)$$

At $x=\frac{\pi}{2}$

$$\text{LHL}=\text{RHL}=f\left(\frac{\pi}{2}\right).....(C)$$

$$\text{Now, } f\left(\frac{\pi}{2}\right)=a\cos 2.\frac{\pi}{2}-b\sin\frac{\pi}{2}=-a-b....(2)$$

$$\text{LHL}=\lim_{x\to\frac{\pi}{2}^-}f(x)=\lim_{h\to 0}f\left(\frac{\pi}{2}-h\right)$$

$$=\lim_{h\to 0}2\left(\frac{\pi}{2}-h\right)\cot\left(\frac{\pi}{2}-h\right)+b=\pi\times 0+b=b$$

using (C), we get,

$$-a-b=b\quad\Rightarrow\ 2b=-a\quad\Rightarrow\ b=\frac{-a}{2}$$

from (B), $a+\frac{a}{2}=\frac{\pi}{4}$

$$\Rightarrow\qquad \frac{3}{2}a=\frac{\pi}{4}$$

$$\Rightarrow\qquad a=\frac{\pi}{6}$$

and $b=\frac{-a}{2}=\frac{-\pi}{12}$

Thus, $a=\frac{\pi}{6}$, $b=\frac{-\pi}{12}$

Chapter 9 Continuity Ex 9.2 Q7

It is given that the $f(x)$ is continuous on $[0,8]$

$f(x)$ is continuous at $x = 2$ and $x = 4$.

Now, At $x = 2$

$LHL = RHL = f(2) \ldots.(A)$

$f(2) = 3 \times 2 + 2 = 8 \ldots.(1)$

$$LHL = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} (2-h)^2 + a(2-h) + b = 4 + 2a + b$$

from (A)

$4 + 2a + b = 8$

$2a + b = 4 \ldots..(B)$

Now, At $x = 4$

$LHL = RHL = f(4) \ldots.(C)$

$f(4) = 3 \times 4 + 2 = 14 \ldots..(2)$

$$RHL = \lim_{x \to 4^+} f(x) = \lim_{h \to 0} f(4+h) = \lim_{h \to 0} 2a(4+h) + 5b = 8a + 5b$$

From (C), we get,

$8a + 5b = 14 \ldots.(D)$

Solving (B) and (D), we get,

$a = 3$ and $b = -2$

Chapter 9 Continuity Ex 9.2 Q8

The function will be continuous on $\left[0, \frac{\pi}{2}\right]$ if it is continuous at every point in $\left[0, \frac{\pi}{2}\right]$

Let us consider the point $x = \frac{\pi}{4}$,

We must have,

$LHL = RHL = f\left(\frac{\pi}{4}\right) \ldots.(A)$

$$LHL = \lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \to 0} \frac{\tan\left(\frac{\pi}{4} - \frac{\pi}{4} + h\right)}{\cot + 2\left(\frac{\pi}{4} - h\right)} = \lim_{h \to 0} \frac{\tan h}{\tan 2h} \qquad \left[\because \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta\right]$$

$$= \lim_{h \to 0} \frac{\frac{\tan h}{h}}{\frac{\tan 2h}{h}} = \frac{1}{2}$$

Thus, using (A) we get,

$f\left(\frac{\pi}{4}\right) = \frac{1}{2}$

Hence, $f(x)$ will be continuous on $\left[0, \frac{\pi}{2}\right]$ if $f\left(\frac{\pi}{4}\right) = \frac{1}{2}$.

Chapter 9 Continuity Ex 9.2 Q9

When $x < 2$, we have

$f(x) = 2x - 1$, which is a polynomial of degree 1.

So, $f(x)$ is continuous for $x < 2$.

When $x > 2$, we have

$f(x) = \frac{3x}{2}$, which is again a polynomial of degree 1.

So, $f(x)$ is continuous for $x > 2$.

