RD Sharma Solutions Class 12 Maths Chapter 28 Ex 28.5

We know that, shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$S.D. = \left|\frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 7\hat{k}) \text{ and}$$
$$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k})$$

$$\Rightarrow \quad \vec{a_1} = (3\hat{i} + 5\hat{j} + 7\hat{k}),\ \vec{b_1} = (\hat{i} - 2\hat{j} + 7\hat{k})$$
$$\vec{a_2} = (-\hat{i} - \hat{j} - \hat{k}),\ \vec{b_2} = (7\hat{i} - 6\hat{j} + \hat{k})$$

So,
$$\vec{a_2} - \vec{a_1} = (-\hat{i} - \hat{j} - \hat{k}) - (3\hat{i} + 5\hat{j} + 7\hat{k})$$
$$= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$$
$$= -4\hat{i} - 6\hat{j} - 8\hat{k} = -2(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 7 \\ 7 & -6 & 1 \end{vmatrix}$$
$$= \hat{i}(-2 + 42) - \hat{j}(1 - 49) + \hat{k}(-6 + 14)$$
$$= 40\hat{i} + 48\hat{j} + 8\hat{k}$$
$$= 8(5\hat{i} + 6\hat{j} + \hat{k})$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = \{-2(2\hat{i} + 3\hat{j} + 4\hat{k})\}\{8(5\hat{i} + 6\hat{j} + \hat{k})\}$$
$$= -16[(2)(5) + (3)(6) + (4)(1)]$$
$$= -16[10 + 18 + 4]$$
$$= -16 \times 32$$
$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = -512$$

$$|\vec{b_1} \times \vec{b_2}| = 8\sqrt{(5)^2 + (6)^2 + (1)^2}$$
$$= 8\sqrt{25 + 36 + 1}$$
$$= 8\sqrt{62}$$

Substituting values of $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get required shortest distance between given lines, so

$$S.D. = \left|\frac{-512}{8\sqrt{62}}\right|$$

$$S.D. = \frac{512}{\sqrt{3968}}$$

Straight Line in Space Ex 28.5 Q1(iii)

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

Now, $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} + 4\hat{j} + 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$

$$= 2\hat{i} + 4\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}\left(15 - 16\right) - \hat{j}\left(10 - 12\right) + \hat{k}\left(8 - 9\right)$$

$$\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\left(-\hat{i} + 2\hat{j} - \hat{k}\right)$$

$$= (1)(-1) + (2)(2) + (2)(-1)$$

$$= -1 + 4 - 2$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = 1$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{1 + 4 + 1}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{6}$$

Substituting values of $\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)$ and $\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{1}{\sqrt{6}}\right|$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q1(iv)

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-t)\hat{k}$

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

Above equations can be rewritten as

$\vec{r} = (i - 2j + 3k) + t(-i + j - k)$

$\vec{r} = (i - j - k) + s(i + 2j - 2k)$

Shortest distance is given by $\left|\frac{(a_2 - a_1)\cdot(b_1 \times b_2)}{|(b_1 \times b_2)|}\right|$

$(b_1 \times b_2) = -3j - 3k$

$(a_2 - a_1) = j - 4k$

$(a_2 - a_1)\cdot(b_1 \times b_2) = 9$

$|(b_1 \times b_2)| = 3\sqrt{2}$

Shortest distance is $\frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$

Straight Line in Space Ex 28.5 Q1(v)

We know that, the shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\vec{r} = \left|\frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|}\right| \qquad ---(i)$$

Given equations of lines are,

$\vec{r} = (\lambda - 1)\hat{i} + (\lambda + 1)\hat{j} - (1 + \lambda)\hat{k}$

$\Rightarrow \quad \vec{r} = (-\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - \hat{k})$ and

$\vec{r} = (1 - \mu)\hat{i} + (2\mu - 1)\hat{j} + (\mu + 2)\hat{k}$

$\Rightarrow \quad \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-\hat{i} + 2\hat{j} + \hat{k})$

So, $\vec{a_1} = (-\hat{i} + \hat{j} - \hat{k})$, $\vec{b_1} = (\hat{i} + \hat{j} - \hat{k})$ and

$\vec{a_2} = (\hat{i} - \hat{j} + 2\hat{k})$, $\vec{b_2} = (-\hat{i} + 2\hat{j} + \hat{k})$

$(\vec{a_2} - \vec{a_1}) = (\hat{i} - \hat{j} + 2\hat{k}) - (-\hat{i} + \hat{j} - \hat{k})$

$= \hat{i} - \hat{j} + 2\hat{k} + \hat{i} - \hat{j} + \hat{k}$

$(\vec{a_2} - \vec{a_1}) = 2\hat{i} - 2\hat{j} + 3\hat{k}$

$$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(1+2) - \hat{j}(1-1) + \hat{k}(2+1)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = 3\hat{i} + 3\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(2\hat{i} - 2\hat{j} + 3\hat{k}\right)\left(3\hat{i} + 3\hat{k}\right)$$

$$= (2)(3) + (-2)(0) + (3)(3)$$

$$= 6 + 0 + 9$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = 15$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(3)^2 + (3)^2}$$

$$= \sqrt{18}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = 3\sqrt{2}$$

Substituting values of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between the given lines, so

$$\text{S.D.} = \left|\frac{15}{3\sqrt{2}}\right|$$

$$\text{S.D.} = \frac{5}{\sqrt{2}} \text{ units}$$

Straight Line in Space Ex 28.5 Q1(vi)

