

**RD Sharma**  
**Solutions**  
**Class 12 Maths**  
**Chapter 28**  
**Ex 28.4**

Let foot of the perpendicular drawn from the point  $P(1, 0, 0)$  to the line  $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$  is  $Q$ . We have to find length of  $PQ$ .

$Q$  is a general point on the line,

$$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8} = \lambda \text{ (say)}$$

Coordinate of  $Q = (2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

Direction ratios line  $PQ$  are

$$= (2\lambda + 1 - 1), (-3\lambda - 1 - 0), (8\lambda - 10 - 0)$$

$$\Rightarrow = (2\lambda), (-3\lambda - 1), (8\lambda - 10)$$

Since, line  $PQ$  is perpendicular to the given line, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$(2)(2\lambda) + (-3)(-3\lambda - 1) + 8(8\lambda - 10) = 0$$

$$4\lambda + 9\lambda + 3 + 64\lambda - 80 = 0$$

$$77\lambda - 77 = 0$$

$$\lambda = 1$$

Therefore, coordinate of  $Q$  is  $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

$$= (2(1) + 1, -3(1) - 1, 8(1) - 10)$$

$$= (3, -4, -2)$$

Therefore, coordinate of  $Q$  is  $(2\lambda + 1, -3\lambda - 1, 8\lambda - 10)$

$$= (2(1) + 1, -3(1) - 1, 8(1) - 10)$$

$$= (3, -4, -2)$$

$$\begin{aligned} PQ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\ &= \sqrt{(1 - 3)^2 + (0 + 4)^2 + (0 + 2)^2} \\ &= \sqrt{4 + 16 + 4} \\ &= \sqrt{24} \\ &= 2\sqrt{6} \end{aligned}$$

So, foot of perpendicular =  $(3, -4, -2)$

length of perpendicular =  $2\sqrt{6}$  units

Let the foot of the perpendicular drawn from  $A(1,0,3)$  to the line joining the points  $B(4,7,1)$

And  $C(3,5,3)$  be  $D$

Equation of line passing through  $B(4,7,1)$  and  $C(3,5,3)$  is

$$\begin{aligned}\frac{x-x_1}{x_2-x_1} &= \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \\ \Rightarrow \frac{x-4}{3-4} &= \frac{y-7}{5-7} = \frac{z-1}{3-1} \\ \Rightarrow \frac{x-4}{-1} &= \frac{y-7}{-2} = \frac{z-1}{2} = \lambda \text{ (say)}\end{aligned}$$

Direction ratio of  $AD$  are

$$\begin{aligned}(-\lambda+4-1), (-2\lambda+7-0), (2\lambda+1-3) \\ = (-\lambda+3), (-2\lambda+7), (2\lambda-2)\end{aligned}$$

Line  $AD$  is perpendicular to  $BC$  so

$$\begin{aligned}a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ \Rightarrow (-1)(-\lambda+3) + (-2)(-2\lambda+7) + 2(2\lambda-2) &= 0 \\ \Rightarrow \lambda - 3 + 4\lambda - 14 + 4\lambda - 4 &= 0 \\ \Rightarrow 9\lambda - 21 &= 0 \\ \Rightarrow \lambda &= \frac{21}{9}\end{aligned}$$

Co-ordinates of  $D$  are

$$\begin{aligned}&= \left( -\frac{21}{9} + 4, (-2)\left(\frac{21}{9} + 7\right), 2\left(\frac{21}{9} + 1\right) \right) \\ &= \left( \frac{15}{9}, \frac{21}{9}, \frac{51}{9} \right) \\ &= \left( \frac{5}{3}, \frac{7}{3}, \frac{17}{3} \right)\end{aligned}$$

### Straight Line in Space Ex 28.4 Q4

Given that  $D$  is the foot of perpendicular from  $A(1, 0, 4)$  on  $BC$ , so

Equation of line passing through  $B, C$  is

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \frac{x-0}{2-0} = \frac{y+11}{-3+11} = \frac{z-3}{1-3}$$

$$\Rightarrow \frac{x}{2} = \frac{y+11}{8} = \frac{z-3}{-2} = \lambda \text{ (say)}$$

