

RD Sharma
Solutions
Class 12 Maths
Chapter 32
Ex 32.2

Mean and Variance of a Random Variable Ex 32.2 Q1(i)

x_i	p_i	$p_i x_i$	$p_i x_i^2$
2	0.2	0.4	0.8
3	0.5	1.5	4.5
4	0.3	1.2	4.8
		$\sum p_i x_i = 3.1$	$\sum p_i x_i^2 = 10.1$

$$\text{Mean} = \sum p_i x_i = 3.1$$

$$\text{Variance} = \sum p_i x_i^2 - (\sum p_i x_i)^2 = 10.1 - (3.1)^2 = 0.49$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = 0.7$$

Mean and Variance of a Random Variable Ex 32.2 Q1(ii)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	0.4	0.4	0.4
3	0.1	0.3	0.9
4	0.2	0.8	3.2
5	0.3	1.5	7.5
		$\sum x p = 3$	$\sum x^2 p = 12$

$$\text{Mean} = \sum x p$$

$$\text{mean} = 3$$

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\sum x^2 p - (\text{mean})^2} \\ &= \sqrt{12 - (3)^2} \\ &= \sqrt{3} \end{aligned}$$

$$\text{Standard Deviation} = 1.732$$

Mean and Variance of a Random Variable Ex 32.2 Q1(iii)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-5	$\frac{1}{4}$	$-\frac{5}{4}$	$\frac{25}{4}$
-4	$\frac{1}{8}$	$-\frac{1}{2}$	2
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$
		$\Sigma xp = -1$	$\Sigma x^2 p = \frac{37}{4}$

$$\text{Mean} = \Sigma xp$$

$$\text{mean} = -1$$

$$\text{Standard deviation} = \sqrt{\Sigma x^2 p - (\text{mean})^2}$$

$$= \sqrt{\frac{37}{4} - (-1)^2}$$

$$= \sqrt{\frac{33}{4}}$$

$$= \sqrt{8.25}$$

$$\text{Standard Deviation} = 2.9$$

Mean and Variance of a Random Variable Ex 32.2 Q1(iv)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-1	0.3	-0.3	0.3
0	0.1	0	0
1	0.1	0.1	0.1
2	0.3	0.6	1.2
3	0.2	0.6	1.8
		$\Sigma xp = 1$	$\Sigma x^2 p = 3.4$

$$\text{Mean} = \Sigma xp$$

$$\text{mean} = 1$$

$$\text{Standard deviation} = \sqrt{\Sigma x^2 p - (\text{mean})^2}$$

$$= \sqrt{(3.4) - (1)^2}$$

$$= \sqrt{2.4}$$

$$\text{Standard Deviation} = 1.5$$

Mean and Variance of a Random Variable Ex 32.2 Q1(v)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	0.4	0.4	0.4
2	0.3	0.6	1.2
3	0.2	0.6	1.8
4	0.1	0.4	1.6
		$\sum x p = 2$	$\sum x^2 p = 5$

$$\text{Mean} = \sum x p$$

$$\text{mean} = 2$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\sum x^2 p - (\text{mean})^2} \\ &= \sqrt{5 - (2)^2}\end{aligned}$$

$$\text{Standard Deviation} = 1$$

Mean and Variance of a Random Variable Ex 32.2 Q1(vi)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	0.2	0	0
1	0.5	0.5	0.5
3	0.2	0.6	1.8
5	0.1	0.5	2.5
		$\sum x p = 1.6$	$\sum x^2 p = 4.8$

$$\text{Mean} = \sum x p$$

$$\text{mean} = 1.6$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\sum x^2 p - (\text{mean})^2} \\ &= \sqrt{4.8 - (1.6)^2} \\ &= \sqrt{4.8 - 2.56} \\ &= \sqrt{2.24}\end{aligned}$$