We know that, the shortest distance between lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

Given equations of lines are,

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 5\hat{j} + 2\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(2\hat{i} - \hat{j} - \hat{k}\right),\ \vec{b_1} = \left(2\hat{i} - 5\hat{j} + 2\hat{k}\right) \text{ and}$$

$$\vec{a_2} = \left(\hat{i} + 2\hat{j} + \hat{k}\right),\ \vec{b_2} = \left(\hat{i} - \hat{j} + \hat{k}\right)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(\hat{i} + 2\hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} - \hat{k}\right)$$

$$= \hat{i} + 2\hat{j} + \hat{k} - 2\hat{i} + \hat{j} + \hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -5 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}(-5+2) - \hat{j}(2-2) + \hat{k}(-2+5)$$

$$= -3\hat{i} + 3\hat{k}$$

$$\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right) = \left(-\hat{i}+3\hat{j}+2\hat{k}\right)\left(-3\hat{i}+3\hat{k}\right)$$
$$= (-1)(-3)+(3)(0)+(2)(3)$$
$$= 3+0+6$$
$$\left(\vec{a_2}-\vec{a_1}\right)\left(\vec{b_1}\times\vec{b_2}\right) = 9$$

$$\left|\vec{b_1}\times\vec{b_2}\right| = \sqrt{(-3)^2+(3)^2}$$
$$= \sqrt{9+9}$$
$$\left|\vec{b_1}\times\vec{b_2}\right| = 3\sqrt{2}$$

Substituting the values of $\left(\vec{a_2}-\vec{a_1}\right).\left(\vec{b_1}\times\vec{b_2}\right)$ and $\left|\vec{b_1}\times\vec{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$S.D. = \left|\frac{9}{3\sqrt{2}}\right|$$

$$S.D. = \frac{3}{\sqrt{2}}$$

Straight Line in Space Ex 28.5 Q1(vii)

Given,

$$\vec{r} = \hat{i}+\hat{j}+\lambda\left(2\hat{i}-\hat{j}+\hat{k}\right) \qquad ------(i)$$

and

$$\vec{r} = 2\hat{i}+\hat{j}-\hat{k}+\mu\left(3\hat{i}-5\hat{j}+2\hat{k}\right) \qquad ------(ii)$$

Comparing (i) and (ii) with $\vec{r} = \vec{a}_1+\lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2+\mu\vec{b}_2$ respectively, we get

$$\vec{a}_1 = \hat{i}+\hat{j}, \qquad \vec{b}_1 = 2\hat{i}-\hat{j}+\hat{k}$$
$$\vec{a}_2 = 2\hat{i}+\hat{j}-\hat{k}, \qquad \vec{b}_2 = 3\hat{i}-5\hat{j}+2\hat{k}$$

$\therefore \quad \vec{a}_2-\vec{a}_1 = \hat{i}-\hat{k}$

and $\quad \vec{b}_1\times\vec{b}_2 = \left(2\hat{i}-\hat{j}+\hat{k}\right)\times\left(3\hat{i}-5\hat{j}+2\hat{k}\right)$

$$= \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 3 & -5 & 2\end{vmatrix} = 3\hat{i}-\hat{j}-7\hat{k}$$

So, $\quad \left|\vec{b}_1\times\vec{b}_2\right| = \sqrt{9+1+49} = \sqrt{59}$

Hence, the shortest distance between the lines l_1 and l_2 is given by

$$d = \left|\frac{\left(\vec{b}_1\times\vec{b}_2\right).\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right| = \frac{|3-0+7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

Straight Line in Space Ex 28.5 Q1(viii)

The equation of lines are

$\vec{r} = (8+3\lambda)\hat{i} - (9+16\lambda)\hat{j} + (10+7\lambda)\hat{k}$ and $\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$

The lines pass through $\vec{a_1} = 8\hat{i} - 9\hat{j} + 10\hat{k}$ and $\vec{a_2} = 15\hat{i} + 29\hat{j} + 5\hat{k}$

and parallel to vectors, $\vec{b_1} = 3\lambda\hat{i} - 16\lambda\hat{j} + 7\lambda\hat{k}$ and $\vec{b_2} = 3\mu\hat{i} + 8\mu\hat{j} - 5\mu\hat{k}$

$$\vec{a_1} - \vec{a_2} = -7\hat{i} - 38\hat{j} + 5\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{i} + 36\hat{j} + 72\hat{k}$$

So, $\left(\vec{a_1} - \vec{a_2}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = -168 - 1368 + 360 = -1176$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{576 + 1296 + 5184} = 84$$

$$\text{S.D.} = \left|\frac{\left(\vec{a_1} - \vec{a_2}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| = \left|\frac{-1176}{84}\right| = 14$$

Straight Line in Space Ex 28.5 Q2(i)

Given lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \text{ (say)}$$

$$x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 4\lambda + 3$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (4\lambda + 3)\hat{k}$$

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right),\ \vec{b_1} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)$$

and, $\quad \frac{x-2}{3} = \frac{y-3}{4} = \frac{z-5}{5} = \mu$ (say)

$$x = 3\mu + 2,\ y = 4\mu + 3,\ z = 5\mu + 5$$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (3\mu + 2)\hat{i} + (4\mu + 3)\hat{j} + (5\mu + 5)\hat{k}$$

$$\vec{r} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right) + \mu\left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(2\hat{i} + 3\hat{j} + 5\hat{k}\right),\ \vec{b_2} = \left(3\hat{i} + 4\hat{j} + 5\hat{k}\right)$$