Coordinate of  $D = (2\lambda, 8\lambda - 11, -2\lambda + 3)$

Direction ratios of  $AD = 2\lambda - 1, 8\lambda - 11 - 0, -2\lambda + 3 - 4$   
 $= (2\lambda - 1), (8\lambda - 11), (-2\lambda - 1)$

Since, line  $AD$  is perpendicular on  $BC$ , so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\begin{aligned} \Rightarrow & (2)(2\lambda - 1) + (8)(8\lambda - 11) + (-2)(-2\lambda - 1) = 0 \\ \Rightarrow & 4\lambda - 2 + 64\lambda - 88 + 4\lambda + 2 = 0 \\ \Rightarrow & 72\lambda - 88 = 0 \\ \Rightarrow & \lambda = \frac{88}{72} \\ & \lambda = \frac{11}{9} \end{aligned}$$

$$\begin{aligned} \text{Coordinate of } D &= (2\lambda, 8\lambda - 11, -2\lambda + 3) \\ &= \left( 2\left(\frac{11}{9}\right), 8\left(\frac{11}{9}\right) - 11, -2\left(\frac{11}{9}\right) + 3 \right) \end{aligned}$$

$$\text{Coordinate of } D = \left( \frac{22}{9}, \frac{-11}{9}, \frac{5}{9} \right)$$

### Straight Line in Space Ex 28.4 Q5

Let foot of the perpendicular from  $P(2, 3, 4)$  is  $\theta$  on the line  $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ , so

Equation of given line is,

$$\begin{aligned} \frac{4-x}{2} &= \frac{y}{6} = \frac{1-z}{3} \\ \frac{x-4}{-2} &= \frac{y}{6} = \frac{z-1}{-3} = \lambda \text{ (say)} \end{aligned}$$

$$\text{Coordinate of } Q = (-2\lambda + 4, 6\lambda, -3\lambda + 1)$$

$$\begin{aligned} \text{Direction ratios of } PQ &= (-2\lambda + 4 - 2), (6\lambda - 3), (-3\lambda + 1 - 4) \\ &= (-2\lambda + 2), (6\lambda - 3), (-3\lambda - 3) \end{aligned}$$

Line  $PQ$  is perpendicular to given line, so

$$\begin{aligned} a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ (-2)(-2\lambda + 2) + (6)(6\lambda - 3) + (-3)(-3\lambda - 3) &= 0 \\ 4\lambda - 4 + 36\lambda - 18 + 9\lambda + 9 &= 0 \\ 49\lambda - 13 &= 0 \\ \lambda &= \frac{13}{49} \end{aligned}$$

$$\begin{aligned} \text{Coordinate of } Q &= (-2\lambda + 4, 6\lambda, -3\lambda + 1) \\ &= \left( -2\left(\frac{13}{49}\right) + 4, 6\left(\frac{13}{49}\right), -3\left(\frac{13}{49}\right) + 1 \right) \\ &= \left( \frac{-26 + 196}{49}, \frac{78}{49}, \frac{-39 + 49}{49} \right) \end{aligned}$$

$$\text{Coordinate of } Q = \left( \frac{170}{49}, \frac{78}{49}, \frac{10}{49} \right)$$

$$\begin{aligned} PQ &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \\ &= \sqrt{\left(\frac{170}{49} - 2\right)^2 + \left(\frac{78}{49} - 3\right)^2 + \left(\frac{10}{49} - 4\right)^2} \\ &= \sqrt{\left(\frac{72}{49}\right)^2 + \left(\frac{69}{49}\right)^2 + \left(-\frac{168}{49}\right)^2} \\ &= \sqrt{\frac{5184 + 4761 + 34596}{2401}} \\ &= \sqrt{\frac{44541}{2401}} \\ &= \sqrt{\frac{909}{49}} \\ &= \frac{3\sqrt{101}}{49} \end{aligned}$$

Perpendicular distance from  $(2, 3, 4)$  to given line is  $\frac{3\sqrt{101}}{49}$  units.

