RD Sharma Solutions Class 12 Maths Chapter 31 Ex 31.6

Probability Ex 31.6 Q1

Given,

Bag A contains 5 white and 6 black balls

Bag B contains 4 white and 3 black balls.

There are two ways of transferring a ball from bag A to bag B

I- By transferring one white ball from bag A to bag B then drawing one black ball from bag B.

II- By transferring one black ball from bag A to bag B, then drawing one black from bag B.

Let, E_1, E_2 and A be events as below:-

E_1 = One white ball drawn from bag A

E_2 = One black ball drawn from bag B

A = One black ball drawn from bag B

$$P(E_1) = \frac{5}{11}$$

$$P(E_2) = \frac{6}{11}$$

$$P(A \mid E_1) = \frac{3}{8} \qquad \left[\text{Since, } E_1 \text{ has increased one white ball in bag } B\right]$$

$$P\left(\frac{A}{E_2}\right) = \frac{4}{8} \qquad \left[\text{Since, } E_2 \text{ has increased one black ball in bag } B\right]$$

By the law of total probability.

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{5}{11} \times \frac{3}{8} + \frac{6}{11} \times \frac{4}{8}$$

$$= \frac{15}{88} + \frac{24}{88}$$

$$= \frac{39}{88}$$

Required probability $= \frac{39}{88}$.

Probability Ex 31.6 Q2

Purse (I) Contains 2 silver and 4 copper coins

Purse (II) Contains 4 silver and 3 copper coins

One coin is drawn from one of the two purse and it is silver

Let, E_1, E_2 and A are defined as

E_1 = Selecting purse I

E_2 = Selecting purse II

A = Drawing a silver coin

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since, there are only 2 purses]}$$

$$P(A \mid E_1) = P(\text{A} \mid \text{ silver coin from purse I})$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

$$P\left(\frac{A}{E_2}\right) = P(\text{A} \mid \text{ silver coin from purse II})$$

$$= \frac{4}{7}$$

By the law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} \times \frac{4}{7}$$

$$= \frac{1}{6} + \frac{4}{14}$$

$$= \frac{7+12}{42}$$

$$= \frac{19}{42}$$

Required probability $= \frac{19}{42}$.

Probability Ex 31.6 Q3

Bag I contains 4 yellow and 5 red balls
Bag II contains 6 yellow and 3 red balls
Transfer can be done in two ways:-
I- A yellow ball is transferred from bag I to bag II and then one yellow ball is drawn from bag II.
II-A red ball is transferred from bag I to bag II and then one yellow ball is drawn from bag II.
Let E_1, E_2 and A be events as:

E_1 = One yellow ball drawn from bag I
E_2 = One red ball drawn from bag I
A = One yellow ball drawn from bag II.

$$P(E_1) = \frac{4}{9}$$

$$P(E_2) = \frac{5}{9}$$

$$P(A \mid E_1) = \frac{7}{10}$$ [Since E_1 has increased one yellow ball in bag II]

$$P\left(\frac{A}{E_2}\right) = \frac{6}{10}$$ [Since E_2 has increased one red ball in bag II]

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{4}{9} \times \frac{7}{10} + \frac{5}{9} \times \frac{6}{10}$$

$$= \frac{28+30}{90}$$

$$= \frac{58}{90}$$

$$= \frac{29}{45}$$

Required probability $= \frac{29}{45}$.

Probability Ex 31.6 Q4

Bag I contains 3 white and 2 black balls

Bag II contains 2 white and 4 black balls

One bag is chosen at random, then one ball is drawn and it is white.

Let E_1, E_2 and A be events as:

E_1 = Selecting bag I

E_2 = Selecting bag II

A = Drawing one white ball

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2} \qquad \text{[Since there are only 2 bags]}$$

$$P(A \mid E_1) = P[\text{Drawing a white ball from bag I}] = \frac{3}{5}$$

$$P\left(\frac{A}{E_2}\right) = P[\text{Drawing a white ball from bag II}] = \frac{2}{6}$$

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{2}{6}$$

$$= \frac{3}{10} + \frac{2}{12}$$

$$= \frac{18 + 10}{60}$$

$$= \frac{28}{60}$$

$$= \frac{7}{15}$$

Required probability $= \frac{7}{15}$.