We know that, the shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left| \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{|\vec{b_1} \times \vec{b_2}|} \right| \qquad ---(i)$$

$$(\vec{a_2} - \vec{a_1}) = (2\hat{i} + 3\hat{j} + 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$$
$$= 2\hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$(\vec{a_2} - \vec{a_1}) = \hat{i} + \hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$
$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9)$$

$$(\vec{b_1} \times \vec{b_2}) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (\hat{i} + \hat{j} + 2\hat{k})(-\hat{i} + 2\hat{j} - \hat{k})$$
$$= (1)(-1) + (1)(2) + (2)(-1)$$
$$= -1 + 2 - 2$$
$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = -1$$

$$|\vec{b_1} \times \vec{b_2}| = \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{1 + 4 + 1}$$
$$= \sqrt{6}$$

Using the values of $(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$ and $|\vec{b_1} \times \vec{b_2}|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.} = \left| \frac{-1}{\sqrt{3}} \right|$$

$$\text{S.D.} = \frac{1}{\sqrt{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q2(ii)

Given equations of line are,

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \quad (\text{say})$$

$$\Rightarrow \qquad x = 2\lambda + 1,\ y = 3\lambda - 1,\ z = \lambda$$

$$\Rightarrow \qquad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (2\lambda + 1)\hat{i} + (3\lambda - 1)\hat{j} + \lambda\hat{k}$$

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$$

$$\Rightarrow \qquad \overrightarrow{a_1} = \hat{i} - \hat{j},\ \overrightarrow{b_1} = 2\hat{i} + 3\hat{j} + \hat{k}$$

and, $$\frac{x+1}{3} = \frac{y-2}{1} = \mu,\ z = 2$$

$$\Rightarrow \qquad x = 3\mu - 1,\ y = \mu + 2,\ z = 2$$

$$\Rightarrow \qquad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (3\mu - 1)\hat{i} + (\mu + 2)\hat{j} + 2\hat{k}$$

$$\vec{r} = (-\hat{i} + 2\hat{j} + 2\hat{k}) + \mu(3\hat{i} + \hat{j})$$

$$\Rightarrow \qquad \overrightarrow{a_2} = (-\hat{i} + 2\hat{j} + 2\hat{k}),\ \overrightarrow{b_2} = (3\hat{i} + \hat{j})$$

We know that, the shortest distance between two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right| \qquad \text{---(i)}$$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)=\left(\hat{i}-\hat{j}\right)-\left(-\hat{i}+2\hat{j}+2\hat{k}\right)$$
$$=\hat{i}-\hat{j}+\hat{i}-2\hat{j}-2\hat{k}$$
$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right)=2\hat{i}-3\hat{j}-2\hat{k}$$

$$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 1\\ 3 & 1 & 0\end{vmatrix}$$
$$=\hat{i}\,(0-1)-\hat{j}\,(0-3)+\hat{k}\,(2-9)$$

$$\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)=-\hat{i}+3\hat{j}-7\hat{k}$$

$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)=\left(2\hat{i}-3\hat{j}-2\hat{k}\right)\left(-\hat{i}+3\hat{j}-7\hat{k}\right)$$
$$=(2)(-1)+(-3)(3)+(-2)(-7)$$
$$=-2-9+14$$
$$\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)=3$$

$$\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\sqrt{(-1)^2+(3)^2+(-7)^2}=\sqrt{59}$$

Substitute the value of $\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)$ and $\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|$ in equation (i) to get the shortest distance between given lines, so

$$\text{S.D.}=\left|\frac{3}{\sqrt{59}}\right|$$

$$\text{S.D.}=\frac{3}{\sqrt{59}}\text{ units}$$

Straight Line in Space Ex 28.5 Q2(iii)

Given equation of lines are,

$$\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2} = \lambda \text{ (say)}$$

$\Rightarrow \quad x = -\lambda + 1,\ y = \lambda - 2,\ z = -2\lambda + 3$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$= (-\lambda + 1)\hat{i} + (\lambda - 2)\hat{j} + (-2\lambda + 3)\hat{k}$$

$$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(-\hat{i} + \hat{j} - 2\hat{k})$$

$\Rightarrow \quad \vec{a_1} = (\hat{i} - 2\hat{j} + 3\hat{k}),\ \vec{b_1} = (-\hat{i} + \hat{j} - 2\hat{k})$

and, $\quad \frac{x-1}{1} = \frac{y+1}{2} = \frac{z+1}{-2} = \mu \text{ (say)}$

$\Rightarrow \quad x = \mu + 1,\ y = 2\mu - 1,\ z = -2\mu - 1$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$$= (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} + (-2\mu - 1)\hat{k}$$

$$\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - 2\hat{k})$$

$\Rightarrow \quad \vec{a_2} = (\hat{i} - \hat{j} - \hat{k}),\ \vec{b_2} = (\hat{i} + 2\hat{j} - 2\hat{k})$

$$(\vec{a_2} - \vec{a_1}) = (\hat{i} - \hat{j} - \hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k})$$

$$= \hat{i} - \hat{j} - \hat{k} - \hat{i} + 2\hat{j} - 3\hat{k}$$

$$= \hat{j} - 4\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}(-2+4) - \hat{j}(2+2) + \hat{k}(-2-1)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$$

$$= \sqrt{29}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{j} - 4\hat{k}\right)\left(2\hat{i} - 4\hat{j} - 3\hat{k}\right)$$

$$= (0)(2) + (1)(-4) + (-4)(-3)$$

$$= 0 - 4 + 12$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = 8$$

We know that, shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

So, shortest distance between given lines is

$$\text{S.D.} = \left|\frac{8}{\sqrt{29}}\right|$$

$$\text{S.D.} = \frac{8}{\sqrt{29}} \text{ units}$$

Straight Line in Space Ex 28.5 Q2(iv)

