RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.8

The Plane 29.8 Q1

Given, equation of plane is

$$2x - 3y + z = 0 \qquad ---(i)$$

We know that equation of a plane parallel the plane (i) is given by

$$2x - 3y + z + \lambda = 0 \qquad ---(ii)$$

Given that, plane (ii) is passing through the point $(1,-1,2)$ so it must satisfy the equation (ii),

$$2(1) - 3(-1) + (2) + \lambda = 0$$
$$2 + 3 + 2 + \lambda = 0$$
$$7 + \lambda = 0$$

$$\lambda = -7$$

Put the value of λ in equation (ii),

$$2x - 3y + z - 7 = 0$$

So, equation of the required plane is,

$$2x - 3y + z = 7$$

The Plane 29.8 Q2

Given, equation of plane is

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 2 = 0$ --- (i)

We know that equation of a plane parallel to the plane (i) is given by

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + \lambda = 0$ --- (ii)

Given that, plane (ii) is passing through vector $\left(3\hat{i} + 4\hat{j} - \hat{k}\right)$ so it must satisfy equation (ii),

$\left(3\hat{i} + 4\hat{j} - \hat{k}\right)\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + \lambda = 0$

$(3)(2) + (4)(-3) + (-1)(5) + \lambda = 0$

$6 - 12 - 5 + \lambda = 0$

$-11 + \lambda = 0$

$\lambda = 11$

Put the value of λ in equation (ii),

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 11 = 0$

Equation of required plane is,

$\vec{r}.\left(2\hat{i} - 3\hat{j} + 5\hat{k}\right) + 11 = 0$

The Plane 29.8 Q3

We know that, equation of a plane passing through the line of intersection of two planes

$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

Given, equations of plane is,

$2x - 7y + 4z - 3 = 0$ and
$3x - 5y + 4z + 11 = 0$

So, equation of plane passing through the line of intersection of given two planes is

$$\left(2x - 7y + 4z - 3\right) + \lambda\left(3x - 5y + 4z + 11\right) = 0$$
$$2x - 7y + 4z - 3 + 3\lambda x - 5\lambda y + 4\lambda z + 11\lambda = 0$$
$$x\left(2 + 3\lambda\right) + y\left(-7 - 5\lambda\right) + z\left(4 + 4\lambda\right) - 3 + 11\lambda = 0 \qquad ---(i)$$

Plane (1) is passing through the points $(-2, 1, 3)$, so it satisfies the equation (i),

$$(-2)\left(2 + 3\lambda\right) + (1)\left(-7 - 5\lambda\right) + (3)\left(4 + 4\lambda\right) - 3 + 11\lambda = 0$$
$$-4 - 6\lambda - 7 - 5\lambda + 12 + 12\lambda - 3 + 11\lambda = 0$$
$$-2 + 12\lambda = 0$$
$$12\lambda = 2$$
$$\lambda = \frac{2}{12}$$

$$\lambda = \frac{1}{6}$$

Put λ in equation (i),

$$x\left(2 + 3\lambda\right) + y\left(-7 - 5\lambda\right) + z\left(4 + 4\lambda\right) - 3 + 11\lambda = 0$$
$$x\left(2 + \frac{3}{6}\right) + y\left(-7 - \frac{5}{6}\right) + z\left(4 + \frac{4}{6}\right) - 3 + \frac{11}{6} = 0$$
$$x\left(\frac{12 + 3}{6}\right) + y\left(\frac{-42 - 5}{6}\right) + z\left(\frac{24 + 4}{6}\right) - \frac{18 + 11}{6} = 0$$

$$\frac{15}{6}x - \frac{47}{6}y + \frac{28}{6}z - \frac{7}{6} = 0$$

Multiplying by 6, we get

$$15x - 47y + 28z - 7 = 0$$

Therefore, equation of required plane is,

$$15x - 47y + 28z = 7$$

The Plane 29.8 Q4

We know that, equation of a plane passing the line of intersection of planes

$\vec{r}.\overrightarrow{n_1} = d_1$ and $\vec{r}.\overrightarrow{n_2} = d_2$ is given by

$\vec{r}.\left(\overrightarrow{n_1} + \lambda \overrightarrow{n_2}\right) = d_1 + \lambda d_2$

So, equation of plane through the line of intersection of planes $\vec{r}.\left(\hat{i} + 3\hat{j} - \hat{k}\right) = 0$

and $\vec{r}.\left(\hat{j} + 2\hat{k}\right) = 0$ is given by

$$\vec{r}.\left[\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{j} + 2\hat{k}\right)\right] = 0 \qquad ---(i)$$