### Straight Line in Space Ex 28.4 Q6

Let  $\theta$  be the foot of the perpendicular drawn from  $P(2, 4, -1)$  to the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$$

Given line is  $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$  (say)

Coordinate of  $Q$  (General point on the line)  
 $= (\lambda - 5, 4\lambda - 3, -9\lambda + 6)$

Direction ratios of  $PQ$  are  
 $= (\lambda - 5 - 2), (4\lambda - 3 - 4), (-9\lambda + 6 + 1)$   
 $= \lambda - 7, 4\lambda - 7, -9\lambda + 7$

Line  $PQ$  is perpendicular to the given line, so

$$\begin{aligned} a_1a_2 + b_1b_2 + c_1c_2 &= 0 \\ (1)(\lambda - 7) + (4)(4\lambda - 7) + (-9)(-9\lambda + 7) &= 0 \\ \lambda - 7 + 16\lambda - 28 + 81\lambda - 63 &= 0 \\ 98\lambda - 98 &= 0 \\ \lambda &= 1 \end{aligned}$$

$\therefore$  Coordinate of  $Q = (\lambda - 5, 4\lambda - 3, -9\lambda + 6)$   
 $= (1 - 5, 4(1) - 3, -9(1) + 6)$

Coordinate of foot of perpendicular =  $(-4, 1, -3)$

So, equation of the perpendicular  $PQ$  is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \frac{x - 2}{-4 - 2} = \frac{y - 4}{1 - 4} = \frac{z + 1}{-3 + 1}$$

$$\Rightarrow \frac{x - 2}{-6} = \frac{y - 4}{-3} = \frac{z + 1}{-2}$$

### Straight Line in Space Ex 28.4 Q7

Let foot of the perpendicular drawn from  $P(5, 4, -1)$  to the given line is  $Q$ , so

Given equation of line is,

$$\vec{r} = \hat{i} + \lambda(2\hat{i} + 9\hat{j} + 5\hat{k})$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) = (1 + 2\lambda)\hat{i} + (9\lambda)\hat{j} + (5\lambda)\hat{k}$$

Equation the coefficients of  $\hat{i}$ ,  $\hat{j}$  and  $\hat{k}$

$$\Rightarrow x = 1 + 2\lambda, y = 9\lambda, z = 5\lambda$$

$$\Rightarrow \frac{x - 1}{2} = \lambda, \frac{y}{9} = \lambda, \frac{z}{5} = \lambda$$

$$\Rightarrow \frac{x - 1}{2} = \frac{y}{9} = \frac{z}{5} = \lambda \text{ (say)}$$

Coordinate of  $Q = (2\lambda + 1, 9\lambda, 5\lambda)$

Direction ratios of line  $PQ$  are

$$(2\lambda + 1 - 5), 9\lambda - 4, 5\lambda + 1$$

$$\Rightarrow 2\lambda - 4, 9\lambda - 4, 5\lambda + 1$$

### Straight Line in Space Ex 28.4 Q8

Let position vector of foot of perpendicular drawn from  $P(\hat{i} + 6\hat{j} + 3\hat{k})$  on

$\vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$  be  $Q(\vec{q})$ . So

$$Q \text{ is on the line } \vec{r} = (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$$

$$\text{So, Position vector of } Q = (\lambda)\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k}$$

$$\begin{aligned}\overrightarrow{PQ} &= \text{Position vector of } Q - \text{Position vector of } P \\ &= \{(\lambda)\hat{i} + (1 + 2\lambda)\hat{j} + (2 + 3\lambda)\hat{k}\} - \{\hat{i} + 6\hat{j} + 3\hat{k}\} \\ &= (\lambda - 1)\hat{i} + (1 + 2\lambda - 6)\hat{j} + (2 + 3\lambda - 3)\hat{k} \\ \overrightarrow{PQ} &= (\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}\end{aligned}$$