Given equation of lines are,

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} = \lambda \quad \text{(say)}$$

$\Rightarrow \quad x = \lambda + 3,\ y = -2\lambda + 5,\ z = \lambda + 7$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (\lambda + 3)\hat{i} + (-2\lambda + 5)\hat{j} + (\lambda + 7)\hat{k}$$
$$\vec{r} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right),\ \vec{b_1} = \left(\hat{i} - 2\hat{j} + \hat{k}\right)$$

and, $\dfrac{x+1}{7} = \dfrac{y+1}{-6} = \dfrac{z+1}{1} = \mu \quad \text{(say)}$

$\Rightarrow \quad x = 7\mu - 1,\ y = -6\mu - 1,\ z = \mu - 1$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$= (7\mu - 1)\hat{i} + (-6\mu - 1)\hat{j} + (\mu - 1)\hat{k}$$
$$\vec{r} = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$$

$$\Rightarrow \quad \vec{a_2} = \left(-\hat{i} - \hat{j} - \hat{k}\right),\ \vec{b_2} = 7\hat{i} - 6\hat{j} + \hat{k}$$

We know that, shortest distance between $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by,

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(-\hat{i} - \hat{j} - \hat{k}\right) - \left(3\hat{i} + 5\hat{j} + 7\hat{k}\right)$$

$$= -\hat{i} - \hat{j} - \hat{k} - 3\hat{i} - 5\hat{j} - 7\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 1 \\ 7 & -6 & 1 \end{vmatrix}$$

$$= \hat{i}\left(-2+6\right) - \hat{j}\left(1-7\right) + \hat{k}\left(-6+14\right)$$

$$\vec{b_1} \times \vec{b_2} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(4)^2 + (6)^2 + (8)^2}$$

$$= \sqrt{16 + 36 + 64}$$

$$= \sqrt{116}$$

$$= 2\sqrt{29}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(-4\hat{i} - 6\hat{j} - 8\hat{k}\right)\left(4\hat{i} + 6\hat{j} + 8\hat{k}\right)$$

$$= (-4)(4) + (-6)(6) + (-8)(8)$$

$$= -16 - 36 - 64$$

$$= -116$$

Substituting the values of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get the shortest distance between the two given lines, so

$$\text{S.D.} = \left|\frac{-116}{2\sqrt{29}}\right|$$

$$= \frac{58}{\sqrt{29}}$$

Straight Line in Space Ex 28.5 Q3(i)

Given equations of lines are,

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right)$$

$$\Rightarrow \quad \overrightarrow{a_1} = \left(\hat{i} - \hat{j}\right),\ \overrightarrow{b_1} = \left(2\hat{i} + \hat{k}\right)$$

and, $\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{j} - \hat{k}\right)$

$$\Rightarrow \quad \overrightarrow{a_2} = \left(2\hat{i} - \hat{j}\right),\ \overrightarrow{b_2} = \left(\hat{i} + \hat{j} - \hat{k}\right)$$

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right).\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} - \hat{j}\right) - \left(\hat{i} - \hat{j}\right)$$
$$= 2\hat{i} - \hat{j} - \hat{i} + \hat{j}$$
$$= \hat{i}$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$
$$= \hat{i}\left(0 - 1\right) - \hat{j}\left(-2 - 1\right) + \hat{k}\left(2 - 0\right)$$
$$= -\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = \left(\hat{i}\right)\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$$
$$= (1)(-1) + (0)(3) + (0)(2)$$
$$= -1 + 0 + 0$$
$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right) = -1$$

$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{(-1)^2 + (3)^2 + (2)^2}$$
$$= \sqrt{1 + 9 + 4}$$
$$\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right| = \sqrt{14}$$

So, shortest distance between the given lines using equation (1) is,

$$\text{S.D.} = \left|\frac{-1}{\sqrt{14}}\right|$$

$$= \frac{1}{\sqrt{14}} \text{ units}$$

$$\text{S.D} \neq 0$$

Since, shortest distance between lines is not zero, so lines are not intersecting.