$$\text{Standard Deviation} = 1.497$$

Mean and Variance of a Random Variable Ex 32.2 Q1(vii)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-2	0.1	-0.2	0.4
-1	0.2	-0.2	0.2
0	0.4	0	0
1	0.2	0.2	0.2
2	0.1	0.2	0.4
		$\sum x p = 0$	$\sum x^2 p = 1.2$

$$\text{Mean} = \sum x p$$

$$\text{mean} = 0$$

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\sum x^2 p - (\text{mean})^2} \\ &= \sqrt{(1.2)^2 - (0)^2} \end{aligned}$$

$$\text{Standard Deviation} = 1.2$$

Mean and Variance of a Random Variable Ex 32.2 Q1(viii)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
-3	0.05	-0.15	0.45
-1	0.45	-0.45	0.45
0	0.20	0	0
1	0.25	0.25	0.25
3	0.05	0.15	0.45
		$\sum x p = -0.2$	$\sum x^2 p = 1.6$

$$\text{Mean} = \sum x p$$

$$\text{mean} = -0.2$$

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\sum x^2 p - (\text{mean})^2} \\ &= \sqrt{1.6 - (-0.2)^2} \\ &= \sqrt{1.6 - 0.04} \\ &= \sqrt{1.56} \end{aligned}$$

$$\text{Standard Deviation} = 1.249$$

Mean and Variance of a Random Variable Ex 32.2 Q1(ix)

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{1}{6}$	0	0
1	$\frac{5}{18}$	$\frac{5}{18}$	$\frac{5}{18}$
2	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{9}$
3	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{2}$
4	$\frac{1}{9}$	$\frac{4}{9}$	$\frac{16}{9}$
5	$\frac{1}{18}$	$\frac{5}{18}$	$\frac{25}{18}$
		$\sum p_i x_i = \frac{35}{18}$	$\sum p_i x_i^2 = \frac{35}{6}$

$$\text{Mean} = \sum p_i x_i = \frac{35}{18}$$

$$\text{Variance} = \sum p_i x_i^2 - (\sum p_i x_i)^2 = \frac{35}{6} - \left(\frac{35}{18}\right)^2 = \frac{665}{324}$$

$$\text{Standard deviation} = \sqrt{\text{Variance}} = \frac{\sqrt{665}}{18}$$

Mean and Variance of a Random Variable Ex 32.2 Q2

(i) We know that,

$$P(0.5) + P(1) + P(1.5) + P(2) = 1$$

$$k + k^2 + 2k^2 + k = 1$$

$$3k^2 + 2k - 1 = 0$$

$$3k^2 + 3k - k - 1 = 0$$

$$(3k - 1)(k + 1) = 0$$

$$k = \frac{1}{3} \text{ or } k = -1$$

We know that $0 \leq P(X) \leq 1$

$$\therefore k = \frac{1}{3}$$

(ii)

x_i	p_i	$p_i x_i$
0.5	$\frac{1}{3}$	$\frac{1}{6}$
1	$\frac{1}{9}$	$\frac{1}{9}$
1.5	$\frac{2}{9}$	$\frac{1}{3}$
2	$\frac{1}{3}$	$\frac{2}{3}$
		$\sum p_i x_i = \frac{23}{18}$

$$\text{Mean} = \sum p_i x_i = \frac{23}{18}$$

Mean and Variance of a Random Variable Ex 32.2 Q3

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
a	p	ap	$a^2 p$
b	q	bq	$b^2 q$

$$\text{Mean} = \sum x p$$

$$\text{mean} = ap + bq$$

$$\text{Variance} = \sum x^2 p - (\text{mean})^2$$

$$= (a^2 p + b^2 q) - (ap + bq)^2$$

$$= a^2 p + b^2 q - a^2 p^2 - b^2 q^2 - 2abpq$$

$$= a^2 pq + b^2 pq - 2abpq$$

$$[\because p + q = 1]$$

$$= pq (a^2 + b^2 - 2ab)$$

$$\text{Variance} = pq (a - b)^2$$

$$\text{Standard deviation} = |a - b| \sqrt{pq}$$

Mean and Variance of a Random Variable Ex 32.2 Q4

We know that in a throw of coin,

$$P(H) = \frac{1}{2}, \quad P(T) = \frac{1}{2}$$

Let X denote the number of heads in three tosses of coin.