Given that plane (i) is passing through the point $\left(2\hat{i} + \hat{j} - \hat{k}\right)$, so

$$\left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\left(\hat{j} + 2\hat{k}\right) = 0$$
$$(2)(1) + (1)(3) + (-1)(-1) + \lambda\left[(2)(0) + (1)(1) + (-1)(2)\right] = 0$$
$$(2 + 3 + 1) + \lambda(1 - 2) = 0$$
$$6 - \lambda = 0$$

$$\lambda = 6$$

Put λ in equation (i),

$$\vec{r}.\left[\left(\hat{i} + 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{j} + 2\hat{k}\right)\right] = 0$$
$$\vec{r}.\left[\hat{i} + 3\hat{j} - \hat{k} + 6\left(\hat{j} + 2\hat{k}\right)\right] = 0$$
$$\vec{r}.\left[\hat{i} + 3\hat{j} - \hat{k} + 6\hat{j} + 12\hat{k}\right] = 0$$

$$\vec{r}.\left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$$

So, equation of required plane is,

$$\vec{r}.\left(\hat{i} + 9\hat{j} + 11\hat{k}\right) = 0$$

The Plane 29.8 Q5

We know that, equation of a plane passing through the line of intersection of
$a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by
$(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$

So, equation of plane passing through the line of intersection of plane
$2x - y = 0$ and $3z - y = 0$ is
$(2x - y) + \lambda(3z - y) = 0$
$2x - y + 3\lambda z - \lambda y = 0$
$x(2) + y(-1 - \lambda) + z(3\lambda) = 0$ --- (i)

We know that, two planes are perpendicular if
$a_1a_2 + b_1b_2 + c_1c_2 = 0$ --- (ii)

Given, plane (i) is perpendicular to plane

$4x + 5y - 3z = 8$ --- (iii)

Using (i) and (iii) in equation (ii),
$(2)(4) + (-1 - \lambda)(5) + (3\lambda)(-3) = 0$
$8 - 5 - 5\lambda - 9\lambda = 0$
$3 - 14\lambda = 0$
$-14\lambda = -3$

$$\lambda = \frac{3}{14}$$

Put the value of λ in equation (i),
$2x + y(-1 - \lambda) + z(3\lambda) = 0$
$$2x + y\left(-1 - \frac{3}{14}\right) + z.3\left(\frac{3}{14}\right) = 0$$
$$2x + y\left(\frac{-14 - 3}{14}\right) + \frac{9z}{14} = 0$$
$$2x + y\left(-\frac{17}{14}\right) + \frac{9z}{14} = 0$$

Multiplying with 14, we get

$28x - 17y + 9z = 0$

Equation of required plane is,

$28x - 17y + 9z = 0$

The Plane 29.8 Q6

We know that, the equation plane passing through the line of intersection of plane $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

Here, equation of plane passing through the intersection of plane $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is given by,

$$\left(x + 2y + 3z - 4\right) + \lambda\left(2x + y - z + 5\right) = 0$$
$$x + 2y + 3z - 4 + 2\lambda x + \lambda y - \lambda z + 5\lambda = 0$$
$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0 \qquad ---(i)$$

We know, that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given that plane (i) is perpendicular to plane,

$$5x + 3y - 6z + 8 = 0 \qquad ---(iii)$$

Using plane (i) and (iii) in equation (ii),

$$(5)\left(1 + 2\lambda\right) + (3)\left(2 + \lambda\right) + (-6)\left(3 - \lambda\right) = 0$$
$$5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$$
$$-7 + 19\lambda = 0$$
$$19\lambda = 7$$

$$\lambda = \frac{7}{19}$$

Put value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0$$
$$x\left(1 + \frac{14}{19}\right) + y\left(2 + \frac{7}{19}\right) + z\left(3 - \frac{7}{19}\right) - 4 + \frac{35}{19} = 0$$
$$x\left(\frac{19 + 14}{19}\right) + y\left(\frac{38 + 7}{19}\right) + z\left(\frac{57 - 7}{19}\right)\frac{-76 + 35}{19} = 0$$

$$x\left(\frac{33}{19}\right) + y\left(\frac{45}{19}\right) + z\left(\frac{50}{19}\right) - \frac{41}{19} = 0$$

