RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.13

The Plane Ex 29.13 Q1

$\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$

We know that the lines,

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ are coplanar if

$\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$

and the equation of the plane containing them is

$\vec{r} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2})$

Here

$\vec{a_1} = 2\hat{j} - 3\hat{k}, \vec{b_1} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{a_2} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ and $\vec{b_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Therefore, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix} = \hat{i}(8-9) - \hat{j}(4-6) + \hat{k}(3-4)$

$\Rightarrow \vec{b_1} \times \vec{b_2} = -\hat{i} + 2\hat{j} - \hat{k}$

$\Rightarrow \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{j} - 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 0 + 4 + 3 = 7$

and

$\vec{a_2} \cdot (\vec{b_1} \times \vec{b_2}) = (2\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = -2 + 12 - 3 = 7$

Since $\vec{a_1} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_2} \cdot (\vec{b_1} \times \vec{b_2})$, the lines are coplanar.

Now the equation of the plane containing the given lines is

$\vec{r} \cdot (\vec{b_1} \times \vec{b_2}) = \vec{a_1} \cdot (\vec{b_1} \times \vec{b_2})$

$\vec{r} \cdot (-\hat{i} + 2\hat{j} - \hat{k}) = 7$

$\Rightarrow \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = -7$

$\Rightarrow \vec{r} \cdot (\hat{i} - 2\hat{j} + \hat{k}) + 7 = 0$

The Plane Ex 29.13 Q2

We know that lines $\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$ and $\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

And equation of plane containing them is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

Here, equation of lines are

$$\frac{x + 1}{-3} = \frac{y - 3}{2} = \frac{z + 2}{1} \text{ and } \frac{x}{1} = \frac{y - 7}{-3} = \frac{z + 7}{2}$$

So, $x_1 = -1$, $y_1 = 3$, $z_1 = -2$, $l_1 = -3$, $m_1 = 2$, $n_1 = 1$
$x_2 = 0$, $y_2 = 7$, $z_2 = -7$, $l_2 = 1$, $m_2 = -3$, $n_2 = 2$

So,

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 0 + 1 & 7 - 3 & -7 + 2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 4 & -5 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix}$$

$$= 1(4 + 3) - 4(-6 - 1) - 5(9 - 2)$$
$$= 7 + 28 - 35$$

$$= 0$$

So, lines are coplanar.

Equation of plane containing line is

$$\begin{vmatrix} x + 1 & y - 3 & z + 2 \\ -3 & 2 & 1 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$$(x + 1)(4 + 3) - (y - 3)(-6 - 1) + (z + 2)(9 - 2) = 0$$
$$7x + 7 + 7y - 21 + 7z + 14 = 0$$
$$7x + 7y + 7z = 0$$

The Plane Ex 29.13 Q3

We know that the plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad \text{---(i)}$$

Required plane is passing through $(0, 7, -7)$, so

$$a(x - 0) + b(y - 7) + c(z + 7) = 0$$
$$ax + b(y - 7) + c(z + 7) = 0 \qquad \text{---(ii)}$$

Plane (ii) also contains line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ so, it passes through point $(-1, 3, -2)$,

$$a(-1) + b(3 - 7) + c(-2 + 7) = 0$$
$$-a - 4b + 5c = 0 \qquad \text{---(iii)}$$

Also, plane (ii) will be parallel to line
so, $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$$(a)(-3) + (b)(2) + (c)(1) = 0$$
$$-3a + 2b + c = 0 \qquad \text{---(iv)}$$

Solving (iii) and (iv) by cross-multiplication,

$$\frac{a}{(-4)(1) - (5)(2)} = \frac{b}{(-3)(5) - (-1)(1)} = \frac{c}{(-1)(2) - (-4)(-3)}$$
$$\frac{a}{-4 - 10} = \frac{b}{-15 + 1} = \frac{c}{-2 - 12}$$

$$\frac{a}{-14} = \frac{b}{-14} = \frac{c}{14} = \lambda\,(\text{Say})$$

$\Rightarrow \quad a = -14\lambda,\ b = -14\lambda,\ c = -14\lambda$

Put a, b, c in equation (ii),

$$ax + b(y-7) + c(z+7) = 0$$
$$(-14\lambda)x + (-14\lambda)(y-7) + (-14\lambda)(z+7) = 0$$