Here,  $\overrightarrow{PQ}$  is perpendicular to given line

So,

$$\{(\lambda - 1)\hat{i} + (2\lambda - 5)\hat{j} + (3\lambda - 1)\hat{k}\} \cdot \{\hat{i} + 2\hat{j} + 3\hat{k}\} = 0$$

$$\Rightarrow (\lambda - 1)(1) + (2\lambda - 5)(2) + (3\lambda - 1)(3) = 0$$

$$\Rightarrow \lambda - 1 + 4\lambda - 10 + 9\lambda - 3 = 0$$

$$\Rightarrow 14\lambda - 14 = 0$$

$$\Rightarrow \lambda = 1$$

$$\begin{aligned}\text{Position vector of } Q &= (\hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}) \\ &= (\hat{j} + 2\hat{k}) + (1)(\hat{i} + 2\hat{j} + 3\hat{k})\end{aligned}$$

$$\text{Foot of perpendicular} = \hat{i} + 3\hat{j} + 5\hat{k}$$

$$\begin{aligned}\overrightarrow{PQ} &= \text{Position vector of } Q - \text{Position vector of } P \\ &= (\hat{i} + 3\hat{j} + 5\hat{k}) - (\hat{i} + 6\hat{j} + 3\hat{k}) \\ &= \hat{i} + 3\hat{j} + 5\hat{k} - \hat{i} - 6\hat{j} - 3\hat{k} \\ &= -3\hat{j} + 2\hat{k}\end{aligned}$$

$$\begin{aligned}|\overrightarrow{PQ}| &= \sqrt{(-3)^2 + (2)^2} \\ &= \sqrt{13} \text{ units}\end{aligned}$$

$$\text{Length of perpendicular} = \sqrt{13} \text{ units}$$

**Straight Line in Space Ex 28.4 Q9**

Let  $Q$  be the perpendicular drawn from  $P(-\hat{i} + 3\hat{j} + 2\hat{k})$  on the line

$$\vec{r} = (2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$$

Let the position vector of  $Q$  be

$$\begin{aligned} & (2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k}) \\ & (2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (3 + 3\lambda)\hat{k} \end{aligned}$$

$$\begin{aligned} \overrightarrow{PQ} &= \text{Position vector of } Q - \text{Position vector of } P \\ &= \{(2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (3 + 3\lambda)\hat{k}\} - \{-\hat{i} + 3\hat{j} + 2\hat{k}\} \\ &= (2\lambda + 1)\hat{i} + (2\lambda - 3)\hat{j} + (3 + 3\lambda - 2)\hat{k} \\ \overrightarrow{PQ} &= (2\lambda + 1)\hat{i} + (\lambda - 1)\hat{j} + (3\lambda + 1)\hat{k} \end{aligned}$$

Since,  $\overrightarrow{PQ}$  is perpendicular to the given line, so

$$\begin{aligned} & \{(2\lambda + 1)\hat{i} + (\lambda - 1)\hat{j} + (3\lambda + 1)\hat{k}\} \cdot \{2\hat{i} + \hat{j} + 3\hat{k}\} = 0 \\ & (2\lambda + 1)(2) + (\lambda - 1)(1) + (3\lambda + 1)3 = 0 \\ & 4\lambda + 2 + \lambda - 1 + 9\lambda + 3 = 0 \\ & 14\lambda + 4 = 0 \\ & \lambda = -\frac{4}{14} \\ & \lambda = -\frac{2}{7} \end{aligned}$$

$$\begin{aligned} \text{Position vector of } Q &= (2\lambda)\hat{i} + (2 + \lambda)\hat{j} + (3 + 3\lambda)\hat{k} \\ &= 2\left(-\frac{2}{7}\right)\hat{i} + \left(2 - \frac{2}{7}\right)\hat{j} + \left(3 + 3\left(-\frac{2}{7}\right)\right)\hat{k} \\ &= -\frac{4}{7}\hat{i} + \frac{12}{7}\hat{j} + \frac{15}{7}\hat{k} \end{aligned}$$

$$\text{Coordinates of foot of the perpendicular} = \left(-\frac{4}{7}, \frac{12}{7}, \frac{15}{7}\right)$$