So, $X = 0, 1, 2, 3$

$$\begin{aligned} P(X = 0) &= P(T)P(T)P(T) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} P(X = 1) &= P(H)P(T)P(T) + P(T)P(H)P(T) + P(T)P(T)P(H) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} P(X = 2) &= P(H)P(H)P(T) + P(H)P(T)P(H) + P(T)P(H)P(H) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} P(X = 3) &= P(H)P(H)P(H) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{8} \end{aligned}$$

So,

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{1}{8}$	0	0
1	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
2	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{12}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{9}{8}$
		$\sum xp = \frac{3}{2}$	$\sum x^2 p = 3$

$$\text{Mean} = \sum xp$$

$$\text{mean} = \frac{3}{2}$$

$$\text{Variance} = \sum x^2 p - (\text{mean})^2$$

$$= 3 - \frac{9}{4}$$

$$\text{Variance} = \frac{3}{4}$$

Two cards are drawn simultaneously from a pack of 52 cards.
Let X denotes the number of kings drawn.

So, $X = 0, 1, 2$

$$\begin{aligned} P(X = 0) &= \frac{48C_2}{52C_2} \\ &= \frac{48 \times 47}{52 \times 51} \\ &= \frac{188}{221} \end{aligned}$$

$$\begin{aligned} P(X = 1) &= \frac{4C_1 \times 48C_1}{52C_2} \\ &= \frac{4 \times 48 \times 2}{52 \times 51} \\ &= \frac{32}{221} \end{aligned}$$

$$\begin{aligned} P(X = 2) &= \frac{4C_2}{52C_2} \\ &= \frac{4 \times 3}{52 \times 51} \\ &= \frac{1}{221} \end{aligned}$$

So,

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{188}{221}$	0	0
1	$\frac{32}{221}$	$\frac{32}{221}$	$\frac{32}{221}$
2	$\frac{1}{221}$	$\frac{2}{221}$	$\frac{4}{221}$
		$\Sigma xp = \frac{34}{221}$	$\Sigma x^2 p = \frac{36}{221}$

$$\text{Mean} = \Sigma xp$$

$$\text{mean} = \frac{34}{221}$$

$$\text{Variance} = \Sigma x^2 p - (\text{mean})^2$$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2$$

$$= \frac{7956 - 1156}{48841}$$

$$= \frac{6800}{48841}$$

$$\text{Variance} = \frac{400}{2873}$$

We know that ,in a throw of coin.

$$P(T) = \frac{1}{2}, \quad P(H) = \frac{1}{2}$$

Let X denote the number of tails in three throws of coins.

So, X can take values from 0,1,2,3

$$P(X = 0) = P(H)P(H)P(H)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

$$P(X = 1) = P(T)P(H)P(H) + P(H)P(T)P(H) + P(H)P(H)P(T)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{8}$$

$$P(X = 2) = P(T)P(T)P(H) + P(T)P(H)P(T) + P(H)P(T)P(T)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{3}{8}$$

$$P(X = 3) = P(T)P(T)P(T)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$

So,

$$\text{Mean} = \sum xp$$

$$\text{mean} = \frac{3}{2}$$

$$\text{Variance} = \sum x^2p - (\text{mean})^2$$

$$= 3 - \left(\frac{3}{2}\right)^2$$

$$= 3 - \frac{9}{4}$$

$$\text{Variance} = \frac{3}{4}$$

$$\text{Standard Deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{\frac{3}{4}}$$

$$\text{Standard Deviation} = 0.87$$

Mean and Variance of a Random Variable Ex 32.2 Q7

Total 12 good and bad eggs. 10 are good and 2 are bad.