Probability Ex 31.6 Q5

Given,

Bag I contains 1 white, 2 black and 3 red balls

Bag II contains 2 white, 1 black and 1 red balls

Bag III contains 4 white, 5 black and 3 red balls.

A bag is chosen at random, then one red and one white ball is drawn.

Let E_1, E_2, E_3 and A be events as:

E_1 = Selecting bag I

E_2 = Selecting bag II

E_3 = Selecting bag III

A = Drawing one red and one white ball

$P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$ [Since there are only three bags]

$P(A \mid E_1) = P$ [Drawing one red and one white ball from bag I]

$$= \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}$$

$$= \frac{1 \times 3}{\frac{6 \times 5}{2}}$$

$$= \frac{1}{5}$$

$P\left(\frac{A}{E_2}\right) = P$ [Drawing one red and one white ball from bag II]

$$= \frac{{}^2C_1 \times {}^1C_1}{{}^4C_2}$$

$$= \frac{2 \times 1}{\frac{4 \times 3}{2}}$$

$$= \frac{1}{3}$$

$p\left(\frac{A}{E_3}\right) = P$ [Drawing one red and one white ball from bag III]

$$= \frac{{}^4C_1 \times {}^3C_1}{{}^{12}C_2}$$

$$= \frac{4 \times 3}{\frac{12 \times 11}{2}}$$

$$= \frac{2}{11}$$

By law of total probability,

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$$= \frac{1}{3}\times\frac{1}{5} + \frac{1}{3}\times\frac{1}{3} + \frac{1}{3}\times\frac{2}{11}$$

$$= \frac{1}{15} + \frac{1}{9} + \frac{2}{33}$$

$$= \frac{33+55+30}{495}$$

$$= \frac{118}{495}$$

Required probability = $\frac{118}{495}$.

Probability Ex 31.6 Q6

An unbiased coin is tossed, then

I:- If head occurs, pair of dice is rolled and sum on them is either 7 or 8.

II:- If tail occurs, a card is drawn from cards numbered 2,3,...,12 and is 7 or 8.

Let E_1, E_2, A be events as

E_1 = Head occurs on the coin

E_2 = Tail occurs on the coin

A = Noted number is 7 or 8

$$P(E_1) = \frac{1}{2}$$

$$P(E_2) = \frac{1}{2}$$

$P(A|E_1) = P$[Pair of dice shows 7 or 8 as sum]

[Sum on dice is 7 or 8 when $(1,6),(2,5),(3,4),(4,3),(5,2),(6,1),(2,6),(3,5),(4,4),(5,3),(6,2)$]

$$P(A|E_1) = \frac{11}{36}$$

$P\left(\frac{A}{E_2}\right) = P$[7 or 8 on card drawn from 11 cards numbered $2,3,4,\dots,12$]

$$= \frac{2}{11}$$

By law of total probability,

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{1}{2}\times\frac{11}{36} + \frac{1}{2}\times\frac{2}{11}$$

$$= \frac{11}{72} + \frac{2}{22}$$

$$= \frac{121+72}{792}$$

$$= \frac{193}{792}$$

Required probability = $\frac{193}{792}$.

Probability Ex 31.6 Q7

Let E_1, E_2, A be defined as,

E_1 = Item produced by machine A

E_2 = Item produced by machine B

A = The item drawn is defective

$P(E_1) = 60\%$

$= \frac{60}{100}$

$P(E_2) = 40\%$

$= \frac{40}{100}$

$P(A|E_1) = P$[Defective item from machine A]

$= 2\%$

$= \frac{2}{100}$

$P\left(\frac{A}{E_2}\right) = P$[Defective item from machine B]

$= 1\%$

$= \frac{1}{100}$

By law of total probability,

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{60}{100} \times \frac{2}{100} + \frac{40}{100} \times \frac{1}{100}$$

$$= \frac{120 + 40}{10000}$$

$$= \frac{160}{10000}$$

$$= 0.016$$

Required probability = 0.016.

