RD Sharma Solutions Class 12 Maths Chapter 28 Ex 28.3

We have equation of first line,

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-1}{5} = \lambda \text{ (Say)} \quad ---(1)$$

General point on line (1) is

$$(3\lambda + 1,\ 2\lambda - 1,\ 5\lambda + 1)$$

Another line is,

$$\frac{x+2}{4} = \frac{y-1}{3} = \frac{z+1}{-2} = \mu \text{ (Say)} \quad ---(2)$$

General point on line (2) is,

$$(4\mu - 2,\ 3\mu + 1,\ -2\mu - 1)$$

If lines (1) and (2) intersect, then they have a common point, so for same value of λ and μ, we must have,

$$3\lambda + 1 = 4\mu - 2 \Rightarrow 3\lambda - 4\mu = -3 \quad ---(3)$$

$$2\lambda - 1 = 3\mu + 1 \Rightarrow 2\lambda - 3\mu = 2 \quad ---(4)$$

$$5\lambda + 1 = -2\mu - 1 \Rightarrow 5\lambda + 2\mu = -2 \quad ---(5)$$

Solving equation (3) and (4) to get λ and μ,

$$6\lambda - 8\mu = -6$$

$$\underset{(-)}{6\lambda} \underset{(+)}{-\ 9\mu} = \underset{(-)}{6}$$

$$\mu = -12$$

Put the value of μ in equation (3),

$$3\lambda - 4(-12) = -3$$

$$3\lambda + 48 = -3$$

$$3\lambda = -3 - 48$$

$$3\lambda = -51$$

$$\lambda = \frac{-51}{3}$$

$$\lambda = -17$$

Put the value of λ and μ in equation (5),

$$5\lambda + 2\mu = -2$$

$$5(-17) + 2(-12) = -2$$

$$-85 - 24 = -2$$

$$-109 \neq -2$$

LHS $\neq$ RHS

Straight Line in Space Ex 28.3 Q3

Given equation of first line is

$$\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7} = \lambda \text{ (Say)} \qquad ---(1)$$

General point on line (1) is

$$(3\lambda - 1, 5\lambda - 3, 7\lambda - 5)$$

Another equation of line is

$$\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5} = \mu \text{ (Say)} \qquad ---(2)$$

General point on line (2) is,

$$(\mu + 2, 3\mu + 4, 5\mu + 6)$$

If lines (1) and (2) are intersecting then, they have a common point. So for same value of λ and μ, we must have,

$$3\lambda - 1 = \mu + 2 \Rightarrow 3\lambda - \mu = 3 \qquad ---(3)$$

$$5\lambda - 3 = 3\mu + 4 \Rightarrow 5\lambda - 3\mu = 7 \qquad ---(4)$$

$$7\lambda - 5 = 5\mu + 6 \Rightarrow 7\lambda - 5\mu = 11 \qquad ---(5)$$

Solving equation (3) and (4) to get λ and μ,

$$15\lambda - 5\mu = 15$$

$$15\lambda - 9\mu = 21$$

$$(-) \quad (+) \quad (-)$$

$$4\mu = -6$$

$$\mu = \frac{-3}{2}$$

Put the value of μ in equation (3),

$$3\lambda - \mu = 3$$

$$3\lambda - \left(-\frac{3}{2}\right) = 3$$

$$3\lambda = 3 - \frac{3}{2}$$

$$\lambda = \frac{1}{2}$$

Put the value of λ and μ in equation (5),

$$7\lambda - 5\mu = 11$$

$$7\left(\frac{1}{2}\right) - 5\left(-\frac{3}{2}\right) = 11$$

$$\frac{7}{2}+\frac{15}{2}=11$$

$$\frac{22}{2}=11$$

$$11=11$$

LHS $\neq$ RHS

Since, the values of λ and μ obtained by solving (3) and (4) satisfy equation (5), Hence

Given lines intersect each other.

Point of intersection $=(3\lambda-1,\ 5\lambda-3,\ 7\lambda-5)$

$$=\left\{\frac{3}{2}-1,\ \left(\frac{5}{2}-3\right),\ \left(\frac{7}{2}-5\right)\right\}$$

$$=\left(\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)$$

Point of intersection is $\left(\frac{1}{2},\ \frac{-1}{2},\ \frac{-3}{2}\right)$.