Dividing by (-14λ), we get

$$x + y - 7 + z + 7 = 0$$
$$x + y + z = 0$$

So, equation of plane containing the given point and line is $x + y + z = 0$

The other line is $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(1)(1) + (1)(-3) + (1)(2) = 0$$
$$1 - 3 + 2 = 0$$

$$0 = 0$$

$LHS = RHS$

So, $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ lie on plane $x + y + z = 0$

The Plane Ex 29.13 Q4

We know that equation of plane passing through (x_1, y_1, z_1) is given by

$$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0 \qquad ---(i)$$

Since, required plane contain lines

$$\frac{x-4}{1} = \frac{y-3}{-4} = \frac{z-2}{5} \text{ and } \frac{x-3}{1} = \frac{y+2}{-4} = \frac{z}{5}$$

So, required plane passes through $(4,3,2)$ and $(3,-2,0)$, so, equation of required plane is,

$$a(x - 4) + b(y - 3) + c(z - 2) = 0 \qquad ---(ii)$$

Plane (ii) also passes through $(3,-2,0)$, so

$$a(3 - 4) + b(-2 - 3) + c(0 - 2) = 0$$
$$-a - 5b - 2c = 0$$
$$a + 5b + 2c = 0 \qquad ---(iii)$$

Now plane (ii) is also parallel to line with direction ratios $1,-4,5$, so,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-4) + (c)(5) = 0$$
$$a - 4b + 5c = 0 \qquad ---(iv)$$

Solving equation (iii) and (iv) by cross-multiplication,

$$\frac{a}{(5)(5)-(-4)(2)} = \frac{b}{(1)(2)-(1)(5)} = \frac{c}{(1)(-4)-(1)(5)}$$

$$\frac{a}{25+8} = \frac{b}{2-5} = \frac{c}{-4-5}$$

$$\frac{a}{33} = \frac{b}{-3} = \frac{c}{-9}$$

Multiplying by 3,

$$\frac{a}{11} = \frac{b}{-1} = \frac{c}{-3} = \lambda\,(\text{Say})$$

$$\Rightarrow \qquad a = 11\lambda,\ b = -\lambda,\ c = -3\lambda$$

Put a, b, c in equation (ii),

$$a(x-4)+b(y-3)+c(z-2) = 0$$

$$(11\lambda)(x-4)+(-\lambda)(y-3)+(-3\lambda)(z-2) = 0$$

$$11\lambda x - 44\lambda - \lambda y + 3\lambda - 3\lambda z + 6\lambda = 0$$

$$11\lambda x - \lambda y - 3\lambda z - 35\lambda = 0$$

Dividing by λ,

$$11x - y - 3z - 35 = 0$$

So, equation of required plane is,

$$11x - y - 3z - 35 = 0$$

The Plane Ex 29.13 Q5

We have, equation of line is

$$\frac{x+4}{3} = \frac{y+6}{5} = \frac{z-1}{-2} = \lambda \text{ (Say)}$$

General point on this line is given by

$$(3\lambda - 4,\ 5\lambda - 6,\ -2\lambda + 1) \qquad ---(i)$$

Another equation of line is

$$3x - 2y + z + 5 = 0 = 2x + 3y + 4z - 4$$

Let a, b, c be the direction ratio of the line so, it will be perpendicular to normal of $3x - 2y + z + 5 = 0$ and $2x + 3y + 4z - 4 = 0$, so

Using $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(3)(a) + (-2)(b) + (1)(c) = 0$$

$$3a - 2b + c = 0 \qquad ---(ii)$$

Again,

$$(2)(a) + (3)(b) + (4)(c) = 0$$

$$2a + 3b + 4c = 0 \qquad ---(iii)$$

Solving (ii) and (iii) by cross-multiplication,

$$\frac{a}{(-2)(4)-(3)(1)} = \frac{b}{(2)(1)-(3)(4)} = \frac{c}{(3)(3)-(-2)(2)}$$

$$\frac{a}{-8-3} = \frac{b}{2-12} = \frac{c}{9+4}$$

$$\frac{a}{-11} = \frac{b}{-10} = \frac{c}{13}$$

Direction ratios are proportional to $-11, -10, 13$

Let $z = 0$, so

$$3x - 2y = -5 \qquad ---(A)$$

$$2x + 3y = 4 \qquad ---(B)$$

Solving (A) and (B),

$$\begin{array}{rcrcl} & 6x & - \; 4y & = & -10 \\ & 6x & + \; 9y & = & 12 \\ - & - & - & & - \\ \hline & & -13y & = & -12 \end{array}$$

$$y = \frac{22}{13}$$

Put y in equation (A),

$$3x - 2\left(\frac{22}{13}\right) = -5$$

$$3x-\frac{44}{13}=-5$$

$$3x=-5+\frac{44}{13}$$

$$3x=-\frac{21}{13}$$

$$x=-\frac{7}{13}$$

So, equation of line (2) in symmetrical form,

$$\frac{x+\frac{7}{13}}{-11}=\frac{y-\frac{22}{13}}{-10}=\frac{z-0}{13}$$

Put the general point of line (1) from equation (1)

$$\frac{3\lambda-4+\frac{7}{13}}{-11}=\frac{5\lambda-6-\frac{22}{13}}{-10}=\frac{-2\lambda+1}{13}$$

$$\frac{39\lambda-52+7}{-11\times13}=\frac{65\lambda-78-22}{-10\times13}=\frac{-2\lambda+1}{13}$$

$$\frac{39\lambda-45}{-11}=\frac{65\lambda-100}{-10}=\frac{-2\lambda+1}{1}$$

The equation of the plane is $45x-17y+25z+53=0$
Their point of intersection is $(2,4,-3)$

The Plane Ex 29.13 Q6

We know that plane $\vec{r}.\vec{n} = d$ contains the line $\vec{r} = \vec{a} + \lambda\vec{b}$ if

(i) $\vec{b}.\vec{n} = 0$ (ii) $\vec{a}.\vec{n} = d$ ---(i)

Given, equation of plane $\vec{r}.\left(\hat{i} + 2\hat{j} - \hat{k}\right) = 3$ and equation of line $\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} + \hat{j} + 4\hat{k}\right)$

so, $\vec{n} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{a} = \hat{i} + \hat{j}$

$d = 3$ $\vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$

$\vec{b}.\vec{n} = \left(2\hat{i} + \hat{j} + 4\hat{k}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$

$= (2)(1) + (1)(2) + (4)(-1)$

$= 2 + 2 - 4$

$\vec{b}.\vec{n} = 0$

$\vec{a}.\vec{n} = \left(\hat{i} + \hat{j}\right)\left(\hat{i} + 2\hat{j} - \hat{k}\right)$

$= (1)(1) + (1)(2) + (0)(-1)$

$= 1 + 2 - 0$

$= 3$

$= d$

since, $\vec{b}.\vec{n} = 0$ and $\vec{a}.\vec{n} = d$, so, from (i)

Given line lie on the given plane.

The Plane Ex 29.13 Q7

Let $L_1 : \dfrac{x+3}{3} = \dfrac{y}{-2} = \dfrac{z-7}{6}$ and

$L_2 : \dfrac{x+6}{1} = \dfrac{y+5}{-3} = \dfrac{z-1}{2}$ be the equations of two lines

Let the plane be $ax + by + cz + d = 0$...(1)

Given that the required plane passes through the intersection of the lines L_1 and L_2.

Hence the normal to the plane is perpendicular to the lines L_1 and L_2.

$\therefore 3a - 2b + 6c = 0$

$a - 3b + 2c = 0$

Using cross-multiplication, we get,

$$\frac{a}{-4+18} = \frac{b}{6-6} = \frac{c}{-9+2}$$

$$\Rightarrow \frac{a}{14} = \frac{b}{0} = \frac{c}{-7}$$

$$\Rightarrow \frac{a}{2} = \frac{b}{0} = \frac{c}{-1}$$

The Plane Ex 29.13 Q8

Let the equation of the plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$........(i)

Plane is passing through (3,4,2) and (7,0,6)

$$\frac{3}{a}+\frac{4}{b}+\frac{2}{c}=1$$

$$\frac{7}{a}+\frac{0}{b}+\frac{6}{c}=1$$

Required plane is perpendicular to $2x-5y-15=0$

$$\frac{2}{a}+\frac{-5}{b}+\frac{0}{c}=0$$

$\Rightarrow 2b=5a$

$\therefore b=2.5a$

$$\frac{3}{a}+\frac{4}{2.5a}+\frac{2}{c}=1$$

$$\frac{7}{a}+\frac{6}{c}=1$$

Solving the above 2 equations,

$a=3.4=\frac{17}{5}$, $b=\ 8.5=\frac{17}{2}$ and $c=\frac{-34}{6}=-\frac{17}{3}$

Substituting the values in (i)

$$\frac{x}{\frac{17}{5}}+\frac{y}{\frac{17}{2}}+\frac{z}{-\frac{17}{3}}=1$$

$$\Rightarrow \frac{5x}{17}+\frac{2y}{17}-\frac{3z}{17}=1$$

$\Rightarrow 5x+2y-3z=17$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(5\hat{i}+2\hat{j}-3\hat{k})=17$

$\Rightarrow \vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$

Vector equation of the plane is $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$.