Now, consider the point $x = 2$

$f(2) = \frac{3 \times 2}{2} = 3$

$$\text{LHL} = \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} 2(2-h) - 1 = 3$$

$$\text{RHL} = \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h) = \lim_{h \to 0} \frac{3(2+h)}{2} = 3$$

LHL = RHL = $f(2) = 3$

Thus, $f(x)$ is continuous at $x = 2$

Hence, $f(x)$ is continuous every where.

Chapter 9 Continuity Ex 9.2 Q10

Let $f(x) = \sin|x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \, o \, h$, where $g(x) = |x|$ and $h(x) = \sin x$

$$\left[\because (goh)(x) = g(h(x)) = g(\sin x) = |\sin x| = f(x)\right]$$

It has to be proved first that $g(x) = |x|$ and $h(x) = \sin x$ are continuous functions.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} (-x) = -c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} x = c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, g is continuous at all points x, such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0$

$\therefore \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} (x) = g(0)$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x) = \sin x$

It is evident that $h(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + k$

If $x \to c$, then $k \to 0$

$h(c) = \sin c$

$h(c) = \sin c$

$\lim\limits_{x \to c} h(x) = \lim\limits_{x \to c} \sin x$

$= \lim\limits_{k \to 0} \sin(c+k)$

$= \lim\limits_{k \to 0} [\sin c \cos k + \cos c \sin k]$

$= \lim\limits_{k \to 0} (\sin c \cos k) + \lim\limits_{h \to 0} (\cos c \sin k)$

$= \sin c \cos 0 + \cos c \sin 0$

$= \sin c + 0$

$= \sin c$

$\therefore \lim\limits_{x \to c} h(x) = g(c)$

Therefore, h is a continuous function.

It is known that for real valued functions g and h, such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = g(h(x)) = g(\sin x) = |\sin x|$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q11

When $x < 0$, we have,

$f(x) = \frac{\sin x}{x}$

We know that the $\sin x$ and the identity function x are continuous for $x < 0$.

So, the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$, we have,

$f(x) = x + 1$, which is a polynomial of degree 1. So, $f(x)$ is continuous for $x > 0$

Now, consider the point $x = 0$.

$f(0) = 0 + 1 = 1$.

$\text{LHL} = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0 - h) = \lim\limits_{h \to 0} \frac{\sin(-h)}{-h} = \lim\limits_{h \to 0} \frac{-\sin h}{-h} = 1$

$\text{RHL} = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 + h) = \lim\limits_{h \to 0} h + 1 = 1$

Thus, LHL = RHL = $f(0) = 1$

So, $f(x)$ is continuous at $x = 0$.

Hence, $f(x)$ is continuous everywhere

Chapter 9 Continuity Ex 9.2 Q12

The given function is $g(x)=x-[x]$

It is evident that g is defined at all integral points.

Let n be an integer.

Then,

$$g(n)=n-[n]=n-n=0$$

The left hand limit of f at $x=n$ is,

$$\lim_{x\to n^-} g(x)=\lim_{x\to n^-}(x-[x])=\lim_{x\to n^-}(x)-\lim_{x\to n^-}[x]=n-(n-1)=1$$

The right hand limit of f at $x=n$ is,

$$\lim_{x\to n^+} g(x)=\lim_{x\to n^+}(x-[x])=\lim_{x\to n^+}(x)-\lim_{x\to n^+}[x]=n-n=0$$

It is observed that the left and right hand limits of f at $x=n$ do not coincide.

Therefore, f is not continuous at $x=n$

Hence, g is discontinuous at all integral points

Chapter 9 Continuity Ex 9.2 Q13

It is known that if g and h are two continuous functions, then

$g+h,\ g-h,$ and $g.h$ are also continuous.

It has to proved first that $g(x) = \sin x$ and $h(x) = \cos x$ are continuous functions.

Let $g(x) = \sin x$

It is evident that $g(x) = \sin x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$g(c) = \sin c$

$$\begin{aligned}\lim_{x\to c} g(x) &= \lim_{x\to c} \sin x \\ &= \lim_{h\to 0} \sin(c+h) \\ &= \lim_{h\to 0}[\sin c \cos h + \cos c \sin h] \\ &= \lim_{h\to 0}(\sin c \cos h) + \lim_{h\to 0}(\cos c \sin h) \\ &= \sin c \cos 0 + \cos c \sin 0 \\ &= \sin c + 0 \\ &= \sin c\end{aligned}$$

$\therefore \lim_{x\to c} g(x) = g(c)$

Therefore, g is a continuous function.

Let $h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let c be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$h(c) = \cos c$

$$\begin{aligned}\lim_{x\to c} h(x) &= \lim_{x\to c} \cos x \\ &= \lim_{h\to 0} \cos(c+h) \\ &= \lim_{h\to 0}[\cos c \cos h - \sin c \sin h] \\ &= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h \\ &= \cos c \cos 0 - \sin c \sin 0 \\ &= \cos c \times 1 - \sin c \times 0 \\ &= \cos c\end{aligned}$$

$\therefore \lim_{x\to c} h(x) = h(c)$

Therefore, h is a continuous function.

Therefore, it can be concluded that

(a) $f(x) = g(x) + h(x) = \sin x + \cos x$ is a continuous function

(b) $f(x) = g(x) - h(x) = \sin x - \cos x$ is a continuous function

(c) $f(x) = g(x) \times h(x) = \sin x \times \cos x$ is a continuous function

Chapter 9 Continuity Ex 9.2 Q14

The given function is $f(x) = \cos(x^2)$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = \cos x$ and $h(x) = x^2$

$$\left[\because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x)\right]$$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that g is defined for every real number.

Let c be a real number.

Then, $g(c) = \cos c$

Put $x = c + h$

If $x \to c$, then $h \to 0$

$$\lim_{x\to c} g(x) = \lim_{x\to c} \cos x$$
$$= \lim_{h\to 0} \cos(c+h)$$
$$= \lim_{h\to 0} [\cos c \cos h - \sin c \sin h]$$
$$= \lim_{h\to 0} \cos c \cos h - \lim_{h\to 0} \sin c \sin h$$
$$= \cos c \cos 0 - \sin c \sin 0$$
$$= \cos c \times 1 - \sin c \times 0$$
$$= \cos c$$

$$\therefore \lim_{x\to c} g(x) = g(c)$$

Therefore, $g(x) = \cos x$ is continuous function.

$h(x) = x^2$

Clearly, h is defined for every real number.

Let k be a real number, then $h(k) = k^2$

$$\lim_{x\to k} h(x) = \lim_{x\to k} x^2 = k^2$$

$$\therefore \lim_{x\to k} h(x) = h(k)$$

Therefore, h is a continuous function.

It is known that for real valued functions g and h,such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

Chapter 9 Continuity Ex 9.2 Q15

The given function is $f(x)=|\cos x|$

This function f is defined for every real number and f can be written as the composition of two functions as,

$f = g \, o \, h$, where $g(x)=|x|$ and $h(x)=\cos x$

$\left[\because (goh)(x)=g(h(x))=g(\cos x)=|\cos x|=f(x)\right]$

It has to be first proved that $g(x)=|x|$ and $h(x)=\cos x$ are continuous functions.

$g(x)=|x|$ can be written as

$$g(x)=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c<0$, then $g(c)=-c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c}(-x)=-c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x<0$

Case II:

If $c>0$, then $g(c)=c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c} x=c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x>0$

Case III:

If $c=0$, then $g(c)=g(0)=0$

$$\lim_{x\to 0^-} g(x)=\lim_{x\to 0^-}(-x)=0$$
$$\lim_{x\to 0^+} g(x)=\lim_{x\to 0^+}(x)=0$$
$$\therefore \lim_{x\to 0^-} g(x)=\lim_{x\to 0^+}(x)=g(0)$$

Therefore, g is continuous at $x=0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x)=\cos x$

It is evident that $h(x)=\cos x$ is defined for every real number.