3 eggs are drawn from this lot

Let X be the random variable that denotes the number of bad eggs in the lot.

$$\begin{aligned}P(X = 0) &= P(3 \text{ good and } 0 \text{ bad}) = {}^3C_0 \cdot {}^{10}C_3 / {}^{12}C_3 \\ &= 1 \times 120 / 220 = 6/11\end{aligned}$$

$$\begin{aligned}P(X = 1) &= P(2 \text{ good and } 1 \text{ bad}) = {}^2C_1 \cdot {}^{10}C_2 / {}^{12}C_3 \\ &= 2 \times 45 / 220 = 9/22\end{aligned}$$

$$\begin{aligned}P(X = 2) &= P(1 \text{ good and } 2 \text{ bad}) = {}^2C_2 \cdot {}^{10}C_1 / {}^{12}C_3 \\ &= 1 \times 10 / 220 = 1/22\end{aligned}$$

The probability distribution of X is

X	0	1	2
$P(X)$	$\frac{6}{11}$	$\frac{9}{22}$	$\frac{1}{22}$

$$\text{The mean} = 0 \times \frac{6}{11} + 1 \times \frac{9}{22} + 2 \times \frac{1}{22} = \frac{11}{22} = 1/2$$

Mean and Variance of a Random Variable Ex 32.2 Q8

A pair of dice is thrown. And X denote minimum of the two number appeared.

So, X can have values 2,3,4,5,6.

$$P(X = 1) = \frac{11}{36} \quad \left[\text{Possible pairs: } (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (3,1), (4,1), (5,1), (6,1) \right]$$

$$P(X = 2) = \frac{9}{36} \quad \left[\text{Possible pairs: } (2,2), (2,3), (2,4), (2,5), (2,6), (3,2), (4,2), (5,2), (6,2) \right]$$

$$P(X = 3) = \frac{7}{36} \quad \left[\text{Possible pairs: } (3,3), (3,4), (3,5), (3,6), (4,3), (5,3), (6,3) \right]$$

$$P(X = 4) = \frac{5}{36} \quad \left[\text{Possible pairs: } (4,4), (4,5), (4,6), (5,4), (6,4) \right]$$

$$P(X = 5) = \frac{3}{36} \quad \left[\text{Possible pairs: } (5,5), (5,6), (6,5) \right]$$

$$P(X = 6) = \frac{1}{36} \quad \left[\text{Possible pairs: } (6,6) \right]$$

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	$\frac{11}{36}$	$\frac{11}{36}$	$\frac{11}{36}$
2	$\frac{9}{36}$	$\frac{18}{36}$	$\frac{36}{36}$
3	$\frac{7}{36}$	$\frac{21}{36}$	$\frac{63}{36}$
4	$\frac{5}{36}$	$\frac{20}{36}$	$\frac{80}{36}$
5	$\frac{3}{36}$	$\frac{15}{36}$	$\frac{75}{36}$
6	$\frac{1}{36}$	$\frac{6}{36}$	$\frac{36}{36}$
		$\Sigma xp = \frac{91}{36}$	$\Sigma x^2 p = \frac{301}{36}$

$$\text{Mean} = \Sigma xp$$

$$\text{Mean} = \frac{91}{36}$$

$$\text{Variance} = \Sigma x^2 p - (\text{mean})^2$$

$$\begin{aligned}
 &= \frac{301}{36} - \left(\frac{91}{36}\right)^2 \\
 &= \frac{10836 - 8281}{1296} \\
 &= \frac{2555}{1296}
 \end{aligned}$$

$$\text{Variance} = 1.97$$

Probability distribution is

$$\begin{array}{cccccc}
 x & : & 1 & 2 & 3 & 4 & 5 & 6 \\
 p(x) & : & \frac{11}{36} & \frac{9}{36} & \frac{7}{36} & \frac{5}{36} & \frac{3}{36} & \frac{1}{36}
 \end{array}$$

Mean and Variance of a Random Variable Ex 32.2 Q9

We know that ,In a toss of coin.

$$P(T) = \frac{1}{2}, \quad P(H) = \frac{1}{2}$$

Let X denote the number of occurring head in 4 throws of coins.