Straight Line in Space Ex 28.5 Q3(ii)

Given equations of lines are,

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$

$$\Rightarrow \quad \vec{a_1} = \left(\hat{i} + \hat{j} - \hat{k}\right), \vec{b_1} = \left(3\hat{i} - \hat{j}\right)$$

and, $\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$

$$\Rightarrow \quad \vec{a_2} = \left(4\hat{i} - \hat{k}\right), \vec{b_2} = \left(2\hat{i} + 3\hat{k}\right)$$

We know that, shortest distance between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$S.D. = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| \quad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(4\hat{i} - \hat{k}\right) - \left(\hat{i} + \hat{j} - \hat{k}\right)$$
$$= 4\hat{i} - \hat{k} - \hat{i} - \hat{j} + \hat{k}$$
$$= 3\hat{i} - \hat{j}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$$
$$= \hat{i}(-3 - 0) - \hat{j}(9 - 0) + \hat{k}(0 + 2)$$
$$= -3\hat{i} - 9\hat{j} + 2\hat{k}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-3)^2 + (-9)^2 + (2)^2}$$
$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{9 + 81 + 4}$$
$$= \sqrt{94}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(3\hat{i} - \hat{j}\right)\left(-3\hat{i} - 9\hat{j} + 2\hat{k}\right)$$
$$= (3)(-3) + (-1)(-9) + (0)(2)$$
$$= -9 + 9 + 0$$
$$= 0$$

Using $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get shortest distance between given lines, so

$$S.D. = \left|\frac{0}{\sqrt{94}}\right|$$

$$S.D = 0$$

Since, shortest distance between the given lines is zero, so lines are intersecting.

Straight Line in Space Ex 28.5 Q3(iii)

Given equations of lines are,

$$\frac{x-1}{2}=\frac{y+1}{3}=\frac{z}{1}=\lambda \ \text{(say)}$$

$\Rightarrow \quad x = 2\lambda+1,\ y = 3\lambda-1,\ z=\lambda$

$$\Rightarrow \quad \vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$$
$$= (2\lambda+1)\hat{i}+(3\lambda-1)\hat{j}+(\lambda)\hat{k}$$
$$\vec{r} = (\hat{i}-\hat{j})+\lambda(2\hat{i}+3\hat{j}+\hat{k})$$

$\Rightarrow \quad \overrightarrow{a_1} = (\hat{i}-\hat{j}),\ \overrightarrow{b_1} = (2\hat{i}+3\hat{j}+\hat{k})$

and, $\quad \frac{x+1}{5}=\frac{y-2}{1}=\mu\ \text{(say)},\ z=2$

$\Rightarrow \quad x = 5\mu-1,\ y=\mu+2,\ z=2$

$$\Rightarrow \quad \vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$$
$$= (5\mu-1)\hat{i}+(\mu+2)\hat{j}+2\hat{k}$$
$$\vec{r} = (-\hat{i}+2\hat{j}+2\hat{k})+\mu(5\hat{i}+\hat{j})$$

$\Rightarrow \quad \overrightarrow{a_2} = (-\hat{i}+2\hat{j}+2\hat{k}),\ \overrightarrow{b_2} = (5\hat{i}+\hat{j})$

We know that, the shortest distance between $\vec{r}=\overrightarrow{a_1}+\lambda\overrightarrow{b_1}$ and $\vec{r}=\overrightarrow{a_2}+\lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{(\overrightarrow{a_2}-\overrightarrow{a_1})\cdot(\overrightarrow{b_1}\times\overrightarrow{b_2})}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(-\hat{i} + 2\hat{j} + 2\hat{k}\right) - \left(\hat{i} - \hat{j}\right)$$
$$= -\hat{i} + 2\hat{j} + 2\hat{k} - \hat{i} + \hat{j}$$
$$= -2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 5 & 1 & 0 \end{vmatrix}$$
$$= \hat{i}\left(0 - 1\right) - \hat{j}\left(0 - 5\right) + \hat{k}\left(2 - 15\right)$$
$$\vec{b_1} \times \vec{b_2} = -\hat{i} + 5\hat{j} - 13\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right)\left(\vec{b_1} \times \vec{b_2}\right) = \left(-2\hat{i} + 3\hat{j} + 2\hat{k}\right)\left(-\hat{i} + 5\hat{j} - 13\hat{k}\right)$$
$$= (-2)(-1) + (3)(5) + (2)(-13)$$
$$= -9$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-1)^2 + (5)^2 + (-13)^2}$$
$$= \sqrt{1 + 25 + 169}$$
$$= \sqrt{195}$$

Substituting the value of $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (i) to get shortest distance between given lines, so

$$\text{S.D.} = \left|\frac{-9}{\sqrt{195}}\right|$$
$$= \frac{9}{\sqrt{195}} \text{ units}$$

Since, shortest distance between given lines is not zero, so lines are not intersecting.

Straight Line in Space Ex 28.5 Q3(iv)

Given lines are,

$$\frac{x-5}{4} = \frac{y-7}{-5} = \frac{z+3}{-5} = \lambda \ \text{(say)}$$

$\Rightarrow \quad x = 4\lambda + 5,\ y = -5\lambda + 7,\ z = -5\lambda - 3$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$= (4\lambda + 5)\hat{i} + (-5\lambda + 7)\hat{j} + (-5\lambda - 3)\hat{k}$

$\vec{r} = \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} - 5\hat{j} - 5\hat{k}\right)$

$\Rightarrow \quad \overrightarrow{a_1} = \left(5\hat{i} + 7\hat{j} - 3\hat{k}\right),\ \overrightarrow{b_1} = \left(4\hat{i} - 5\hat{j} - 5\hat{k}\right)$

and, $\quad \frac{x-8}{7} = \frac{y-7}{1} = \frac{z-5}{3} = \mu \ \text{(say)}$

$\Rightarrow \quad x = 7\mu + 8,\ y = \mu + 7,\ 3\mu + 5$

$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$= (7\mu + 8)\hat{i} + (\mu + 7)\hat{j} + (3\mu + 5)\hat{k}$