Let c be a real number. Put $x=c+h$

If $x\to c$, then $h\to 0$

$h(c)=\cos c$

$$\begin{aligned}\lim_{x\to c} h(x)&=\lim_{x\to c}\cos x\\&=\lim_{h\to 0}\cos(c+h)\\&=\lim_{h\to 0}\left[\cos c\cos h-\sin c\sin h\right]\\&=\lim_{h\to 0}\cos c\cos h-\lim_{h\to 0}\sin c\sin h\\&=\cos c\cos 0-\sin c\sin 0\\&=\cos c\times 1-\sin c\times 0\\&=\cos c\end{aligned}$$
$$\therefore \lim_{x\to c} h(x)=h(c)$$

Therefore, $h(x)=\cos x$ is a continuous function.

It is known that for real valued functions g and h,such that $(g \circ h)$ is defined at c, if g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c.

Therefore, $f(x)=(goh)(x)=g(h(x))=g(\cos x)=|\cos x|$ is a continuous function

Chapter 9 Continuity Ex 9.2 Q16

The given function is $f(x)=|x|-|x+1|$

The two functions, g and h, are defined as

$g(x)=|x|$ and $h(x)=|x+1|$

Then, $f=g-h$

The continuity of g and h is examined first.

$g(x)=|x|$ can be written as

$$g(x)=\begin{cases}-x, & \text{if } x<0\\ x, & \text{if } x\geq 0\end{cases}$$

Clearly, g is defined for all real numbers.

Let c be a real number.

Case I:

If $c<0$, then $g(c)=-c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c}(-x)=-c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x<0$

Case II:

If $c>0$, then $g(c)=c$ and $\lim\limits_{x\to c} g(x)=\lim\limits_{x\to c} x=c$

$\therefore \lim\limits_{x\to c} g(x)=g(c)$

Therefore, g is continuous at all points x, such that $x>0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$$
$$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0$$
$$\therefore \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} (x) = g(0)$$

Therefore, g is continuous at $x = 0$

From the above three observations, it can be concluded that g is continuous at all points.

$h(x) = |x+1|$ can be written as

$$h(x) = \begin{cases} -(x+1), & \text{if, } x < -1 \\ x+1, & \text{if } x \geq -1 \end{cases}$$

Clearly, h is defined for every real number.

Let c be a real number.

Case I:

If $c < -1$, then $h(c) = -(c+1)$ and $\lim_{x \to c} h(x) = \lim_{x \to c} [-(x+1)] = -(c+1)$

$$\therefore \lim_{x \to c} h(x) = h(c)$$

Therefore, h is continuous at all points x, such that $x < -1$

Case II:

If $c > -1$, then $h(c) = c+1$ and $\lim_{x \to c} h(x) = \lim_{x \to c} (x+1) = c+1$

$$\therefore \lim_{x \to c} h(x) = h(c)$$

Therefore, h is continuous at all points x, such that $x > -1$

Case III:

If $c=-1$, then $h(c)=h(-1)=-1+1=0$

$$\lim_{x\to -1^-} h(x)=\lim_{x\to -1^-}\left[-(x+1)\right]=-(-1+1)=0$$

$$\lim_{x\to -1^+} h(x)=\lim_{x\to -1^+}(x+1)=(-1+1)=0$$

$$\therefore \lim_{x\to -1^-} h(x)=\lim_{h\to -1^+} h(x)=h(-1)$$

Therefore, h is continuous at $x=-1$

From the above three observations, it can be concluded that h is continuous at all points of the real line.

g and h are continuous functions. Therefore, $f=g-h$ is also a continuous function.

Therefore, f has no point of discontinuity.

Chapter 9 Continuity Ex 9.2 Q17

He given function f is $f(x)=\begin{cases} x^2\sin\frac{1}{x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$

It is evident that f is defined at all points of the real line.

Let c be a real number.