So, X can take values from $X = 0, 1, 2, 3, 4$

$$\begin{aligned} P(X = 0) &= P(T)P(T)P(T)P(T) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} P(X = 1) &= P(H)P(T)P(T)P(T) \times {}^4C_1 \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 4 \\ &= \frac{4}{16} \end{aligned}$$

$$\begin{aligned} P(X = 2) &= P(H)P(H)P(T)P(T) \times {}^4C_2 \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 6 \\ &= \frac{6}{16} \end{aligned}$$

$$\begin{aligned} P(X = 3) &= P(H)P(H)P(H)P(T) \times {}^4C_3 \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 4 \\ &= \frac{4}{16} \end{aligned}$$

$$\begin{aligned} P(X = 4) &= P(H)P(H)P(H)P(H) \\ &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1}{16} \end{aligned}$$

So,

$$\text{Mean} = \sum xp$$

$$\text{mean} = 2$$

$$\text{Variance} = \sum x^2p - (\text{mean})^2$$

$$= 5 - (2)^2$$

$$\text{Variance} = 1$$

Probability distribution is

x	: 0	1	2	3	4
$P(x)$: $\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$

Mean and Variance of a Random Variable Ex 32.2 Q10

X denotes twice the number appearing on the die.

So, $X = 2, 4, 6, 8, 10, 12$.

Probability distribution is

X	: 2	4	6	8	10	12
$P(x)$: $\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\text{Mean} = \sum xp$$

$$\text{mean} = 7$$

$$\text{Variance} = \sum x^2p - (\text{mean})^2$$

$$= \left(\frac{364}{6} \right) - (7)^2$$

$$= \frac{364 - 294}{6}$$

$$= \frac{70}{6}$$

$$\text{Variance} = 11.7$$

Mean and Variance of a Random Variable Ex 32.2 Q11

$$\text{Probability of even number} = P(E) = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow P(O) = \frac{1}{2}$$

Here, X have values 1 or 3 according as an odd or even number.

So,

$$X : 1 \quad 3$$

$$P(X) : \frac{1}{2} \quad \frac{1}{2}$$

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
3	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{9}{2}$
		$\sum xp = 2$	$\sum x^2p = 5$

$$\text{Mean} = \sum xp$$

$$\text{mean} = 2$$

$$\text{Variance} = \sum x^2p - (\text{mean})^2$$

$$= 5 - 4$$

$$\text{Variance} = 1$$

Mean and Variance of a Random Variable Ex 32.2 Q12

Let the event of getting a head = H and getting a tail = T
 Let X denote the variable longest consecutive heads occurring in 4 tosses. The possible values are

- X = 0 (no head) {T, T, T, T}
 X = 1 (1 heads) {H, T, T, T}
 X = 2 (2 heads) {H, H, T, T}
 X = 3 (3 heads) {H, H, H, T}
 X = 4 (4 heads) {H, H, H, H}

n(S) = {(HHHH),(HHHT),(HHTT),(HTHH),(HTHT),(HTTH),(HTTT),
 (THHH),(THTH),(THTT),(THTT),(THTT),(TTHH),(TTHT)
 (TTTH),(TTTT)}

$$P(X=0) = \frac{1}{16}$$

$$P(X=1) = \frac{7}{16}$$

$$P(X=2) = \frac{5}{16}$$

$$P(X=3) = \frac{2}{16}$$

$$P(X=4) = \frac{1}{16}$$

Probability distribution is

X	0	1	2	3	4
$p_i = P(X)$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{2}{16}$	$\frac{1}{16}$
$p_i x_i^2$	0	$\frac{7}{16}$	$\frac{20}{16}$	$\frac{18}{16}$	1