Equation of  $PQ$  is

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

$$\Rightarrow \vec{r} = (-\hat{i} + 3\hat{j} + 2\hat{k}) + \lambda\left(\left(-\frac{4}{7}\hat{i} + \frac{12}{7}\hat{j} + \frac{15}{7}\hat{k}\right) - (-\hat{i} + 3\hat{j} + 2\hat{k})\right)$$

**Straight Line in Space Ex 28.4 Q10**



Let foot of the perpendicular drawn from  $(0, 2, 7)$  to the line  $\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2}$  be  $Q$ .

Given equation of the line is

$$\frac{x+2}{-1} = \frac{y-1}{3} = \frac{z-3}{-2} = \lambda \text{ (say)}$$

Coordinate of  $Q$  is  $(-\lambda - 2, 3\lambda + 1, -2\lambda + 3)$

Direction ratios of  $PQ$  are  $(-\lambda - 2 - 0), (3\lambda + 1 - 2), (-2\lambda + 3 - 7)$   
 $= (-\lambda - 2), (3\lambda - 1), (-2\lambda - 4)$

Since,  $PQ$  is perpendicular to given line, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$(-1)(-\lambda - 2) + (3)(3\lambda - 1) + (-2)(-2\lambda - 4) = 0$$

$$\Rightarrow \lambda + 2 + 9\lambda - 3 + 4\lambda + 8 = 0$$

$$\Rightarrow 14\lambda + 7 = 0$$

$$\lambda = -\frac{1}{2}$$

Foot of the perpendicular =  $(-\lambda - 2, 3\lambda + 1, -2\lambda + 3)$

$$= \left( -\left(-\frac{1}{2}\right) - 2, 3\left(-\frac{1}{2}\right) + 1, -2\left(-\frac{1}{2}\right) + 3 \right)$$

Foot of the perpendicular =  $\left(-\frac{3}{2}, -\frac{1}{2}, 4\right)$

Let foot of the perpendicular from  $P(1, 2, -3)$  to the line  $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$  be  $Q$

Given equation of the line is

$$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = \lambda$$

$$\Rightarrow x = 2\lambda - 1, y = -2\lambda + 3, z = -\lambda$$

Coordinate of  $Q$   $(2\lambda - 1, -2\lambda + 3, -\lambda)$

Direction ratios of  $PQ$  are

$$(2\lambda - 1 - 1), (-2\lambda + 3 - 2), (-\lambda + 3)$$

$$\Rightarrow (2\lambda - 2), (-2\lambda + 1), (-\lambda + 3)$$

Let  $PQ$  is perpendicular to given line, so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow (2)(2\lambda - 2) + (-2)(-2\lambda + 1) + (-1)(-\lambda + 3) = 0$$

$$\Rightarrow 4\lambda - 4 + 4\lambda - 2 + \lambda - 3 = 0$$

$$\Rightarrow 9\lambda - 9 = 0$$

$$\Rightarrow \lambda = 1$$

Coordinate of foot of perpendicular

$$= (2\lambda - 1, -2\lambda + 3, -\lambda)$$

$$= (2(1) - 1, -2(1) + 3, -1)$$

$$= (1, 1, -1)$$

Equation of line  $AB$  is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \frac{x - 0}{-3 - 0} = \frac{y - 6}{-6 - 6} = \frac{z + 9}{3 + 9}$$

$$\Rightarrow \frac{x}{-3} = \frac{y - 6}{-12} = \frac{z + 9}{12} = \lambda \text{ (say)}$$

Coordinate of point  $D = (-3\lambda, -12\lambda + 6, 12\lambda - 9)$

Direction ratios of  $CD = (-3\lambda - 7), (-12\lambda + 6 - 4), (12\lambda - 9 + 1)$   
 $= (-3\lambda - 7), (-12\lambda + 2), (12\lambda - 8)$