Straight Line in Space Ex 28.3 Q4

Equation of the line passing through $A(0,-1,-1)$ and $B(4,5,1)$ is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\frac{x-0}{4-0}=\frac{y+1}{5+1}=\frac{z+1}{1+1}$$

$$\frac{x}{4}=\frac{y+1}{6}=\frac{z+1}{2}=\lambda \text{ (say)}$$

So, general point on line AB is

$$(4\lambda,\ 4\lambda,\ 2\lambda-1)$$

Now, equation of the line passing through $C(3,9,4)$ and $D(-4,4,4)$ is

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}$$

$$\frac{x-3}{-4-3}=\frac{y-9}{4-9}=\frac{z-4}{4-4}$$

$$\frac{x-3}{-7}=\frac{y-9}{-5}=\frac{z-4}{0}=\mu \text{ (say)}$$

So, general point on line CD is

$$(-7\mu+3,\ -5\mu+9,\ 0.\mu+4)$$

$$(-7\mu+3,\ -5\mu+9,\ 4)$$

If lines AB and CD intersect, there must be a common point to them. So we have to find λ and μ such that

$$4\lambda=-7\mu+3 \quad \Rightarrow 4\lambda+7\mu=3 \qquad ---(1)$$

$$6\lambda-1=-5\mu+9 \quad \Rightarrow 6\lambda+5\mu=10 \qquad ---(2)$$

$$2\lambda-1=4 \quad \Rightarrow 2\lambda-1=4 \qquad ---(3)$$

From equation (3),

$$2\lambda=4+1$$

$$\lambda=\frac{5}{2}$$

Put $\lambda=\frac{5}{2}$ in equation (2),

$$6\left(\frac{5}{2}\right)+5\mu=10$$

$$5\mu=10-15$$

$$5\mu=-5$$

$$\mu=-1$$

Now, put values of λ and μ in equation (1),

$$4\lambda + 7(\mu) = 3$$

$$4\left(\frac{5}{2}\right) + 7(-1) = 3$$

$$10 - 7 = 3$$

$$3 = 3$$

LHS $\neq$ RHS

Since, the values of λ and μ by solving (2) and (3) satisfy equation (1), so

Line AB and CD are intersecting lines

Point of intersection of AB and CD

$$= (-7\mu + 3, -5\mu + 9, 4)$$

$$= (-7(-1) + 3, -5(-1) + 9, 4)$$

$$= (7 + 3, 5 + 9, 4)$$

$$= (10, 14, 4)$$

So, point of intersection of AB and CD $= (10, 14, 4)$.

Straight Line in Space Ex 28.3 Q5

Given equations of lines are

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right)$$

$$\vec{r} = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$

If these lines intersect, they must have a common point, so, for some value of λ and μ we must have,

$$\left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j}\right) = \left(4\hat{i} - \hat{k}\right) + \mu\left(2\hat{i} + 3\hat{k}\right)$$

$$(1 + 3\lambda)\hat{i} + (1 - \lambda)\hat{j} - \hat{k} = (4 + 2\mu)\hat{i} + (-1 + 3\mu)\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}, \hat{k}$, we get

$$1 + 3\lambda = 4 + 2\mu \quad \Rightarrow 3\lambda - 2\mu = 3 \quad ---(1)$$

$$1 - \lambda = 0 \quad \Rightarrow \lambda = 1 \quad ---(2)$$

$$-1 = -1 + 3\mu \quad \Rightarrow \mu = 0 \quad ---(3)$$

Put the value of λ and μ in equation (1),

$$3\lambda - 2\mu = 3$$

$$3(1) - 2(0) = 3$$

$$3 = 3$$

$$\text{LHS} = \text{RHS}$$

The value of λ and μ satisfy equation (1), so

Lines are intersecting.

Put value of λ in equation (1) to get point of intersection

$$\vec{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + (1)\left(3\hat{i} - \hat{j}\right)$$

$$= \hat{i} + \hat{j} - \hat{k} + 3\hat{i} - \hat{j}$$

$$= 4\hat{i} - \hat{k}$$

So, point of intersection is $(4, 0, -1)$.