Multiplying by 19, we get

$$33x + 45y + 50z - 41 = 0$$

Equation of required plane is,

$$33x + 45y + 50z - 41 = 0$$

The Plane 29.8 Q7

We know that, equation of a plane passing through the line of intersection of planes $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ is given by,

$$(a_1x+b_1y+c_1z+d_1)+\lambda(a_2x+b_2y+c_2z+d_2)=0$$

So, equation of plane passing through the line of intersection of planes $x+2y+3z+4=0$ and $x-y+z+3=0$ is

$$(x+2y+3z+4)+\lambda(x-y+z+3)=0$$
$$x(1+\lambda)+y(2-\lambda)+z(3+\lambda)+4+3\lambda=0 \qquad ---(i)$$

Equation (i) is passing through origin, so

$$(0)(1+\lambda)+(0)(2-\lambda)+(0)(3+\lambda)+4+3(\lambda)=0$$
$$0+0+0+4+3\lambda=0$$
$$3\lambda=-4$$

$$\lambda=-\frac{4}{3}$$

Put the value of λ in equation (i),

$$x(1+\lambda)+y(2-\lambda)+z(3+\lambda)+4+3\lambda=0$$
$$x\left(1-\frac{4}{3}\right)+y\left(2+\frac{4}{3}\right)+z\left(3-\frac{4}{3}\right)+4-\frac{12}{3}=0$$
$$x\left(\frac{3-4}{3}\right)+y\left(\frac{6+4}{3}\right)+z\left(\frac{9-4}{3}\right)+4-4=0$$

$$-\frac{x}{3}+\frac{10y}{3}+\frac{5z}{3}=0$$

Multiplying by 3, we get

$$-x+10y+5z=0$$
$$x-10y-5z=0$$

The equation of required plane is,

$$x-10y-5z=0$$

The Plane 29.8 Q8

We know that equation of plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by,

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes $x - 3y + 2z - 5 = 0$ and $2x - y + 3z - 1 = 0$ is given by

$$\left(x - 3y + 2z - 5\right) + \lambda\left(2x - y + 3z - 1\right) = 0$$
$$x\left(1 + 2\lambda\right) + y\left(-3 - \lambda\right) + z\left(2 + 3\lambda\right) - 5 - \lambda = 0 \qquad ---(i)$$

Plane (i) is passing through the point $(1, -2, 3)$ so,

$$(1)\left(1 + 2\lambda\right) + (-2)\left(-3 - \lambda\right) + (3)\left(2 + 3\lambda\right) - 5 - \lambda = 0$$
$$1 + 2\lambda + 6 + 2\lambda + 6 + 9\lambda - 5 - \lambda = 0$$
$$8 + 12\lambda = 0$$
$$12\lambda = -8$$
$$\lambda = -\frac{8}{12}$$
$$\lambda = -\frac{2}{3}$$

Put the value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(-3 - \lambda\right) + z\left(2 + 3\lambda\right) - 5 - \lambda = 0$$
$$x\left(1 - \frac{4}{3}\right) + y\left(-3 + \frac{2}{3}\right) + z\left(2 - \frac{6}{3}\right) - 5 + \frac{2}{3} = 0$$
$$x\left(\frac{3-4}{3}\right) + y\left(\frac{-9+2}{3}\right) + z\left(\frac{6-6}{3}\right)\frac{-15+2}{3} = 0$$

$$-\frac{1}{3}x - \frac{7}{3}y + z(0) - \frac{13}{3} = 0$$

Multiplying by (-3),

$$x + 7y + 13 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

$$\vec{r}\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

Equation of required plane is,

$$\vec{r}\left(\hat{i} + 7\hat{j}\right) + 13 = 0$$

The Plane 29.8 Q9

We know that, equation of plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the line of intersection of planes is $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is given by,

$$\left(x + 2y + 3z - 4\right) + \lambda\left(2x + y - z + 5\right) = 0$$

$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0 \qquad ---(i)$$

We know that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given that plane (i) is perpendicular to plane,

$$5x + 3y + 6z + 8 = 0 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),

$$(5)\left(1 + 2\lambda\right) + (3)\left(2 + \lambda\right) + (6)\left(3 - \lambda\right) = 0$$

$$5 + 10\lambda + 6 + 3\lambda + 18 - 6\lambda = 0$$

$$29 + 7\lambda = 0$$

$$7\lambda = -29$$

$$\lambda = -\frac{29}{7}$$

Put the value of λ in equation (i),

$$x\left(1 + 2\lambda\right) + y\left(2 + \lambda\right) + z\left(3 - \lambda\right) - 4 + 5\lambda = 0$$