$\vec{r} = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right) + \mu\left(7\hat{i} + \hat{j} + 3\hat{k}\right)$

$\Rightarrow \quad \overrightarrow{a_2} = \left(8\hat{i} + 7\hat{j} + 5\hat{k}\right),\ \overrightarrow{b_2} = \left(7\hat{i} + \hat{j} + 3\hat{k}\right)$

We know that, shortest distance between lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ is given by

$$\text{S.D.} = \left|\frac{\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\cdot\left(\overrightarrow{b_1} \times \overrightarrow{b_2}\right)}{\left|\overrightarrow{b_1} \times \overrightarrow{b_2}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2}-\vec{a_1}\right) = \left(8\hat{i}+7\hat{j}+5\hat{k}\right)-\left(5\hat{i}+7\hat{j}-3\hat{k}\right)$$
$$= 8\hat{i}+7\hat{j}+5\hat{k}-5\hat{i}-7\hat{j}+3\hat{k}$$
$$= 3\hat{i}+8\hat{k}$$

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -5 \\ 7 & 1 & 3 \end{vmatrix}$$
$$= \hat{i}\left(-15+5\right)-\hat{j}\left(12+35\right)+\hat{k}\left(4+35\right)$$
$$= -10\hat{i}-47\hat{j}+39\hat{k}$$

$$\left(\vec{a_2}-\vec{a_1}\right)\left(\vec{b_1}\times\vec{b_2}\right) = \left(3\hat{i}+8\hat{k}\right)\left(-10\hat{i}-47\hat{j}+39\hat{k}\right)$$
$$= (3)(-10)+(0)(-4)+(8)(39)$$
$$= -30+312$$
$$= 282$$

Using equation (i) to get the shortest distance between the given lines, so

$$S.D. = \left|\frac{282}{\left|\vec{b_1}\times\vec{b_2}\right|}\right|$$

$$S.D. \neq 0$$

Since, the shortest distance between given lines is not equal to zero, so

Given lines are not intersecting.

Straight Line in Space Ex 28.5 Q4(i)

Given, equation of lines are,

$$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(1)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(-\hat{i} + \hat{j} - \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \mu\left(\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu'\left(\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(2)$$

These two lines passes through the points having position vectors $\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$ respectively and both are parallel to the vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$

We know that, shortest distance between parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ is given by

$$\text{S.D.} = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right).\vec{b}}{\left|\vec{b}\right|}\right| \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

$$= 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \hat{i} - 3\hat{j} - 4\hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & -4 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(-3 - 4\right) - \hat{j}\left(1 + 4\right) + \hat{k}\left(-1 + 3\right)$$

$$\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = -7\hat{i} - 5\hat{j} + 2\hat{k}$$

$$\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right| = \sqrt{\left(-7\right)^2 + \left(-5\right)^2 + \left(2\right)^2}$$

$$= \sqrt{49 + 25 + 4}$$

$$= \sqrt{78}$$

$$\left|\vec{b}\right| = \sqrt{\hat{i} - \hat{j} + \hat{k}}$$

$$= \sqrt{\left(1\right)^2 + \left(-1\right)^2 + \left(1\right)^2}$$

$$\left|\vec{b}\right| = \sqrt{3}$$

Using $\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (1) to get the shortest distance between parallel lines, so

$$\text{S.D.} = \frac{\sqrt{78}}{\sqrt{3}}$$

$$\text{S.D.} = \sqrt{\frac{78}{3}}$$

$$\text{S.D.} = \sqrt{26} \text{ units}$$

Straight Line in Space Ex 28.5 Q4(ii)

Given, equation of lines are,

$$\vec{r} = \left(\hat{i} + \hat{j}\right) + \lambda\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(1)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(4\hat{i} - 2\hat{j} + 2\hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + 2\mu\left(2\hat{i} - \hat{j} + \hat{k}\right)$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu'\left(2\hat{i} - \hat{j} + \hat{k}\right) \qquad ---(2)$$

So, $\overrightarrow{a_1} = \left(\hat{i} + \hat{j}\right)$, $\overrightarrow{a_2} = 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

We know that, the shortest distance between the parallel lines $\vec{r} = \overrightarrow{a_1} + \lambda\vec{b}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\vec{b}$ is given by

$$\text{S.D.} = \frac{\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right)\vec{b}\right|}{\left|\vec{b}\right|} \qquad ---(i)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \left(2\hat{i} + \hat{j} - \hat{k}\right) - \left(\hat{i} + \hat{j}\right)$$

$$= 2\hat{i} + \hat{j} - \hat{k} - \hat{i} - \hat{j}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) = \hat{i} - \hat{k}$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(0 - 1\right) - \hat{j}\left(1 + 2\right) + \hat{k}\left(-1 - 0\right)$$

$$\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b} = -\hat{i} - 3\hat{j} - \hat{k}$$

$$\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right| = \sqrt{(-1)^2 + (-3)^2 + (-1)^2}$$

$$= \sqrt{1 + 9 + 1}$$

$$\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right| = \sqrt{11}$$

$$\left|\vec{b}\right| = \sqrt{(2)^2 + (-1)^2 + (1)^2}$$

$$= \sqrt{4 + 1 + 1}$$

$$\left|\vec{b}\right| = \sqrt{6}$$

Using $\left|\left(\overrightarrow{a_2} - \overrightarrow{a_1}\right) \times \vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (1) to get the shortest distance between the given lines, so

$$\text{S.D.} = \frac{\sqrt{11}}{\sqrt{6}}$$

$$\text{S.D.} = \sqrt{\frac{11}{6}} \text{ units}$$

Straight Line in Space Ex 28.5 Q5

Equation of line passing through $(0,0,0)$ and $(1,0,2)$ is given by $\vec{r} = \vec{a} + \lambda\left(\vec{b} - \vec{a}\right)$

$$\vec{r} = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right) + \lambda\left((1-0)\hat{i} + (0-0)\hat{j} + (2-0)\hat{k}\right)$$

$$\vec{r} = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{k}\right) \qquad ---(1)$$