Line  $CD$  is perpendicular to line  $AB$ , so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow (-3)(-3\lambda - 7) + (-12)(-12\lambda + 2) + (12)(12\lambda - 8) = 0$$

$$\Rightarrow 9\lambda + 21 + 144\lambda - 24 + 144\lambda - 96 = 0$$

$$\Rightarrow 297\lambda - 99 = 0$$

$$\Rightarrow \lambda = \frac{1}{3}$$

Coordinate of  $D = (-3\lambda, -12\lambda + 6, 12\lambda - 9)$

$$= \left( -3\left(\frac{1}{3}\right), -12\left(\frac{1}{3}\right) + 6, 12\left(\frac{1}{3}\right) - 9 \right)$$

Coordinate of  $D = (-1, 2, -5)$

Equation of  $CD$  is,

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\Rightarrow \frac{x - 7}{-1 - 7} = \frac{y - 4}{2 - 4} = \frac{z + 1}{-5 + 1}$$

$$\Rightarrow \frac{x - 7}{-8} = \frac{y - 4}{-2} = \frac{z + 1}{-4}$$

or 
$$\frac{x - 7}{4} = \frac{y - 4}{1} = \frac{z + 1}{2}$$

Let  $P \equiv (2, 4, -1)$ .

In order to find the distance we need to find a point  $Q$  on the line.

We see that line is passing through the point  $Q(-5, -3, 6)$ .

So, let take this point as required point.

Also line is parallel to the vector  $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$ .

$$\text{Now, } \vec{PQ} = (-5\hat{i} - 3\hat{j} + 6\hat{k}) - (2\hat{i} + 4\hat{j} - \hat{k}) = -7\hat{i} - 7\hat{j} + 7\hat{k}$$

$$\vec{b} \times \vec{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & -9 \\ -7 & -7 & 7 \end{vmatrix} = -35\hat{i} + 56\hat{j} + 21\hat{k}$$

$$|\vec{b} \times \vec{PQ}| = \sqrt{1225 + 3136 + 441} = \sqrt{4802}$$

$$|\vec{b}| = \sqrt{1 + 16 + 81} = \sqrt{98}$$

$$d = \frac{|\vec{b} \times \vec{PQ}|}{|\vec{b}|} = \frac{\sqrt{4802}}{\sqrt{98}} = 7$$

Let L be the foot of the perpendicular drawn from A(1, 8, 4) on the line joining the points B (0, -1, 3) and C (2, -3, -1).

Equation of the line passing through the points B (0, -1, 3) and C (2, -3, -1) is given by,

$$\vec{r} = \vec{b} + \lambda(\vec{c} - \vec{b})$$

$$\vec{r} = (0 + 2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k}$$

Let position vector of L be,

$$\vec{r} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} \dots\dots\dots(i)$$

Then,  $\vec{AL}$  = Position vector of L - position vector of A

$$\Rightarrow \vec{AL} = (2\lambda)\hat{i} + (-1 - 2\lambda)\hat{j} + (3 - 4\lambda)\hat{k} - (1\hat{i} + 8\hat{j} + 4\hat{k})$$

$$\Rightarrow \vec{AL} = (-1 + 2\lambda)\hat{i} + (-9 - 2\lambda)\hat{j} + (-1 - 4\lambda)\hat{k}$$

Since  $\vec{AL}$  is perpendicular to the given line which is parallel to  $\vec{b} = 2\hat{i} - 2\hat{j} - 4\hat{k}$

$$\therefore \vec{AL} \cdot \vec{b} = 0$$

$$\Rightarrow 2(-1 + 2\lambda) - 2(-9 - 2\lambda) - 4(-1 - 4\lambda) = 0$$

$$\Rightarrow -2 + 4\lambda + 18 + 4\lambda + 4 + 16\lambda = 0$$

$$\Rightarrow 24\lambda = -20$$

$$\Rightarrow \lambda = \frac{-5}{6}$$

Putting value of  $\lambda = \frac{-5}{6}$  in (i) we get

$$\vec{r} = -\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{19}{3}\hat{k}$$

Coordinates of the foot of the perpendicular are  $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ .