RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.4

The Plane Ex 29.4 Q1

Here, it is given that, the required plane is at a distanceof 3 unit from origin and k is unit vector normal to it. We know that, vector equation of a plane normal to unit vector $\hat{n}$ and at distance d from origin, is

$$\vec{r}.\hat{n} = d$$

So, here $d = 3$ unit

$\hat{n} = \hat{k}$

The equation of the required plane is,

$$\vec{r}.\hat{k} = 3$$

The Plane Ex 29.4 Q2

We know that, vector equation of a plane which is at a distance d unit from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(i)$$

Here, $d = 5$ unit

$\bar{n} = \hat{i} - 2\hat{j} - 2\hat{k}$

$$\hat{n} = \frac{\bar{n}}{|\bar{n}|}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}}$$

$$= \frac{\hat{i} - 2\hat{j} - 2\hat{k}}{\sqrt{9}}$$

$$\hat{n} = \frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right)$$

Put, value of d and $\hat{n}$ in equation (i),

The equation of required plane is,

$$\vec{r}.\frac{1}{3}\left(\hat{i} - 2\hat{j} - 2\hat{k}\right) = 5$$

The Plane Ex 29.4 Q3

Given equation of plane is,

$$2x - 3y - 6z = 14$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right).\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right) = 14$$

Dividing the equation by $\sqrt{(2)^2 + (-3)^2 + (-6)^2}$

$$\vec{r}.\frac{\left(2\hat{i} - 3\hat{j} - 6\hat{k}\right)}{\sqrt{4+9+36}} = \frac{14}{\sqrt{4+9+36}}$$

$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = \frac{14}{7}$$

$$\vec{r}.\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2 \qquad ---(i)$$

We know that the vector equation of a plane with distance d from origin and normal to unit vector $\hat{n}$ is given by

$$\vec{r}.\hat{n} = d \qquad ---(ii)$$

Comparing (i) and (ii),

$d = 2$ and

$\hat{n} = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$

So, distance of plane from origin = 2 unit

Direction cosine of normal to plane = $\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}$

The Plane Ex 29.4 Q4

Given equation of plane is,

$$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) + 6 = 0$$

$$\vec{r}.\left(\hat{i} - 2\hat{j} + 2\hat{k}\right) = -6$$

Multiplying both the sides by (-1),

$$\vec{r}.\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = 6$$

$$\vec{r}.\vec{n} = 6 \qquad ---(i)$$

Here, $\vec{n} = -\hat{i} + 2\hat{j} - 2\hat{k}$

$$|\vec{n}| = \sqrt{(-1)^2 + (2)^2 + (-2)^2}$$

$$= \sqrt{1 + 4 + 4}$$

$$= \sqrt{9}$$

$$= 3$$

Dividing equation (i) by $|\vec{n}| = 3$ both the sides,

$$\vec{r}.\frac{\vec{n}}{|\vec{n}|} = \frac{6}{|\vec{n}|}$$

$$\vec{r}.\frac{1}{3}\left(-\hat{i} + 2\hat{j} - 2\hat{k}\right) = \frac{6}{3}$$

$$\vec{r}\left(-\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right) = 2 \qquad ---(ii)$$

We know that, equation of a plane at distance d from origin and normal to unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d \qquad ---(iii)$$

Comparing equation (ii) and (iii),

$d = 2$

$$\hat{n} = -\frac{\hat{i}}{3} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$$

The Plane Ex 29.4 Q5

Given equation of plane is,

$$2x - 3y + 6z + 14 = 0$$
$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = -14$$
$$\vec{r}.\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right) = -14$$

Multiplying by (-1) both the sides,

$$\vec{r}.\left(-2\hat{i} + 3\hat{j} - 6\hat{k}\right) = 14 \qquad \text{---(i)}$$

So, $\vec{r}.\vec{n} = 14$

$$\vec{n} = -2\hat{i} + 3\hat{j} - 6\hat{k}$$
$$\left|\vec{n}\right| = \sqrt{(-2)^2 + (3)^2 + (-6)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$

$$\left|\vec{n}\right| = 7$$

Dividing equation (i) by $\left|\vec{n}\right| \Rightarrow$ both the sides,

$$\vec{r}.\frac{\left(-2\hat{i} + 3\hat{j} - 6\hat{k}\right)}{7} = \frac{14}{7}$$

$$\vec{r}.\left(-\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 2$$

$$-\frac{2}{7}x + \frac{3}{7}y - \frac{6}{7}z = 2$$

The Plane Ex 29.4 Q6

Given, direction ratios of perpendicular from origin to a plane is 12, − 3, 4
So,