The line passes through $B(1,3,-2)$.

$5(1)+2(3)-3(-2)=17$

The point B lies on the plane.

$\therefore$ The line $\vec{r}=\hat{i}+3\hat{j}-2\hat{k}+\lambda(\hat{i}-\hat{j}+\hat{k})$ lies on the plane $\vec{r}.(5\hat{i}+2\hat{j}-3\hat{k})=17$.

The Plane Ex 29.13 Q9

The direction ratio of the line $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ is

$r_1=(-3,-2k,2)$

The direction ratio of the line $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ is

$r_2=(k,1,5)$

Since the lines $\frac{x-1}{-3}=\frac{y-2}{-2k}=\frac{z-3}{2}$ and $\frac{x-1}{k}=\frac{y-2}{1}=\frac{z-3}{5}$ are perpendicular so

$$r_1\cdot r_2=0$$
$$(-3,-2k,2)\cdot(k,1,5)=0$$
$$-3k-2k+10=0$$
$$-5k=-10$$
$$k=2$$

Therefore the equation of the lines are $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{5}$

The equation of the plane containing the perpendicular lines $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ and $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{5}$ is

$$\begin{vmatrix} x & y & z \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$$\begin{vmatrix} x & y & z \\ -3 & -4 & 2 \\ 2 & 1 & 5 \end{vmatrix}=0$$

$$(-20-2)x-y(-15-4)+z(-3+8)+d=0$$
$$-22x+19y+5z+d=0$$

The line $\frac{x-1}{-3}=\frac{y-2}{-4}=\frac{z-3}{2}$ pass through the point $(1,2,3)$ so putting $x=1,y=2,z=3$ in the equation $-22x+19y+5z+d=0$ we get

$$-22(1)+19(2)+5(3)+d=0$$
$$d=22-38-15$$
$$d=-31$$

Therefore the equation of the plane containing the lines is

$-22x+19y+5z=31$

The Plane Ex 29.13 Q10

Any point on the line

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=k$$

is of the form, $(3k+2,\ 4k-1,\ 2k+2)$.

If the point $P(3k+2,\ 4k-1,\ 2k+2)$ lies in the plane $x-y+z-5=0$, we have,

$(3k+2)-(4k-1)+(2k+2)-5=0$

$\Rightarrow 3k+2-4k+1+2k+2-5=0$

$\Rightarrow k=0$

Thus, the coordinates of the point of intersection of the line and the plane are: $P(3\times 0+2,\ 4\times 0-1,\ 2\times 0+2)=P(2,-1,2)$

Let θ be the angle between the line and the plane.

Thus,

$$\sin\theta=\frac{al+bm+cn}{\sqrt{a^2+b^2+c^2}\sqrt{l^2+m^2+n^2}}$$, where, l,m and n are the direction ratios of the line and a,b and c are the direction ratios of the normal to the plane.

Here, $l=3$, $m=4$, $n=2$, $a=1$, $b=-1$, and $c=1$

Hence,

$$\sin\theta=\frac{1\times 3+(-1)\times 4+1\times 2}{\sqrt{1^2+(-1)^2+1^2}\sqrt{3^2+4^2+2^2}}$$

$$\Rightarrow \sin\theta=\frac{1}{\sqrt{3}\sqrt{29}}$$

$$\Rightarrow \theta=\sin^{-1}\left(\frac{1}{\sqrt{3}\sqrt{29}}\right)$$

The Plane Ex 29.13 Q11

Let A, B and C be three points with position vectors
$\hat{i}+\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+\hat{k}$ *and* $\hat{i}+2\hat{j}+\hat{k}$.