Probability Ex 31.6 Q8

Bag A contains 8 white and 7 black balls

Bag B contains 5 white and 4 black balls

Transfer can be done in two ways:-

I-A white ball is transferred from bag A to bag B and then one white ball is drawn from bag B.

II-A black ball is transferred from bag A to bag B, then one white ball is drawn from bag B.

Let E_1, E_2 and A be events as:-

E_1 = One white ball from bag A

E_2 = One black ball from bag A

A = One white ball from bag B

$$P(E_1) = \frac{8}{15}$$

$$P(E_2) = \frac{7}{15}$$

$$P(A \mid E_1) = \frac{6}{10}$$ [Since E_1 has increased white balls in bag B]

$$P\left(\frac{A}{E_2}\right) = \frac{5}{10}$$ [Since E_2 has increased black ball in bag B]

By law of total probability,

$$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P\left(\frac{A}{E_2}\right)$$

$$= \frac{8}{15} \times \frac{6}{10} + \frac{7}{15} \times \frac{5}{10}$$

$$= \frac{48}{150} + \frac{35}{150}$$

$$= \frac{83}{150}$$

Required probability $= \frac{83}{150}$.

Probability Ex 31.6 Q9

There are two bags.

Bag (1) contain 4 white and 5 black balls

Bag (2) contain 3 white and 4 black balls.

A ball is taken from bag (i) and without seeing its colour is put in second bag. Then a ball is drawn from bag 2 and is white in colour.

$$P(\text{White ball from bag 1}) = \frac{4}{9}$$

$$P(W_1) = \frac{4}{9}$$

$$P(\text{Black ball from bag 1}) = \frac{5}{9}$$

$$P(B_1) = \frac{5}{9}$$

P (White ball from bag 2 given B_1 transfer)

$$P\left(\frac{W_2}{B_1}\right) = \frac{3}{8}$$

P (White from bag 2 given W_1 transfer)

$$P\left(\frac{W_2}{W_1}\right) = \frac{4}{8}$$

$$= \frac{1}{2}$$

P (White from bag 2)

$$= P(B_1)P\left(\frac{W_2}{B_1}\right) + P(W_1)P\left(\frac{W_2}{W_1}\right)$$

$$= \frac{5}{9} \times \frac{3}{8} + \frac{4}{9} \times \frac{1}{2}$$

$$= \frac{15}{72} + \frac{4}{18}$$

$$= \frac{31}{72}$$

Required probability $= \dfrac{31}{72}$

Probability Ex 31.6 Q10

There are two bags.

Bag (1) contain 4 white and 5 black balls

Bag (2) contain 6 white and 7 black balls.

A ball is taken from bag (1) and without seeing its colour is put in bag (2). Then a ball is drawn from bag (2) and is found white in colour.

$$P\left(1 \text{ white ball from bag } 1\right) = \frac{4}{9}$$

$$P\left(W_1\right) = \frac{4}{9}$$

$$P\left(1 \text{ black ball from bag } 1\right) = \frac{5}{9}$$

$$P\left(B_1\right) = \frac{5}{9}$$

$P\left(1 \text{ white ball from bag 2 given } W_1 \text{ is put in bag 2}\right)$

$$P\left(\frac{W_2}{W_1}\right) = \frac{7}{14}$$

$$P\left(\frac{W_2}{W_1}\right) = \frac{1}{2}$$

$P\left(1 \text{ white ball from bag 2 given } B_1 \text{ is put in bag 2}\right)$

$$P\left(\frac{W_2}{B_1}\right) = \frac{6}{14}$$

$P\left(1 \text{ white from bag } 2\right)$

$$= P\left(W_1\right)P\left(\frac{W_2}{W_1}\right) + P\left(B_1\right)P\left(\frac{W_2}{B_1}\right)$$

