RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.7

The Plane 29.7 Q1(i)

Here, $\vec{r} = \left(2\hat{i} - \hat{k}\right) + \lambda\hat{i} + \mu\left(\hat{i} - 2\hat{j} - \hat{k}\right)$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represent a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = 2\hat{i} - \hat{k}$, $\vec{b} = \hat{i}$, $\vec{c} = \hat{i} - 2\hat{j} - \hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= \hat{i}\left(0 - 0\right) - \hat{j}\left(-1 - 0\right) + \hat{k}\left(-2 - 0\right)$$

$$= 0\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{n} = \hat{j} - 2\hat{k}.$$

We know that vector equation of plane in scalar product form is,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put $\vec{n}$ and $\vec{a}$ in equation (i),

$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = \left(2\hat{i} - \hat{k}\right)\left(\hat{j} - 2\hat{k}\right)$$
$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = (2)(0) + (0)(1) + (-1)(-2)$$
$$= 0 + 0 + 2$$
$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = 2$$

The equation in required form is,

$$\vec{r}.\left(\hat{j} - 2\hat{k}\right) = 2$$

The Plane 29.7 Q1(ii)

Here, $\vec{r} = (1 + s - t)\hat{i} + (2 - s)\hat{j} + (3 - 2s + 2t)\hat{k}$

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + s\left(\hat{i} - \hat{j} - 2\hat{k}\right) + t\left(-\hat{i} + 2\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represent a plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} - 2\hat{k}$, $\vec{c} = -\hat{i} + 2\hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{vmatrix}$$

$$= \hat{i}(-2 - 0) - \hat{j}(2 - 2) + \hat{k}(0 - 1)$$

$$\vec{n} = -2\hat{i} - \hat{k}$$

We know that, vector equation of a plane in scalar product form is,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n} \qquad ---(i)$$

Put value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(-2\hat{i} - \hat{k}\right) = \left(-2\hat{i} - \hat{k}\right)\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$\vec{r}.\left(-2\hat{i} - \hat{k}\right) = (-2)(1) + (0)(2) + (-1)(3)$$
$$= -2 + 0 - 3$$
$$\vec{r}.\left(-2\hat{i} - \hat{k}\right) = -5$$

Multiplying both the sides by (-1),

$$\vec{r}.\left(2\hat{i} + \hat{k}\right) = 5$$

The equation in the required form,

$$\vec{r}.\left(2\hat{i} + \hat{k}\right) = 5$$

The Plane 29.7 Q1(iii)

Given, equation of plane,

$$\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right) + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ is the equation of a plane passing through point $\vec{a}$ and parallel to $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{c} = -\hat{i} + \hat{j} - 2\hat{k}$

The given plane is perpendicular to a vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix}$$

$$= \hat{i}\left(-4 + 1\right) - \hat{j}\left(-2 - 1\right) + \hat{k}\left(1 + 2\right)$$

$$= -3\hat{i} + 3\hat{j} + 3\hat{k}$$

We know that, the equation of plane in scalar product form is given by,
$\vec{r}.\vec{n} = \vec{a}\vec{n}$

$$\vec{r}.\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right) = \left(\hat{i} + \hat{j}\right)\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right)$$
$$= (1)(-3) + (1)(3) + (0)(3)$$
$$= -3 + 3$$

$$\vec{r}.\left(-3\hat{i} + 3\hat{j} + 3\hat{k}\right) = 0$$

Dividing by 3, we get

$$\vec{r}.\left(-\hat{i} + \hat{j} + \hat{k}\right) = 0$$

Equation in required form is,

$$\vec{r}.\left(-\hat{i} + \hat{j} + \hat{k}\right) = 0$$

The Plane 29.7 Q1(iv)

$\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(4i - 2\hat{j} + 3k)$

Plane is passing through $(\hat{i} - \hat{j})$and parallel to

$b(\hat{i} + \hat{j} + \hat{k})$ *and* $c\,(4i - 2\hat{j} + 3k)$

$$n = b \times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 4 & -2 & 3 \end{vmatrix}$$

$n = 5i + j - 6k$

$\vec{r}.n = (\hat{i} - \hat{j}).(5i + j - 6k)$=5-1=4

$\vec{r}.(5i + j - 6k) = 4$

The Plane 29.7 Q2(i)

Here, given equation of plane is,

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + s\left(-\hat{i} + \hat{j} + 2\hat{k}\right) + t\left(\hat{i} + 2\hat{j} + \hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = -\hat{i} + \hat{j} + 2\hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} + \hat{k}$

Given plane is perpendicular to vector

$\vec{n} = \vec{b} \times \vec{c}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}\left(1 - 4\right) - \hat{j}\left(-1 - 2\right) + \hat{k}\left(-2 - 1\right)$$

$\vec{n} = -3\hat{i} + 3\hat{j} - 3\hat{k}$

We know that, equation of plane in the scalar product form,

$\vec{r}.\vec{n} = \vec{a}.\vec{n}$ ---(i)