Normal vector = $12\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{n} = 12\hat{i} - 3\hat{j} + 4\hat{k}$

$$|\vec{n}| = \sqrt{(12)^2 + (-3)^2 + (4)^2}$$
$$= \sqrt{144 + 9 + 16}$$
$$= \sqrt{169}$$

$$|\vec{n}| = 13$$

Normal unit vector $\hat{n} = \frac{\vec{n}}{|\vec{n}|}$

$$= \frac{1}{13}\left(12\hat{i} - 3\hat{j} + 4\hat{k}\right)$$

Given that, perpendicular distance of plane from origin is 5 unit.

$\Rightarrow \qquad d = 5$ unit

We know that, equation of a plane at a distance d from origin and normal unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d$$

So, vector equation of required plane is

$$\vec{r}.\left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{4}{13}\hat{k}\right) = 5$$

Put, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} + \frac{4}{13}\hat{k}\right) = 5$$

$$(x)\left(\frac{12}{13}\right) + (y)\left(-\frac{3}{13}\right) + (z)\left(\frac{4}{13}\right) = 5$$

$$\frac{12}{13}x - \frac{3}{13}y + \frac{4}{13}z = 5$$

The Plane Ex 29.4 Q7

Given equation of plane is

$$x + 2y + 3z - 6 = 0$$

$$\left(x\hat{i} + y\hat{j} + z\hat{k}\right)\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - 6 = 0$$

$$\vec{r}.\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) = 6 \qquad ---(i)$$

$$\vec{r}.\vec{n} = 6$$

So, $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\left|\vec{n}\right| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$

$$= \sqrt{1 + 4 + 9}$$

$$\left|\vec{n}\right| = \sqrt{14}$$

Dividing equation (i) by $\sqrt{14}$, we get

$$\vec{r}.\left(\frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}\right) = \frac{6}{\sqrt{14}} \qquad ---(ii)$$

We know that, vector equation of a plane at distance d unit from origin and normal to unit vector $\hat{n}$ is

$$\vec{r}.\hat{n} = d \qquad ---(iii)$$

Comparing (ii) and (iii), we get

Normal unit vector $= \hat{n} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$

The Plane Ex 29.4 Q8

We know that, vector equation of a plane which is at a distance d from origin and normal to unit vector $\hat{n}$ is given by

$\vec{r}.\hat{n} = d$ ---(i)

Here, given $\vec{d} = 3\sqrt{3}$ unit.

Let, $\vec{a} = \left(p\hat{i} + q\hat{j} + r\hat{k}\right)$

Where $\vec{a}$ is normal vector.

Given that, $\vec{a}$ is equally inclined to the coordinate axes

If l, m, n are direction cosines of $\vec{n}$,

Here, $l = m = n$ ---(ii)

We know that,

$l^2 + m^2 + n^2 = 1$

$l^2 + l^2 + l^2 = 1$ [Using (ii)]

$3l^2 = 1$

$l = \dfrac{1}{\sqrt{3}}$

So, $l = m = n = \dfrac{1}{\sqrt{3}}$

Here,

$$l = \frac{p}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

$$m = \frac{q}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

$$n = \frac{r}{|\vec{a}|} = \frac{1}{\sqrt{3}}$$

Now,

$$\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{p}{|\vec{a}|}\hat{i} + \frac{q}{|\vec{a}|}\hat{j} + \frac{r}{|\vec{a}|}\hat{k}$$

$$\hat{a} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

Put the value of $d = 3\sqrt{3}$ unit and $\hat{n} = \hat{a} = \frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$ in equation (i),

vector equation of the required plane is

$$\vec{r}.\frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right) = 3\sqrt{3}$$

$$\vec{r}\left(\hat{i} + \hat{j} + \hat{k}\right) = 9$$

$$x + y + z = 9$$

The Plane Ex 29.4 Q9

Here, we have to find equating a plane passing through $A(1,2,1)$ and perpendicular to line joining $B(1,4,2)$ and $C(2,3,5)$.