$$= \frac{4}{9}\times\frac{1}{2} + \frac{5}{9}\times\frac{6}{14}$$

$$= \frac{4}{18} + \frac{30}{126}$$

$$= \frac{58}{126}$$

$$= \frac{29}{63}$$

Required probability $= \dfrac{29}{63}$

Probability Ex 31.6 Q11

Urn '1'	Urn '2'
10W 3B	3W 5B

Let $U1_{2W}$, $U1_{1W1B}$, $U1_{2B}$ be the events of transferring 2 white balls, 1 white & 1black ball, 2 black balls from first Urn1 to second Urn2.

$P(U1_{2W}) = {}^{10}C_2/{}^{13}C_2 = 45/78$

$P(U1_{1W1B}) = {}^{10}C_1{}^3C_1/{}^{13}C_2 = 10x3/78$

$P(U1_{2B}) = {}^3C_2/{}^{13}C_2 = 3/78$

Let $U2_W$ be the event that a white ball is drawn from the Urn 2. There are three scenarios for Urn 2 based on the events $U1_{2W}$ $U1_{1W1B}$ $U1_{2B}$

	5W	4W	3W
	5B	6B	7B
Total	10	10	10

$P(U1_{2W}U2_W) = \frac{{}^5C_1}{{}^{10}C_1} = 1/2$

$P(U1_{1W1B}U2_W) = \frac{{}^4C_1}{{}^{10}C_1} = 2/5$

$P(U1_{2B}U2_W) = \frac{{}^3C_1}{{}^{10}C_1} = 3/10$

$P(U2_W) = P(U1_{2W}\ U2_W) + P(U1_{1W1B}\ U2_W) + P(U1_{2B}U2_W)$

$= P(U1_{2W}) x P(U1_{2W}\ U2_W) + P(U1_{1W1B}) x P(U1_{1W1B}\ U2_W) +$

$P(U1_{2B}) x P(U1_{2B}U2_W)$

$= \frac{45}{78} \times \frac{1}{2} + \frac{30}{78} \times \frac{2}{5} + \frac{3}{78} \times \frac{3}{10} = \frac{114}{780} = \frac{59}{130}$

Probability Ex 31.6 Q12

Given,

Bag (1) contains 6 red (R_1) and 8 black (B_1) balls

Bag (2) contains 8 red (R_2) and 6 black (B_2) balls

A ball is drawn from the first bag and without noticing its colour is put in the bag (2). Then a ball is drawn from second bag and it is red.

P (One red ball from bag 2)

$= P\left((B_1 \cap R_2) \cup (R_1 \cap R_2)\right)$

$= P(B_1 \cap R_2) + P(R_1 \cap R_2)$

$= P(B_1) P\left(\frac{R_2}{B_1}\right) + P(R_1) P\left(\frac{R_2}{R_1}\right)$

$= \frac{8}{14} \cdot \frac{8}{15} + \frac{6}{14} \cdot \frac{9}{15}$

$= \frac{64 + 54}{210}$

$= \frac{118}{210}$

$= \frac{59}{105}$

Required probability $= \frac{59}{105}$

Probability Ex 31.6 Q13

Let D be the event that the picked up tube is defective.

Let A_1, A_2 and A_3 be the events that the tube is produced on machines E_1, E_2 and E_3 respectively.

$$P(D) = P(A_1)P(D|A_1) + P(A_2)P(D|A_2) + P(A_3)P(D|A_3) \ldots (i)$$

$$P(A_1) = \frac{50}{100} = \frac{1}{2}, P(A_2) = \frac{25}{100} = \frac{1}{4}, P(A_3) = \frac{25}{100} = \frac{1}{4}$$

$$P(D|A_1) = P(D|A_2) = \frac{4}{100} = \frac{1}{25}$$

$$P(D|A_3) = \frac{5}{100} = \frac{1}{20}$$

Putting these values in (i), we get

$$P(D) = \frac{1}{2} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{1}{20}$$

$$P(D) = \frac{17}{400}$$