Thus, $\overrightarrow{AB}=\vec{b}-\vec{a}=(2\hat{i}-\hat{j}+\hat{k})-(\hat{i}+\hat{j}-2\hat{k})=\hat{i}-2\hat{j}+3\hat{k}$

$\overrightarrow{AC}=\vec{c}-\vec{a}=(\hat{i}+2\hat{j}+\hat{k})-(\hat{i}+\hat{j}-2\hat{k})=\hat{j}+3\hat{k}$

Now consider $\overrightarrow{AB}\times\overrightarrow{AC}$:

$$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 3\\ 0 & 1 & 3\end{vmatrix}$$

$\vec{n}=\hat{i}(-6-3)-3\hat{j}+\hat{k}=-9\hat{i}-3\hat{j}+\hat{k}$

So, the equation of the required plane is

$(\vec{r}-\vec{a})\cdot\vec{n}=0$

$\Rightarrow(\vec{r}\cdot\vec{n})=(\vec{a}\cdot\vec{n})$

$\Rightarrow(\vec{r}\cdot(-9\hat{i}-3\hat{j}+\hat{k}))=(\hat{i}+\hat{j}-2\hat{k})\cdot(-9\hat{i}-3\hat{j}+\hat{k})$

$\Rightarrow\vec{r}\cdot(9\hat{i}+3\hat{j}-\hat{k})=14$

Also, find the coordinates of the point of intersection of this plane and the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$

Any point on the line $\vec{r}=3\hat{i}-\hat{j}-\hat{k}+\lambda(2\hat{i}-2\hat{j}+\hat{k})$ *is of the form,*

$(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$

If the point $P(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$ *lies in the plane,*

$\vec{r}\cdot(9\hat{i}+3\hat{j}-\hat{k})=14$, *we have,*

$9(3+2\lambda)-3(1+2\lambda)-(-1+\lambda)=14$

$\Rightarrow 27+18\lambda-3-6\lambda+1-\lambda=14$

$\Rightarrow 11\lambda=-11$

$\Rightarrow\lambda=-1$

Thus, the required point of intersection is

$P(3+2\lambda,\ -1-2\lambda,\ -1+\lambda)$

$\Rightarrow P(3+2(-1),\ -1-2(-1),\ -1+(-1))$

$\Rightarrow P(1,\ 1,\ -2)$

The Plane Ex 29.13 Q12

$$\frac{5-x}{-4}=\frac{y-7}{4}=\frac{z+3}{-5}$$
$$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}\text{.......(i)}$$

$$\frac{x-8}{7}=\frac{2y-8}{2}=\frac{z-5}{3}$$
$$\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}\text{.....(ii)}$$

Here,

$a_1 = 4, b_1 = 4, c_1 = -5$

$a_2 = 7, b_2 = 1, c_2 = 3$

$x_1 = 5, y_1 = 7, z_1 = -3$

$x_2 = 8, y_2 = 4, z_2 = 5$

Condition for two lines to be coplanar,

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\therefore \begin{vmatrix} 8-5 & 4-7 & 5+3 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$

$= 3(12+5)+3(12+35)+8(4-28)$

$= 3\times 17+3\times 47+8\times(-24)$

$= 51+141-192$

$= 192-192$

$= 0$

$\therefore$ The lines are coplanar to each other.

The Plane Ex 29.13 Q13

Required equation of plane is passing through the point (3, 2, 0),

$\therefore\ a(x-3)+b(y-2)+c(z-0)=0$

$\Rightarrow a(x-3)+b(y-2)+cz=0$..........(i)

Required equation of plane also contains the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$,

so it passes through the point (3, 2, 0)

$\Rightarrow a(3-3)+b(6-2)+c4=0$

$\Rightarrow 4b+4c=0$..................................(ii)

Also plane will be parallel to,

$a(1)+b(5)+c(4)=0$

$a+5b+4c=0$..............(iii)

Solving (ii) and (iii) by cross multiplication,

$$\frac{a}{16\text{-}20}=\frac{b}{4-0}=\frac{c}{0-4}=\lambda(\text{say})$$

$$-\frac{a}{4}=\frac{b}{4}=-\frac{c}{4}=\lambda(\text{say})$$

$\Rightarrow a=-\lambda, b=\lambda,\ c=-\lambda$

Put $a=-\lambda, b=\lambda,\ c=-\lambda$ in equation (i) we get

$(-\lambda)(x-3)+(\lambda)(y-2)+(-\lambda)z=0$

$\Rightarrow -x+3+y-2-z=0$

$\Rightarrow x-y+z-1=0$