$$x\left(1 - \frac{58}{7}\right) + y\left(2 - \frac{29}{7}\right) + z\left(3 + \frac{29}{7}\right) - 4 - \frac{145}{7} = 0$$

$$x\left(\frac{7 - 58}{7}\right) + y\left(\frac{14 - 29}{7}\right) + z\left(\frac{21 + 29}{7}\right)\frac{-28 - 145}{7} = 0$$

$$x\left(-\frac{51}{7}\right) + y\left(-\frac{15}{7}\right) + z\left(\frac{50}{7}\right) - \frac{173}{7} = 0$$

The Plane 29.8 Q10

$\vec{r}.(\hat{i}+3\hat{j})+6=0$ and $\vec{r}.(3\hat{i}-\hat{j}-4\hat{k})=0$

$x+3y+6=0;\ 3x-y-4z=0$

$x+3y+6+\lambda(3x-y-4z)=0$

$x(1+3\lambda)+y(3-\lambda)+-4z\lambda+6=0$

Distance from origin to plane $=\left|\dfrac{6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(4\lambda)^2}}\right|=1$

$36=(1+3\lambda)^2+(3-\lambda)^2+(4\lambda)^2$

$36=1+6\lambda+9\lambda^2+9-6\lambda+\lambda^2+16\lambda^2$

$26=26\lambda^2$

$\lambda^2=1$

$\lambda=\pm1$

Case : 1 $\lambda=1$

$x+3y+6+1(3x-y-4z)=0$

$4x+2y-4z+6=0$

Case : 2 $\lambda=-1$

$x+3y+6-1(3x-y-4z)=0$

$2x-4y-4z-6=0$

The Plane 29.8 Q11

We know that equation of a plane passing through the line of intersection of two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is given by

$$\left(a_1x + b_1y + c_1z + d_1\right) + \lambda\left(a_2x + b_2y + c_2z + d_2\right) = 0$$

So, equation of plane passing through the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ is

$$\left(2x + 3y - z + 1\right) + \lambda\left(x + y - 2z + 3\right) = 0$$

$$x\left(2 + \lambda\right) + y\left(3 + \lambda\right) + z\left(-1 - 2\lambda\right) + 1 + 3\lambda = 0 \qquad ---(i)$$

We know that two planes are perpendicular if

$$a_1a_2 + b_1b_2 + c_1c_2 = 0 \qquad ---(ii)$$

Given, plane (i) is perpendicular to the plane,

$$3x - y - 2z - 4 = 0 \qquad ---(iii)$$

Using (i) and (iii) in equation (ii),

$$\left(3\right)\left(2 + \lambda\right) + \left(-1\right)\left(3 + \lambda\right) + \left(-2\right)\left(-1 - 2\lambda\right) = 0$$

$$6 + 3\lambda - 3 - \lambda + 2 + 4\lambda = 0$$

$$6\lambda + 5 = 0$$

$$6\lambda = -5$$

$$\lambda = -\frac{5}{6}$$

Put the value of λ in equation (i),

$$x\left(2 + \lambda\right) + y\left(3 + \lambda\right) + z\left(-1 - 2\lambda\right) + 1 + 3\lambda = 0$$

$$x\left(2 - \frac{5}{6}\right) + y\left(3 - \frac{5}{6}\right) + z\left(-1 + \frac{10}{6}\right) + 1 - \frac{15}{6} = 0$$

$$x\left(\frac{12 - 5}{6}\right) + y\left(\frac{18 - 5}{6}\right) + z\left(\frac{-6 + 10}{6}\right) + \frac{6 - 15}{6} = 0$$

$$\frac{7x}{6} + \frac{13y}{6} + \frac{4z}{6} - \frac{9}{6} = 0$$

The Plane 29.8 Q12

We know that, equation of a plane passing through the line of intersection of plane

$\vec{r}.\overrightarrow{n_1} - d_1 = 0$ and $\vec{r}.\overrightarrow{n_2} - d_2 = 0$ is

$\left(\vec{r}.\overrightarrow{n_1} - d_1\right) + \lambda\left(\vec{r}.\overrightarrow{n_2} - d_2\right) = 0$

So, equation of plane passing through the line of intersection of plane $\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4 = 0$ and $\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5 = 0$ is given by

$\left[\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 4\right] + \lambda\left[\vec{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) + 5\right] = 0$