We know that, the vector equation of a plane passing through a point $\vec{a}$ and perpendicular to vector $\vec{n}$ is given by,

$$(\vec{r}-\vec{a}).\vec{n}=0 \qquad ---(i)$$

Here, $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$

$\vec{n}=\overrightarrow{BC}$

$= \text{Position vector of } C - \text{Position vector of } B$

$=\left(2\hat{i}+3\hat{j}+5\hat{k}\right)-\left(\hat{i}+4\hat{j}+2\hat{k}\right)$

$=2\hat{i}+3\hat{j}+5\hat{k}-\hat{i}-4\hat{j}-2\hat{k}$

$\vec{n}=\hat{i}-\hat{j}+3\hat{k}$

Put, $\vec{a}$ and $\vec{n}$ in equation (i),

Vector equation of plane is

$$\left[\vec{r}-\left(\hat{i}+2\hat{j}+\hat{k}\right)\right].\left(\hat{i}-\hat{j}+3\hat{k}\right)=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-\left(\hat{i}+2\hat{j}+\hat{k}\right)\left(\hat{i}-\hat{j}+3\hat{k}\right)=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-\left[(1)(1)+(2)(-1)+(1)(3)\right]=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-\left[1-2+3\right]=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-(4-2)=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)-2=0$$

$$\vec{r}.\left(\hat{i}-\hat{j}+3\hat{k}\right)=2 \qquad ---(ii)$$

$$|\vec{n}| = \sqrt{(1)^2 + (-1)^2 + (3)^2}$$
$$= \sqrt{1+1+9}$$
$$= \sqrt{11}$$

Dividing equation (i) by $\sqrt{11}$,

$$\vec{r}.\left(\frac{1}{\sqrt{11}}\hat{i} - \frac{1}{\sqrt{11}}\hat{j} + \frac{3}{\sqrt{11}}\hat{k}\right) = \frac{2}{\sqrt{11}}$$

$$\vec{r}.\hat{n} = d$$

So, perpendicular distance of plane from origin = $\frac{2}{\sqrt{11}}$ units

Equation of plane, $\vec{r}.\left(\hat{i} - \hat{j} + 3\hat{k}\right) = 2$

Equation of plane, $x - y + 3z - 2 = 0$

The Plane 29.4 Q10

We know that the vector equation of a plane at a distance 'p' from the origin and normal to the unit vector $\hat{n}$ is $\vec{r} \cdot \hat{n} = p$

Vector normal to the plane is $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

The unit vector normal to the plane is

$$\hat{n} = \frac{2}{\sqrt{2^2 + (-3)^2 + 4^2}}\hat{i} - \frac{3}{\sqrt{2^2 + (-3)^2 + 4^2}}\hat{j} + \frac{4}{\sqrt{2^2 + (-3)^2 + 4^2}}\hat{k}$$

$$\Rightarrow \hat{n} = \frac{2}{\sqrt{4+9+16}}\hat{i} - \frac{3}{\sqrt{4+9+16}}\hat{j} + \frac{4}{\sqrt{4+9+16}}\hat{k}$$

$$\Rightarrow \hat{n} = \frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}$$

Here, given that $p = \frac{6}{\sqrt{29}}$

Thus, the vector equation of the plane is

$$\vec{r} \cdot \left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

The Cartesian equation of the plane is

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right) = \frac{6}{\sqrt{29}}$$

$$\Rightarrow \left(\frac{2x}{\sqrt{29}} - \frac{3y}{\sqrt{29}} + \frac{4z}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$$

$$\Rightarrow \left(\frac{2x - 3y + 4z}{\sqrt{29}}\right) = \frac{6}{\sqrt{29}}$$

$$\Rightarrow 2x - 3y + 4z = 6$$

The Plane 29.4 Q11

The Cartesian equation of the given plane is

$2x - 3y + 4z - 6 = 0.$

The above equation can be rewritten as

$2x - 3y + 4z = 6$

Therefore, the vector equation of the plane is

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 6$

$\Rightarrow \vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 6 \ldots .(1)$

We know that the vector equation of a plane at a distance 'p' from the origin and normal to unit vector $\hat{n}$ is $\vec{r} \cdot \hat{n} = p$

We have, $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}.$

Thus $|\vec{n}| = \sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{29}$

Dividing the equation (1) by $|\vec{n}| = \sqrt{29}$, we have,

$$\vec{r} \cdot \left(\frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}} \right) = \frac{6}{\sqrt{29}}$$

Hence the normal form of the equation of the plane is

$$\vec{r} \cdot \left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k} \right) = \frac{6}{\sqrt{29}}$$

Hence the perpendicular distance of the origin from the plane is $p = \frac{6}{\sqrt{29}}$.