Equation of another line passing through $(1,3,0)$ and $(0,3,0)$ is

$$\vec{r} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right) + \mu\left((0-1)\hat{i} + (3-3)\hat{j} + (0-0)\hat{k}\right)$$

$$\vec{r} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right) + \mu\left(-\hat{i}\right) \qquad ---(2)$$

From equation (1) and (2)

$$\vec{a_1} = \left(0.\hat{i} + 0.\hat{j} + 0.\hat{k}\right),\ \vec{b_1} = \left(\hat{i} + 2\hat{k}\right)$$

$$\vec{a_2} = \left(\hat{i} + 3\hat{j} + 0\hat{k}\right), \quad \vec{b_2} = -\hat{i}$$

We know that, shortest distance between the lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$ is given by

$$\text{S.D.} = \frac{\left|\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)\right|}{\left|\vec{b_1} \times \vec{b_2}\right|} \qquad ---(3)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(\hat{i} + 3\hat{j} + 0.\hat{k}\right) - \left(0.\hat{i} + 0.\hat{j} + 0\hat{k}\right)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(\hat{i} + 3\hat{j}\right)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -1 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(0+2) + \hat{k}(-2)$$

$$\left(\vec{b_1} \times \vec{b_2}\right) = -2\hat{j}$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{i} + 3\hat{j}\right)\left(-2\hat{j}\right)$$

$$= (1)(0) + (3)(-2)$$

$$\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right) = -6$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{(-2)^2}$$

$$\left|\vec{b_1} \times \vec{b_2}\right| = 2$$

Using $\left(\vec{a_2} - \vec{a_1}\right).\left(\vec{b_1} \times \vec{b_2}\right)$ and $\left|\vec{b_1} \times \vec{b_2}\right|$ in equation (1) to get shortest distance between the lines, so

$$\text{S.D.} = \left|\frac{-6}{2}\right|$$

$$\text{S.D.} = 3 \text{ units}$$

Straight Line in Space Ex 28.5 Q6

Given equations of lines are,

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6} = \lambda \ \text{(say)}$$

$\Rightarrow \quad x = 2\lambda + 1,\ y = 3\lambda + 2,\ z = 6\lambda - 4$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (2\lambda + 1)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda - 4)\hat{k}$$

$$\vec{r} = \left(\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$\Rightarrow \quad \overrightarrow{a_1} = \hat{i} + 2\hat{j} - 4\hat{k},\ \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

Another equation of line is,

$$\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12} = \mu \ \text{(say)}$$

$\Rightarrow \quad x = 4\mu + 3,\ y = 6\mu + 3,\ 12\mu - 5$

$$\Rightarrow \quad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= (4\mu + 3)\hat{i} + (6\mu + 3)\hat{j} + (12\mu - 5)\hat{k}$$

$$= \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu\left(4\hat{i} + 6\hat{j} + 12\hat{k}\right)$$

$$= \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + 2\mu\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$$\vec{r} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) + \mu'\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$$

$\Rightarrow \quad \overrightarrow{a_2} = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right),\ \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

We know that, shortest distance between parallel lines $\vec{r} = \vec{a_1} + \lambda\vec{b}$ and $\vec{r} = \vec{a_2} + \mu\vec{b}$ is given by

$$S.D. = \frac{\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|}{\left|\vec{b}\right|} \qquad ---(i)$$

$$\left(\vec{a_2} - \vec{a_1}\right) = \left(3\hat{i} + 3\hat{j} - 5\hat{k}\right) - \left(\hat{i} + 2\hat{j} - 4\hat{k}\right)$$
$$= 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$$
$$\left(\vec{a_2} - \vec{a_1}\right) = 2\hat{i} + \hat{j} - \hat{k}$$

$$\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$
$$= \hat{i}\left(6 + 3\right) - \hat{j}\left(12 + 2\right) + \hat{k}\left(6 - 2\right)$$
$$= 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right| = \sqrt{\left(9\right)^2 + \left(-14\right)^2 + \left(4\right)^2}$$
$$= \sqrt{81 + 196 + 16}$$
$$= \sqrt{293}$$

$$\left|\vec{b}\right| = \sqrt{\left(2\right)^2 + \left(3\right)^2 + \left(6\right)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49}$$
$$\left|\vec{b}\right| = 7$$

Using $\left|\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}\right|$ and $\left|\vec{b}\right|$ in equation (i) to get the shortest distance between given lines, so

$$S.D. = \frac{\sqrt{293}}{7} \text{ units}$$

Straight Line in Space Ex 28.5 Q7(i)