$\vec{r}.\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right] - 4 + 5\lambda = 0$ ---(i)

We know that two planes are perpendicular if

$\overrightarrow{n_1}.\overrightarrow{n_2} = 0$ ---(ii)

Given that plane (i) is perpendicular to plane

$\vec{r}.\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) + 8 = 0$ ---(iii)

Using (i) and (iii) in equation (ii),

$\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right]\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) = 0$

$\left[\hat{i}\left(1 + 2\lambda\right) + \hat{j}\left(2 + \lambda\right) + \hat{k}\left(3 - \lambda\right)\right]\left(5\hat{i} + 3\hat{j} - 6\hat{k}\right) = 0$

$\left(1 + 2\lambda\right)\left(5\right) + \left(2 + \lambda\right)\left(3\right) + \left(3 - \lambda\right)\left(-6\right) = 0$

$5 + 10\lambda + 6 + 3\lambda - 18 + 6\lambda = 0$

$19\lambda - 7 = 0$

$\lambda = \frac{7}{19}$

Put value of λ in equation (i),

$\vec{r}.\left[\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + \hat{j} - \hat{k}\right)\right] - 4 + 5\lambda = 0$

$\vec{r}\left[\hat{i} + 2\hat{j} + 3\hat{k} + \frac{14}{19}\hat{i} + \frac{7}{19}\hat{j} - \frac{7}{19}\hat{k}\right] - 4 + 5\left(\frac{7}{19}\right) = 0$

$\vec{r}\left[\frac{33\hat{i}}{19} + \frac{45\hat{j}}{19} - \frac{50\hat{k}}{19}\right]\frac{-76 + 35}{19} = 0$

$\vec{r}\left(\frac{33\hat{i} + 45\hat{j} + 50\hat{k}}{19}\right) - \frac{41}{19} = 0$

Multiplying by 19,

$\vec{r}\left(33\hat{i} + 45\hat{j} + 50\hat{k}\right) - 41 = 0$

Equation of required plane is,

$\vec{r}\left(33\hat{i} + 45\hat{j} + 50\hat{k}\right) - 41 = 0$

$33x + 45y + 50z - 41 = 0$

The Plane 29.8 Q13

The equation of a plane passing through the intersection of

$\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=6$ and $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=-5$ is

$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})-6]+\lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})+5]=0$

$\Rightarrow \vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=(6-5\lambda)\ldots(1)$

$\Rightarrow [x\hat{i}+y\hat{j}+z\hat{k}]\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=(6-5\lambda)$

$\Rightarrow [x(1+2\lambda)+y(1+3\lambda)+z(1+4\lambda)]=(6-5\lambda)\ldots(2)$

The requried plane also passes through the point(1, 1, 1).

Substiuting $x=1, y=1, z=1$ in equation (2), we have,

$1\times(1+2\lambda)+1\times(1+3\lambda)+1\times(1+4\lambda)=(6-5\lambda)$

$\Rightarrow 1+2\lambda+1+3\lambda+1+4\lambda=6-5\lambda$

$\Rightarrow 3+9\lambda=6-5\lambda$

$\Rightarrow 14\lambda=6-3$

$\Rightarrow 14\lambda=3$

$\Rightarrow \lambda=\frac{3}{14}$

Substituting the value $\lambda=\frac{3}{14}$ in equation (1), we have,

$\vec{r}\cdot\left[\left(1+2\left(\frac{3}{14}\right)\right)\hat{i}+\left(1+3\left(\frac{3}{14}\right)\right)\hat{j}+\left(1+4\left(\frac{3}{14}\right)\right)\hat{k}\right]=\left(6-5\left(\frac{3}{14}\right)\right)$

$\Rightarrow \vec{r}\cdot\left[\frac{20}{14}\hat{i}+\frac{23}{14}\hat{j}+\frac{26}{14}\hat{k}\right]=\frac{69}{14}$

$\Rightarrow \vec{r}\cdot\left[20\hat{i}+23\hat{j}+26\hat{k}\right]=69$

The Plane 29.8 Q14

We know that, equation of the plane passing through the line of intersection of planes

$\vec{r}.\overrightarrow{n_1} - d_1 = 0$ and $\vec{r}.\overrightarrow{n_2} - d_2 = 0$ is

$\left(\vec{r}.\overrightarrow{n_1} - d_1\right) + \lambda\left(\vec{r}.\overrightarrow{n_2} - d_2\right) = 0$