Case I:

If $c\neq 0$, then $f(c)=c^2\sin\frac{1}{c}$

$$\lim_{x\to c} f(x)=\lim_{x\to c}\left(x^2\sin\frac{1}{x}\right)=\left(\lim_{x\to c} x^2\right)\left(\lim_{x\to c}\sin\frac{1}{x}\right)=c^2\sin\frac{1}{c}$$

$$\therefore \lim_{x\to c} f(x)=f(c)$$

Therefore, f is continuous at all points $x\neq 0$

Case II:

If $c = 0$, then $f(0) = 0$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-}\left(x^2 \sin\frac{1}{x}\right) = \lim_{x \to 0}\left(x^2 \sin\frac{1}{x}\right)$$

It is known that, $-1 \le \sin\frac{1}{x} \le 1,\ x \ne 0$

$$\Rightarrow -x^2 \le \sin\frac{1}{x} \le x^2$$

$$\Rightarrow \lim_{x \to 0}\left(-x^2\right) \le \lim_{x \to 0}\left(x^2 \sin\frac{1}{x}\right) \le \lim_{x \to 0} x^2$$

$$\Rightarrow 0 \le \lim_{x \to 0}\left(x^2 \sin\frac{1}{x}\right) \le 0$$

$$\Rightarrow \lim_{x \to 0}\left(x^2 \sin\frac{1}{x}\right) = 0$$

$$\therefore \lim_{x \to 0^-} f(x) = 0$$

Similarly, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+}\left(x^2 \sin\frac{1}{x}\right) = \lim_{x \to 0}\left(x^2 \sin\frac{1}{x}\right) = 0$

$$\therefore \lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$$

Therefore, f is continuous at $x = 0$

From the above observations, it can be concluded that f is continuous at every point of the real line.

Thus, f is a continuous function.

Chapter 9 Continuity Ex 9.2 Q18

$$f(x) = \frac{1}{x+2}$$

$$\lim_{x \to -2^-} f(x) = \lim_{h \to 0} \frac{1}{-2-h+2} = \lim_{h \to 0} -\frac{1}{h} \to -\infty$$

$$\lim_{x \to -2^+} f(x) = \lim_{h \to 0} \frac{1}{-2+h+2} = \lim_{h \to 0} \frac{1}{h} \to \infty$$

$\therefore$ $f(x)$ is discontinuous at $x = -2$

Let $g(x) = f(f(x)) = \dfrac{x+2}{2x+5}$

$$\lim_{x \to -\frac{5}{2}^-} g(x) = \lim_{h \to 0} \frac{-\frac{5}{2}-h+2}{-5-h+5} = \lim_{h \to 0} -\frac{-\frac{5}{2}-h+2}{h} \to -\infty$$

$$\lim_{x \to -\frac{5}{2}^+} f(x) = \lim_{h \to 0} \frac{-\frac{5}{2}+h+2}{-5+h+5} = \lim_{h \to 0} \frac{-\frac{5}{2}-h+2}{h} \to \infty$$

$\therefore$ $g(x)$ is discontinuous at $x = -\dfrac{5}{2}$

$\therefore$ $f(f(x))$ is discontinuous at $x = -\dfrac{5}{2}$

$\therefore$ $f(x)$ is discontinuous at $x = -2$ and $-\dfrac{5}{2}$.

Chapter 9 Continuity Ex 9.2 Q19

$f(t) = \dfrac{1}{t^2+t-2}$, where $t = \dfrac{1}{x-1}$

Clearly $t = \dfrac{1}{x-1}$ is discontinuous at $x = 1$.

For $x \neq 1$, we have

$$f(t) = \frac{1}{t^2+t-2} = \frac{1}{(t+2)(t-1)}$$

This is discontinuous at $t = -2$ and $t = 1$

For $t = -2$, $t = \dfrac{1}{x-1} \Rightarrow x = \dfrac{1}{2}$

For $t = 1$, $t = \dfrac{1}{x-1} \Rightarrow x = 2$

Hence f is discontinuous at $x = \dfrac{1}{2}$, $x = 1$ and $x = 2$.