RD Sharma Solutions Class 12 Maths Chapter 29 Ex 29.12

The Plane Ex 29.12 Q1(i)

Direction ratios of the given line are

$(5-3, 1-4, 6-1) = (2, -3, 5)$

Hence, equation of the line is

$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$$

$\Rightarrow x = 2r+5, y = -3r+1, z = 5r+6$

For any point on the yz − plane $x = 0$

$\Rightarrow 2r+5 = 0 \Rightarrow r = -\frac{5}{2}$

$y = -3(-\frac{5}{2}) + 1 = \frac{17}{2}$

$z = 5(-\frac{5}{2}) + 6 = -\frac{13}{2}$

Hence, the coordinates of the point are $\left(0, \frac{17}{2}, -\frac{13}{2}\right)$.

The Plane Ex 29.12 Q1(ii)

Direction ratios of the given line are
$(5-3, 1-4, 6-1) = (2, -3, 5)$
Hence, equation of the line is
$$\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-6}{5} = r$$
$\Rightarrow x = 2r+5, y = -3r+1, z = 5r+6$
For any point on zx − plane $y = 0$
$\Rightarrow -3r+1 = 0 \Rightarrow r = \frac{1}{3}$
$$x = 2\left(\frac{1}{3}\right) + 5 = \frac{17}{3}$$
$$z = 5\left(\frac{1}{3}\right) + 6 = \frac{23}{3}$$
Hence, the coordinates of the point are $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$.

The Plane Ex 29.12 Q2

Let the coordinates of the points A and B be
$(3, -4, -5)$ *and* $(2, -3, 1)$ *repectively.*
Equation of the line joining the points (x_1, y_1, z_1) *and* (x_2, y_2, z_2) *is*
$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = r, \text{ where } r \text{ is some constant.}$$
Thus equation of AB is
$$\frac{x-3}{2-3} = \frac{y-(-4)}{(-3)-(-4)} = \frac{z-(-5)}{1-(-5)} = r$$
$$\Rightarrow \frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = r$$
Any point on the line AB is of the form
$-r+3, r-4, 6r-5$
Let P be the point of intersection of the line AB and the plane $2x+y+z=7$
Thus, we have,
$2(-r+3) + r - 4 + 6r - 5 = 7$
$\Rightarrow -2r + 6 + r - 4 + 6r - 5 = 7$
$\Rightarrow 5r = 10$
$\Rightarrow r = 2$
Substituting the value of r in $-r+3, r-4, 6r-5$*, the coordinates of P are:*
$(-2+3, 2-4, 6\times 2-5) = (1, -2, 7)$

The Plane Ex 29.12 Q3

The equation of the given line is

$$\vec{r} \cdot = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right) \qquad ...(1)$$

The equation of the given plane is

$$\vec{r}.\left(\hat{i} - \hat{j} + \hat{k}\right) = 5 \qquad ...(2)$$

Substituting the value of $\vec{r}$ from equation (1) in equation (2), we obtain

$$\left[2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)\right] \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$

$$\Rightarrow \left[(3\lambda + 2)\hat{i} + (4\lambda - 1)\hat{j} + (2\lambda + 2)\hat{k}\right] \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 5$$

$$\Rightarrow (3\lambda + 2) - (4\lambda - 1) + (2\lambda + 2) = 5$$

$$\Rightarrow \lambda = 0$$

Substituting this value in equation (1), we obtain the equation of the line as

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

This means that the position vector of the point of intersection of the line and the plane is $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$

This shows that the point of intersection of the given line and plane is given by the coordinates, (2, −1, 2). The point is (−1, −5, −10).

The distance d between the points, (2, −1, 2) and (−1, −5, −10), is

$$d = \sqrt{(-1-2)^2 + (-5+1)^2 + (-10-2)^2} = \sqrt{9+16+144} = \sqrt{169} = 13$$

The Plane Ex 29.12 Q4

To find the point of intersection of the line

$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k}) = 0$,

we substitute $\vec{r}$ of line in the plane.

$$[2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})].(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow [(2 + 3\lambda)\hat{i} + (-4 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}].(\hat{i} - 2\hat{j} + \hat{k}) = 0$$

$$\Rightarrow 2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0$$

$$\Rightarrow 3\lambda = 12 \Rightarrow \lambda = 4$$

$$\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + 4(3\hat{i} + 4\hat{j} + 2\hat{k}) = 14\hat{i} + 12\hat{j} + 10\hat{k}$$

Hence, the distance of the point $2\hat{i} + 12\hat{j} + 5\hat{k}$ from $14\hat{i} + 12\hat{j} + 10\hat{k}$ is

$$\sqrt{(14-2)^2 + (12-12)^2 + (10-5)^2} = \sqrt{12^2 + 5^2} = \sqrt{169} = 13$$

The Plane Ex 29.12 Q5

Equation of the line through the points A(2, − 1,2)

and B(5,3,4) is $\frac{x-2}{5-2} = \frac{y+1}{3+1} = \frac{z-2}{4-2} = r$

$$\Rightarrow \frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = r$$

$$\Rightarrow x = 3r + 2, y = 4r - 1, z = 2r + 2$$

Substituting these in the plane equation we get

$$(3r + 2) - (4r - 1) + (2r + 2) = 5$$

$$\Rightarrow r = 0$$

$$\Rightarrow x = 2, y = -1, z = 2$$

Distance of (2, − 1,2) from (− 1, − 5, − 10) is

$$= \sqrt{(2-(-1))^2 + (-1-(-5))^2 + (2-(-10))^2} = \sqrt{3^2 + 4^2 + 12^2}$$

$$= \sqrt{169} = 13$$

The Plane Ex 29.12 Q6

The equation of a line joining the points A(3, -4, -5) and B(2, -3, 1)is

$$\frac{x-3}{2-3} = \frac{y+4}{-3+4} = \frac{z+5}{1+5} = r$$

$$\Rightarrow x = 3-r, y = -4+r, z = -5+6r$$

Substituting this into the given plane equation we get,

$$2(3-r)+(-4+r)+(-5+6r) = 7$$

$$\Rightarrow r = 2$$

$$\Rightarrow x = 1,\ y = -2,\ z = 7$$

Distance of (1, -2, 7) from (3, 4, 4) is

$$=\sqrt{(3-1)^2+(4+2)^2+(4-7)^2}$$

$$= \sqrt{4+36+9}$$

$$= \sqrt{49}$$

$$= 7$$