So, equation of plane passing through the line of intersection of plane $\vec{r}.\left(2\hat{i} + \hat{j} + 3\hat{k}\right) - 7 = 0$ and $\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) - 9 = 0$ is given by

$$\left[\vec{r}.\left(2\hat{i} + \hat{j} + 3\hat{k}\right) - 7\right] + \lambda\left[\vec{r}.\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right) - 9\right] = 0$$

$$\vec{r}\left[\left(2\hat{i} + \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right)\right] - 7 - 9\lambda = 0$$

$$\vec{r}\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0 \qquad ---(i)$$

Given that plane (i) is passing through

$\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$, so

$$\left(2\hat{i} + \hat{j} + 3\hat{k}\right)\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0$$

$$(2)(2 + 2\lambda) + (1)(1 + 5\lambda) + (3)(3 + 3\lambda) - 7 - 9\lambda = 0$$

$$4 + 4\lambda + 1 + 5\lambda + 9 + 9\lambda - 7 - 9\lambda = 0$$

$$9\lambda + 7 = 0$$

$$9\lambda = -7$$

$$\lambda = -\frac{7}{9}$$

Put value of λ in equation (i),

$$\vec{r}.\left[(2 + 2\lambda)\hat{i} + (1 + 5\lambda)\hat{j} + (3 + 3\lambda)\hat{k}\right] - 7 - 9\lambda = 0$$

$$\vec{r}.\left[\left(2 + \frac{14}{9}\right)\hat{i} + \left(1 - \frac{35}{9}\right)\hat{j} + \left(3 - \frac{21}{9}\right)\hat{k}\right] - 7 + \frac{63}{9} = 0$$

$$\vec{r}.\left[\left(\frac{18 - 14}{9}\right)\hat{i} + \left(\frac{9 - 35}{9}\right)\hat{j} + \left(\frac{27 - 21}{9}\right)\hat{k}\right] - 7 + 7 = 0$$

$$\vec{r}.\left[\left(\frac{4}{9}\right)\hat{i} - \frac{26}{9}\hat{j} + \frac{6\hat{k}}{9}\right] + 0 = 0$$

$$\vec{r}.\left[\frac{4}{9}\hat{i} - \frac{26}{9}\hat{j} + \frac{6}{9}\hat{k}\right] = 0$$

Multiplying by $\left(\frac{9}{2}\right)$, we get

$$\vec{r}\left[2\hat{i} - 13\hat{j} + 3\hat{k}\right] = 0$$

Equation of required plane is,

$$\vec{r}.\left(2\hat{i} - 13\hat{j} + 3\hat{k}\right) = 0$$

The Plane 29.8 Q15

The equation of the family of planes through the intersection of planes
$3x - y + 2z = 4$ and $x + y + z = 2$ is,
$(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0.........(i)$

If it passes through $(2, 2, 1)$, then
$(6 - 2 + 2 - 4) + \lambda(2 + 2 + 1 - 2) = 0$
$\Rightarrow \lambda = -\frac{2}{3}$

Substituting $\lambda = -\frac{2}{3}$ in (i) we get, $7x - 5y + 4z = 0$ as the equation of the required plane.

The Plane 29.8 Q16

The equation of the family of planes through the line of intersection of planes
$x + y + z = 1$ and $2x + 3y + 4z = 5$ is,
$(x + y + z - 1) + \lambda(2x + 3y + 4z - 5) = 0..........(i)$
$(2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)z = 5\lambda + 1$

It is perpendicular to the plane $x - y + z = 0$.
$\therefore (2\lambda + 1)(1) + (3\lambda + 1)(-1) + (4\lambda + 1)(1) = 5\lambda + 1$
$\Rightarrow 2\lambda + 1 - 3\lambda - 1 + 4\lambda + 1 = 5\lambda + 1$
$\Rightarrow \lambda = -\frac{1}{3}$

Substituting $\lambda = -\frac{1}{3}$ in (i), we get, $x - z + 2 = 0$ as the equation of the required plane
and its vector equation is $\vec{r}\cdot(\hat{i} - \hat{k}) + 2 = 0$.

The equation of the family of planes parallel to $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 2$ is,
$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = d.......(i)$

If it passes through (a, b, c) then
$(a\hat{i} + b\hat{j} + c\hat{k})\cdot(\hat{i} + \hat{j} + \hat{k}) = d$
$\Rightarrow a + b + c = d$

Substituting $a + b + c = d$ in (i), we get,
$\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = a + b + c$

$x + y + z = a + b + c$ as the equation of the required plane.