Here,

$a_1 = \hat{i} + 2\hat{j} + \hat{k}$

$b_1 = \hat{i} - \hat{j} + \hat{k}$

$a_2 = 2\hat{i} - \hat{j} - \hat{k}$

$b_2 = 2\hat{i} + \hat{j} + 2\hat{k}$

$$\vec{a}_2 - \vec{a}_1 = 2\hat{i} - \hat{j} - \hat{k} - \hat{i} - 2\hat{j} - \hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = -3\hat{i} + 3\hat{k}$$

The shortest distance between the two lines,

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

$$d = \left|\frac{(-3\hat{i} + 3\hat{k}).(\hat{i} - 3\hat{j} - 2\hat{k})}{|-3\hat{i} - 3\hat{k}|}\right| = \left|\frac{-3-6}{\sqrt{(-3)^2 + (-3)^2}}\right| = \frac{9}{3\sqrt{2}}$$

The shortest distance between the two lines $= \frac{3}{\sqrt{2}}$ units

Straight Line in Space Ex 28.5 Q7(ii)

Here,

$\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}$

$\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$

$\vec{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$

$\vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$= \hat{i}(-6+2) - \hat{j}(7-1) + \hat{k}(-14+6)$

$= -4\hat{i} - 6\hat{j} - 8\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i}(3+1) + \hat{j}(5+1) + \hat{k}(7+1)$

$= 4\hat{i} + 6\hat{j} + 8\hat{k}$

The shortest distance between two lines,

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|b_1 \times b_2|} \right|$$

$$= \left| \frac{(-4\hat{i} - 6\hat{j} - 8\hat{k}).(4\hat{i} + 6\hat{j} + 8\hat{k})}{\sqrt{(-4)^2 + (-6)^2 + (-8)^2}} \right|$$

$$= \left| \frac{-16 - 36 - 64}{\sqrt{116}} \right|$$

$$= \left| \frac{-116}{\sqrt{116}} \right|$$

$= 2\sqrt{29}$ units

Straight Line in Space Ex 28.5 Q7(iii)

Here,

$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$,

$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$,

$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

$(\vec{a}_2 - \vec{a}_1) = 4\hat{i} + 5\hat{j} + 6\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$\vec{b}_1 \times \vec{b}_2$

$= (\hat{i} - 3\hat{j} + 2\hat{k}) \times (2\hat{i} + 3\hat{j} + \hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= \hat{i}(-3-6) - \hat{j}(1-4) + \hat{k}(3+6)$

$= -9\hat{i} + 3\hat{j} + 9\hat{k}$

Shortest distance between the two lines $= \left| \frac{(a_2 - a_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$

$$= \left| \frac{(3\hat{i} + 3\hat{j} + 3\hat{k}).(-9\hat{i} + 3\hat{j} + 9\hat{k})}{|-9\hat{i} + 3\hat{j} + 9\hat{k}|} \right|$$

$$= \left| \frac{3 \times (-9) + 3 \times 3 + 3 \times 9}{\sqrt{(-9)^2 + 3^2 + 9^2}} \right|$$

$$= \left| \frac{-27 + 9 + 27}{\sqrt{(-9)^2 + 3^2 + 9^2}} \right|$$

$$= \left| \frac{9}{\sqrt{171}} \right| = \frac{3}{\sqrt{19}} \text{ units}$$

Straight Line in Space Ex 28.5 Q7(iv)

Here,

$\vec{a}_1 = 6\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{a}_2 = -4\hat{i} - \hat{k}$

$\vec{b}_1 = \hat{i} - 2\hat{j} + 2\hat{k}$

$\vec{b}_2 = 3\hat{i} - 2\hat{j} - 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = -4\hat{i} - \hat{k} - 6\hat{i} - 2\hat{j} - 2\hat{k}$

$= -10\hat{i} - 2\hat{j} - 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix}$$

$= \hat{i}(4+4) - \hat{j}(-2-6) + \hat{k}(-2+6)$

$= 8\hat{i} + 8\hat{j} + 4\hat{k}$

$$\text{Shortest Distance} = \left| \frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \left| \frac{(-10\hat{i} - 2\hat{j} - 3\hat{k}).(8\hat{i} + 8\hat{j} + 4\hat{k})}{|8\hat{i} - 8\hat{j} + 4\hat{k}|} \right|$$

$$= \left| \frac{(-10) \times 8 + (-2) \times 8 + (-3) \times 4}{\sqrt{8^2 + (-8)^2 + (-4)^2}} \right|$$

$$= \left| \frac{-80 - 16 - 12}{\sqrt{64 + 64 + 16}} \right| = \left| \frac{-108}{\sqrt{144}} \right| = 9 \text{ units}$$

Straight Line in Space Ex 28.5 Q8

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} - 5\hat{k} - \hat{i} - 2\hat{j} + 4\hat{k}$

$= 2\hat{i} + \hat{j} - \hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$= \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2)$

$= 9\hat{i} - 14\hat{j} + 4\hat{k}$

Shortest distance betweeen 2lines

$$= \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

$$= \left| \frac{9\hat{i} - 14\hat{j} + 4\hat{k}}{\left|\sqrt{2^2 + 3^2 + 6^2}\right|} \right|$$

$$= \left| \frac{9\hat{i} - 14\hat{j} + 4\hat{k}}{\sqrt{49}} \right|$$

$$= \left| \frac{\sqrt{9^2 + (-14)^2 + 4^2}}{\sqrt{49}} \right|$$

$$= \left| \frac{\sqrt{293}}{\sqrt{49}} \right| = \frac{\sqrt{293}}{7} \text{units}$$