Put the value of $\vec{a}$ and $\vec{n}$ in equation (i),

$$\vec{r}.\left(\hat{i} - \hat{j}\right) = \left(\hat{i} - \hat{j}\right)\left(-3\hat{i} + 3\hat{j} - 3\hat{k}\right)$$

$$\vec{r}.\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=(1)(-3)+(-1)(3)+(0)(-3)$$
$$=-3-3+0$$

$$\vec{r}.\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=-6$$

Put $\vec{r}.=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)=-6$$
$$(x)(-3)+(y)(3)+(z)(-3)=-6$$

$$-3x+3y-3z=-6$$

Dividing by (-3), we get

$$x-y+z=2$$

Equation in required form is,

$$x-y+z=2$$

The Plane 29.7 Q2(ii)

Given, equation of plane,

$$\vec{r}=(1+s+t)\hat{i}+(2-s+t)\hat{j}+(3-2s+2t)\hat{k}$$
$$=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+s\left(\hat{i}-\hat{j}-2\hat{k}\right)+t\left(\hat{i}+\hat{j}+2\hat{k}\right)$$

We know that, $\vec{r}=\vec{a}+\lambda\vec{b}+\mu\vec{c}$ represents the equation of a plane passing through the vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$
$\vec{b}=\hat{i}-\hat{j}-2\hat{k}$
$\vec{c}=\hat{i}+\hat{j}+2\hat{k}$

The given plane is perpendicular to vector

$$\vec{n}=\vec{b}\times\vec{c}$$
$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -2\\ 1 & 1 & 2\end{vmatrix}$$

$$=\hat{i}(-2+2)-\hat{j}(2+2)+\hat{k}(1+1)$$
$$=0.\left(\hat{i}\right)-4\hat{j}+2\hat{k}$$
$$\vec{n}=-4\hat{j}+2\hat{k}$$

We know that, equation of plane in scalar product form is given by,

$$\vec{r}.\vec{n}=\vec{a}.\vec{n} \qquad ---(i)$$

Put, the value of $\vec{a}$ and $\vec{n}$ in (i),

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right)=\left(\hat{i}+2\hat{j}+3\hat{k}\right)\left(-4\hat{j}+2\hat{k}\right)$$

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right)=(1)(0)+(2)(-4)+(3)(2)$$

$$=0-8+6$$

$$\vec{r}.\left(-4\hat{j}+2\hat{k}\right)=-2$$

Put $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$$\left(x\hat{i}+y\hat{j}+z\hat{k}\right)\left(-4\hat{j}+2\hat{k}\right)=-2$$

$$(x)(0)+(y)(-4)+(z)(2)=-2$$

$$-4y+2z=-2$$

Dividing by (-2), we get

$$2y-z=1$$

The equation in required form is,

$$2y-z=1$$

The Plane 29.7 Q3(i)

Given, equation of plane is,

$$\vec{r} = (\lambda - 2\mu)\hat{i} + (3 - \mu)\hat{j} + (2\lambda + \mu)\hat{k}$$
$$\vec{r} = (3\hat{j}) + \lambda(\hat{i} + 2\hat{k}) + \mu(-2\hat{i} - \hat{j} + \hat{k})$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a point $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Given, $\vec{a} = 3\hat{j}$

$\vec{b} = \hat{i} + 2\hat{k}$

$\vec{c} = -2\hat{i} - \hat{j} + \hat{k}$

The given plane is perpendicular to

$$\vec{n} = \vec{b} \times \vec{c}$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}\,(0 + 2) - \hat{j}\,(1 + 4) + \hat{k}\,(-1 - 0)$$

$$\vec{n} = 2\hat{i} - 5\hat{j} - \hat{k}$$

Vector equation of plane in non-parametric form is,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$
$$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) = (3\hat{j})(2\hat{i} - 5\hat{j} - \hat{k})$$

$$= (0)(2) + (3)(-5) + (0)(-1)$$
$$= 0 - 15 + 0$$

$$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) = -15$$
$$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) + 15 = 0$$

The required form of equation is,

$$\vec{r}.(2\hat{i} - 5\hat{j} - \hat{k}) + 15 = 0$$

The Plane 29.7 Q3(ii)

Given, equation of plane is,

$$\vec{r} = \left(2\hat{i} + 2\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \mu\left(5\hat{i} - 2\hat{j} + 7\hat{k}\right)$$

We know that, $\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$ represents the equation of a plane passing through a vector $\vec{a}$ and parallel to vector $\vec{b}$ and $\vec{c}$.

Here, $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$
$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$
$\vec{c} = 5\hat{i} - 2\hat{j} + 7\hat{k}$

The given plane is perpendicular to vector

$$\vec{n} = \vec{b} \times \vec{c}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix}$$

$$= \hat{i}\,(14 + 6) - \hat{j}\,(7 - 15) + \hat{k}\,(-2 - 10)$$

$$\vec{n} = 20\hat{i} + 8\hat{j} - 12\hat{k}$$

We know that, equation of a plane in non-parametric form is given by,

$$\vec{r}.\vec{n} = \vec{a}.\vec{n}$$

$$\vec{r}.\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right) = \left(2\hat{i} + 2\hat{j} - \hat{k}\right)\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right)$$

$$= (2)(20) + (2)(8) + (-1)(-12)$$
$$= 40 + 16 + 12$$

$$\vec{r}.\left(20\hat{i} + 8\hat{j} - 12\hat{k}\right) = 68$$

Dividing by 4,

$$\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$$

Equation of plane in required form is,

$$\vec{r}.\left(5\hat{i} + 2\hat{j} - 3\hat{k}\right) = 17$$