$$\text{Mean} = \sum_{i=1 \text{ to } n} X_i \times P(X_i)$$

$$\begin{aligned} \text{Mean, } \mu &= 0 \times \frac{1}{16} + 1 \times \frac{7}{16} + 2 \times \frac{5}{16} + 3 \times \frac{2}{16} + 4 \times \frac{1}{16} \\ &= 0 + \frac{7}{16} + \frac{10}{16} + \frac{6}{16} + \frac{4}{16} \\ &= \frac{27}{16} = 1.7 \end{aligned}$$

$$\begin{aligned} \text{Variance } \text{Var}(X) &= \sum p_i x_i^2 - (\sum p_i x_i)^2 \\ &= \frac{61}{16} - 1.7^2 \\ &= 3.825 - 2.89 \\ &= 0.935 \end{aligned}$$

Mean and Variance of a Random Variable Ex 32.2 Q13

Box contains five cards 1,1,2,2,3.

Here,

X denotes the sum of two number on cards drawn.

Y denotes the maximum of the two number.

So, $X = 2,3,4,5$

$Y = 1,2,3$

$$P(X = 2) = P(1)P(1)$$

$$= \frac{2}{5} \times \frac{1}{4}$$

$$= 0.1$$

$$P(X = 3) = P(1)P(2) + P(2)P(1)$$

$$= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4}$$

$$= 0.4$$

$$P(X = 4) = P(2)P(2) + P(1)P(3) + P(3)P(1)$$

$$= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$

$$= 0.3$$

$$P(X = 5) = P(2)P(3) + P(3)P(2)$$

$$= \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4}$$

$$= 0.2$$

Probability Distribution for X

$X :$	2	3	4	5
$P(x) :$	0.1	0.4	0.3	0.2

$$\text{Mean} = \sum xp$$

$$\text{mean} = 3.6$$

$$\begin{aligned}\text{Variance} &= \sum x^2p - (\text{mean})^2 \\ &= 13.8 - (3.6)^2 \\ &= 13.8 - 12.96\end{aligned}$$

$$\text{Variance} = 0.84$$

$$\begin{aligned}P(Y = 1) &= P(1)P(1) \\ &= \frac{2}{5} \times \frac{1}{4} \\ &= \frac{2}{20} \\ &= 0.1\end{aligned}$$

$$\begin{aligned}P(Y = 2) &= P(1)P(2) + P(2)P(1) + P(2)P(2) \\ &= \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}\end{aligned}$$

$$P(Y = 2) = 0.5$$

$$\begin{aligned}P(Y = 3) &= P(1)P(3) + P(2)P(3) + P(3)P(1) + P(3)P(2) \\ &= \frac{2}{5} \times \frac{1}{4} + \frac{2}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{2}{4} + \frac{1}{5} \times \frac{2}{4} \\ &= 0.4\end{aligned}$$

Probability distribution for Y is

$$\begin{array}{l} X : \quad 1 \quad 2 \quad 3 \\ P(x) : \quad 0.1 \quad 0.5 \quad 0.4 \end{array}$$

Y_i	p_i	$Y_i p_i$	$Y_i^2 p_i$
1	0.1	0.1	0.1
2	0.5	1.0	2.0
3	0.4	1.2	3.6
		$\sum xp = 2.6$	$\sum x^2p = 5.7$

$$\text{Mean} = \sum xp$$

$$\text{mean} = 2.3$$

$$\begin{aligned}\text{Variance} &= \sum x^2p - (\text{mean})^2 \\ &= 5.7 - (2.3)^2\end{aligned}$$

$$\text{Variance} = 0.41$$

Mean and Variance of a Random Variable Ex 32.2 Q14

$$\text{Probability of getting an odd number} = P(O) = \frac{1}{2}$$

$$\Rightarrow P(E) = \frac{1}{2}$$

Die is tossed twice. Let X denote the number of times an odd number occurs.