Straight Line in Space Ex 28.3 Q6(i)

Given equations of lines are

$$\vec{r} = \left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$

If these lines intersect each other, there must be some common point, so, we must have λ and μ such that

$$\left(\hat{i} - \hat{j}\right) + \lambda\left(2\hat{i} + \hat{k}\right) = \left(2\hat{i} - \hat{j}\right) + \mu\left(\hat{i} + \hat{i} - \hat{k}\right)$$

$$\left(1 + 2\lambda\right)\hat{i} - \hat{j} + \lambda\hat{k} = \left(2 + \mu\right)\hat{i} + \left(-1 + \mu\right)\hat{j} - \mu\hat{k}$$

Equation the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$,

$$1 + 2\lambda = 2 + \mu \quad \Rightarrow 2\lambda - \mu = 1 \quad ---(1)$$

$$-1 = -1 + \mu \quad \Rightarrow \mu = 0 \quad ---(2)$$

$$\lambda = -\mu \quad \Rightarrow \lambda = 0 \quad ---(3)$$

Put value of λ and μ in equation (1),

$$2\lambda - \mu = 1$$

$$2(0) - (0) = 1$$

$$0 = 1$$

$$\text{LHS} \neq \text{RHS}$$

Since, the values of λ and μ form equation (2) and (3) does not satisfy equation (1),

Hence, given lines do not intersect each other.

Straight Line in Space Ex 28.3 Q6(ii)

Given, equations of first line is

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1} = \lambda \text{ (say)} \quad ---(1)$$

General point on line (1) is

$$(2\lambda+1,\ 3\lambda-1,\ \lambda)$$

Another equation of line is

$$\frac{x-1}{5} = \frac{y-2}{1},\ z = 3 \quad ---(2)$$

$$\frac{x-1}{5} = \frac{y-2}{1} = \mu,\ \text{(say)},\ z = 3$$

General point on line (2) is

$$(5\mu+1,\ \mu+2,\ 3)$$

If line (1) and (2) intersect each other then, there is a common point to them, so, we must have value of λ and μ such that

$$2\lambda+1 = 5\mu+1 \quad \Rightarrow 2\lambda-5\mu = 0 \quad ---(3)$$

$$3\lambda-1 = \mu+2 \quad \Rightarrow 3\lambda-\mu = 3 \quad ---(4)$$

$$\lambda = 3 \quad \Rightarrow \lambda = 3 \quad ---(5)$$

Put value of λ in equation (4),

$$3\lambda-\mu = 3$$

$$3(3)-\mu = 3$$

$$-\mu = 3-9$$

$$\mu = 6$$

Put the value of λ and μ in equation (3), so

$$2\lambda-5\mu = 0$$

$$2(3)-5(6) = 0$$

$$6-30 = 0$$

$$-24 \neq 0$$

$$\text{LHS} \neq \text{RHS}$$

Since the values of λ and μ obtained from equation (4) and (5) does not satisfy equation (3), so,

Given lines are not intersecting.

Straight Line in Space Ex 28.3 Q6(iii)

Given, equations of first line is,

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda \ (\text{say}) \qquad ---(1)$$

General point on line (1) is,

$$(3\lambda + 1, -\lambda + 1, -1)$$

Another equation of line is

$$\frac{x-4}{2} = \frac{y-0}{0} = \frac{z+1}{3} = \mu \ (\text{say}) \qquad ---(2)$$

General point on line (2) is,

$$(2\mu + 4, 0, 3\mu - 1)$$

If line (1) and (2) intersecting then there must be a common point, so, we must have the value of λ and μ as

$$3\lambda + 1 = 2\mu + 4 \quad \Rightarrow 3\lambda - 2\mu = 3 \qquad ---(1)$$

$$-\lambda + 1 = 0 \quad \Rightarrow \lambda = 1 \qquad ---(2)$$

$$3\mu - 1 = -1 \quad \Rightarrow \mu = 0 \qquad ---(3)$$

Put the value of λ and μ in equation (1), so

$$3\lambda - 2\mu = 3$$

$$3(1) - 2(0) = 3$$

$$3 = 3$$

$$\text{LHS} \neq \text{RHS}$$

Since the values of λ and μ obtained by equation (2) and (3) satisfy equation (1), so,

Given lines are intersecting.

