RD Sharma Solutions Class 12 Maths Chapter 5 Ex 5.5

Algebra of Matrices Ex 5.5 Q1

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\left(A - A^T\right) = \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T\right)$$

$$= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix}$$

$$\left(A - A^T\right) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \text{---(i)}$$

$$-\left(A - A^T\right)^T = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$-\left(A - A^T\right)^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii),

$$\left(A - A^T\right) = -\left(A - A^T\right)^T$$

We know that, x is a skew symmetric matrix if $x = -x^T$

So, $\left(A - A^T\right)$ is skew symmetric.

Algebra of Matrices Ex 5.5 Q2

Given,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$-\left(A - A^T\right)^T = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$$-\left(A - A^T\right)^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$\left(A - A^T\right) = -\left(A - A^T\right)^T$$

We know that, x is skewsymmetric matrx if $x = -x^T$

So, $\left(A - A^T\right)$ is skewsymmetric matrix.

Algebra of Matrices Ex 5.5 Q3

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix} \text{ is a symmetric matrix.}$$

We know that $A = [aij]_{m \times n}$ is a symmetric matrix if $aij = aji$

So,
$$x = a_{13} = a_{31} = 4$$
$$y = a_{21} = a_{12} = 2$$
$$z = a_{22} = a_{22} = z$$
$$t = a_{32} = a_{32} = -3$$

Hence,

$x = 4, y = 2, t = -3$ and z can have any value.

Algebra of Matrices Ex 5.5 Q4

Given,

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\left(A + A^T\right)$$
$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$
$$= \frac{1}{2}\begin{bmatrix} 3+3 & 2+1 & 7-2 \\ 1+2 & 4+4 & 3+5 \\ -2+7 & 5+3 & 8+8 \end{bmatrix}$$
$$= \frac{1}{2}\begin{bmatrix} 6 & 3 & 5 \\ 3 & 8 & 8 \\ 5 & 8 & 16 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}$$

Now, $Y = \frac{1}{2}\left(A - A^T\right)$

$$= \frac{1}{2}\left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 3-3 & 2-1 & 7+2 \\ 1-2 & 4-4 & 3-5 \\ -2-7 & 5-3 & 8-8 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & 1 & 9 \\ -1 & 0 & -2 \\ -9 & 2 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Now,

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} = X$$

$\Rightarrow$ x is a symmetric matrix

Now,

$$-Y^T = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & \frac{1}{2} & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$\Rightarrow \quad -Y^T = Y$

$\therefore \quad Y$ is skew symmetric matrix.

$$X+Y=\begin{bmatrix}3 & \frac{3}{2} & \frac{5}{2}\\ \frac{3}{2} & 4 & 4\\ \frac{5}{2} & 4 & 8\end{bmatrix}+\begin{bmatrix}0 & \frac{1}{2} & \frac{9}{2}\\ \frac{-1}{2} & 0 & -1\\ \frac{-9}{2} & 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3+0 & \frac{3}{2}+\frac{1}{2} & \frac{5}{2}+\frac{9}{2}\\ \frac{3}{2}-\frac{1}{2} & 4+0 & 4-1\\ \frac{5}{2}-\frac{9}{2} & 4+1 & 8-0\end{bmatrix}$$

$$=\begin{bmatrix}3 & 2 & 7\\ 1 & 4 & 3\\ -2 & 5 & 8\end{bmatrix}$$

$$=A$$

Hence,

$$X=\begin{bmatrix}3 & \frac{3}{2} & \frac{5}{4}\\ \frac{3}{2} & 4 & 4\\ \frac{5}{2} & 4 & 8\end{bmatrix},\ Y=\begin{bmatrix}0 & \frac{1}{2} & \frac{9}{2}\\ \frac{-1}{2} & 0 & -1\\ \frac{-9}{2} & 1 & 0\end{bmatrix}$$