So, $X = 0, 1, 2$.

$$P(X = 0) = P(E)P(E)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

$$P(X = 1) = P(O)P(E) + P(E)P(O)$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2}$$

$$P(X = 2) = P(O)P(O)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$P(X = 2) = \frac{1}{4}$$

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	$\frac{1}{4}$	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{4}{4}$
		$\Sigma x p = 1$	$\Sigma x^2 p = \frac{3}{2}$

$$\text{Mean} = \Sigma x p = 1$$

$$\text{Variance} = \Sigma x^2 p - (\text{mean})^2$$

$$= \frac{3}{2} - 1$$

$$\text{Variance} = \frac{1}{2}$$

Out of 13 bulbs 5 are defective \Rightarrow 8 bulbs are good.

3 bulbs are drawn without replacement ,

Let X denote number of defective bulbs,

So, X can have values 0,1,2,3

$$P(X = 0) = p(\text{No defective})$$

$$= \frac{{}^8C_3}{{}^{13}C_3}$$

$$= \frac{8 \times 7 \times 6}{13 \times 12 \times 11}$$

$$= \frac{28}{143}$$

$$P(X = 1) = p(\text{Only one defective})$$

$$= \frac{{}^8C_2 \times {}^5C_1}{{}^{13}C_3}$$

$$= \frac{8 \times 7 \times 5}{2} \times \frac{3 \times 2 \times 1}{13 \times 12 \times 11}$$

$$= \frac{70}{143}$$

$$P(X = 2) = p(\text{Only two defective})$$

$$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3}$$

$$= \frac{8 \times 5 \times 4}{2} \times \frac{3 \times 2 \times 1}{13 \times 12 \times 11}$$

$$= \frac{40}{143}$$

$$P(X = 3) = p(\text{all three are defective})$$

$$= \frac{{}^5C_3}{{}^{13}C_3}$$

$$= \frac{4 \times 5}{2} \times \frac{3 \times 2 \times 1}{13 \times 12 \times 11}$$

$$= \frac{5}{143}$$

So, Probability distribution is

$X :$	0	1	2	3
$P(x) :$	$\frac{28}{143}$	$\frac{70}{143}$	$\frac{40}{143}$	$\frac{5}{143}$

Mean and Variance of a Random Variable Ex 32.2 Q16

$$P(\text{win}) = \frac{1}{13} \Rightarrow P(\text{lose}) = \frac{12}{13}$$

He gains Rs 90 if he wins and loses Rs 10 if his number does not appear.

Let X denote total loss or gain, so,

$$\begin{array}{r} X : \quad 90 \quad -10 \\ P(X) : \quad \frac{1}{13} \quad \frac{12}{13} \\ XP : \quad \frac{90}{13} \quad \frac{-120}{13} \end{array}$$

$$\begin{aligned} E(X) &= \sum XP \\ &= \frac{90}{13} - \frac{120}{13} \end{aligned}$$

$$E(X) = -\frac{30}{13}$$

Mean and Variance of a Random Variable Ex 32.2 Q17

Let 'X' be the random variable which can assume values from 0 to 3.

$$P(X=0) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2600}{22100} = \frac{2}{17}$$

$$P(X=1) = \frac{{}^{26}C_1 \times {}^{26}C_2}{{}^{52}C_3} = \frac{8450}{22100} = \frac{13}{34}$$

$$P(X=2) = \frac{{}^{26}C_2 \times {}^{26}C_1}{{}^{52}C_3} = \frac{8450}{22100} = \frac{13}{34}$$

$$P(X=3) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2600}{22100} = \frac{2}{17}$$

Probability distribution of X:

$$\begin{array}{r} X = x_i \quad 0 \quad 1 \quad 2 \quad 3 \\ p(X = x_i) \quad \frac{2}{17} \quad \frac{13}{34} \quad \frac{13}{34} \quad \frac{2}{17} \end{array}$$

$$\begin{aligned} \text{Mean} &= \sum_{i=0}^3 (x_i \times p_i) \\ &= x_0 p_0 + x_1 p_1 + x_2 p_2 + x_3 p_3 \\ &= 0 \times \frac{2}{17} + 1 \times \frac{13}{34} + 2 \times \frac{13}{34} + 3 \times \frac{2}{17} \\ &= \frac{13 + 26 + 12}{34} \\ &= \frac{51}{34} \\ &= \frac{3}{2} \\ &= 1.5 \end{aligned}$$

Mean and Variance of a Random Variable Ex 32.2 Q18

X can assume values 0, 1, 2.

Yes X is a random variable.

$$P(X = 0) = (\text{Probability of getting no black ball}) = \frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{1 \times \frac{5 \times 4}{2 \times 1}}{\frac{7 \times 6}{2 \times 1}} = \frac{20}{42}$$

$$P(X = 1) = (\text{Probability of getting one black ball}) = \frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{2 \times 5}{\frac{7 \times 6}{2 \times 1}} = \frac{20}{42}$$

$$P(X = 2) = (\text{Probability of getting two black balls}) = \frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{1 \times 1}{\frac{7 \times 6}{2 \times 1}} = \frac{2}{42}$$

Thus, probability distribution of random variable X is,

X	0	1	2
P(X)	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{2}{42}$

x_i	p_i	$p_i x_i$	$p_i x_i^2$
0	$\frac{20}{42}$	0	0
1	$\frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$
2	$\frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$
		$\sum p_i x_i = \frac{4}{7}$	$\sum p_i x_i^2 = \frac{2}{3}$

$$\text{Mean} = \sum p_i x_i = \frac{4}{7}$$

$$\text{Variance} = \sum p_i x_i^2 - (\sum p_i x_i)^2 = \frac{2}{3} - \left(\frac{4}{7}\right)^2 = \frac{50}{147}$$

Mean and Variance of a Random Variable Ex 32.2 Q19

We can select two positive in $6 \times 5 = 30$ different ways.

X denotes the larger number so, X can assume values 3, 4, 5, 6 and 7.

Yes X is a random variable.

$$P(X = 3) = P(\text{larger number is 3}) = \{(2, 3), (3, 2)\} = \frac{2}{30}$$

$$P(X = 4) = P(\text{larger number is 4}) = \{(2, 4), (4, 2), (3, 4), (4, 3)\} = \frac{4}{30}$$

$$P(X = 5) = P(\text{larger number is 5}) = \{(2, 5), (5, 2), (3, 5), (5, 3), (4, 5), (5, 4)\} = \frac{6}{30}$$

$$P(X = 6) = P(\text{larger number is 6}) = \{(2, 6), (6, 2), (3, 6), (6, 3), (4, 6), (6, 4), (5, 6), (6, 5)\} = \frac{8}{30}$$

$$P(X = 7) = P(\text{larger number is 7}) = \{(2, 7), (7, 2), (3, 7), (7, 3), (4, 7), (7, 4), (5, 7), (7, 5), (6, 7), (7, 6)\} = \frac{10}{30}$$

Thus, probability distribution of random variable X is,

x_i	p_i	$p_i x_i$	$p_i x_i^2$
3	$\frac{2}{30}$	$\frac{6}{30}$	$\frac{18}{30}$
4	$\frac{4}{30}$	$\frac{16}{30}$	$\frac{64}{30}$
5	$\frac{6}{30}$	$\frac{30}{30}$	$\frac{150}{30}$
6	$\frac{8}{30}$	$\frac{48}{30}$	$\frac{288}{30}$
7	$\frac{10}{30}$	$\frac{70}{30}$	$\frac{490}{30}$
		$\Sigma p_i x_i = \frac{17}{3}$	$\Sigma p_i x_i^2 = \frac{101}{3}$

$$\text{Mean} = \Sigma p_i x_i = \frac{17}{3}$$

$$\text{Variance} = \Sigma p_i x_i^2 - (\Sigma p_i x_i)^2 = \frac{101}{3} - \left(\frac{17}{3}\right)^2 = \frac{14}{9}$$