Straight Line in Space Ex 28.3 Q6(iv)

Given, equation of line is

$$\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}=\lambda \text{ (say)} \qquad ---(1)$$

General point on line (1) is,

$$(4\lambda+5,\ 4\lambda+7,\ -5\lambda-3)$$

Another equation of line is,

$$\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}=\mu \text{ (say)} \qquad ---(2)$$

General point on line (2) is

$$(7\mu+8,\ \mu+4,\ 3\mu+5)$$

If line (1) and (2) intersecting, then there must have some common point to them, so, we must have value of λ and μ such that

$$4\lambda+5=7\mu+8 \Rightarrow 4\lambda-7\mu=3 \qquad ---(3)$$

$$4\lambda+5=\mu+4 \Rightarrow 4\lambda-\mu=-3 \qquad ---(4)$$

$$-5\lambda-3=3\mu+5 \Rightarrow -5\lambda-3\mu=8 \qquad ---(5)$$

Solving equation (3) and (4) to find λ and μ,

$$4\lambda-7\mu=3$$

$$\underset{(-)}{4\lambda}\ \underset{(+)}{-}\ \mu=\underset{(-)}{-3}$$

$$-6\mu=6$$

$$\mu=-1$$

Put value of λ in equation (3),

$$4\lambda-7\mu=3$$

$$4\lambda-7(-1)=3$$

$$4\lambda=3-7$$

$$\lambda=-1$$

Put the value of λ and μ in equation (5),

$$-5\lambda-3\mu=8$$

$$-5(-1)-3(-1)=8$$

$$5+3=8$$

$$\text{LHS}=\text{RHS}$$

Straight Line in Space Ex 28.3 Q7

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

If the lines intersect eachother, then the shortest distance between the lines should be zero.

Here,

$\vec{a_1} = 3\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{a_2} = 5\hat{i} - 2\hat{j}$

$\vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$$(\vec{b_1} \times \vec{b_2}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix}$$

$= \vec{i}(12 - 4) - \vec{j}(6 - 6) + \vec{k}(2 - 6)$

$= 8\vec{i} - 0\vec{j} - 4\vec{k}$

$(\vec{a_2} - \vec{a_1}) = (5\hat{i} - 2\hat{j} - 3\hat{i} - 2\hat{j} + 4\hat{k}) = (2\hat{i} - 4\hat{j} + 4\hat{k})$

$$\text{Shortest Distance, } d = \left|\frac{(\vec{b_1} \times \vec{b_2}).(\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|}\right|$$

$$= \left|\frac{(8\vec{i} - 0\vec{j} - 4\vec{k}).(2\hat{i} - 4\hat{j} + 4\hat{k})}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right|$$

$$= \left|\frac{8 \times 2 - 0 \times 4 + (-4) \times 4}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right|$$

$$= \left|\frac{0}{|8\vec{i} - 0\vec{j} - 4\vec{k}|}\right| = 0$$

Since the shortest distance is zero, the lines are intersect each other.

Point of intersection of the lines,

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

Lines in the Cartesian form,

$$\frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} = \lambda$$

$x = \lambda + 3, y = 2\lambda + 2, z = 2\lambda - 4$

$$\frac{x-5}{3} = \frac{y+2}{2} = \frac{z}{6} = \mu$$

$x = 3\mu + 5, y = 2\mu - 2, z = 6\mu$

From coordinates of x,

$\lambda + 3 = 3\mu + 5$

$\lambda = 3\mu + 2$.....(i)

From coordinates of y,

$2\lambda + 2 = 2\mu - 2$

$\lambda = \mu - 2$.......(ii)

Solving (i) and (ii),

$\lambda = -4, \mu = -2$

Coordinates of the point of intersection,

$x = 3(-2) + 5, y = 2(-2) - 2, z = 6(-2)$

$x = -1, y = -6, z = -12$

$(-1, -6, -12)$