RD Sharma Solutions Class 12 Maths Chapter 28 Ex 28.2

The direction ratios of a line passing through the points $(1, -1, 2)$ and $(3, 4, -2)$ are

$(3-1, 4+1, -2-2)$

$= (2, 5, -4)$

The direction ratios of a line passing through the points $(0, 3, 2)$ and $(3, 5, 6)$ are

$(3-0, 5-3, 6-2)$

$= (3, 2, 4)$

Angle between the lines

$$\cos\theta = \frac{(a_1a_2 + b_1b_2 + c_1c_2)}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{[2 \times 3 + 5 \times 2 + (-4) \times 4]}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$$\cos\theta = \frac{0}{\sqrt{2^2 + 5^2 + (-4)^2}\sqrt{3^2 + 2^2 + 4^2}}$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

The lines are mutually perpendicular.

Straight Line in Space Ex 28.2 Q3

The direction ratios of a line passing through the points $(4, 7, 8)$ and $(2, 3, 4)$ are

$(4-2, 7-3, 8-4)$

$= (2, 4, 4)$

The direction ratios of a line passing through the points $(-1, -2, 1)$ and $(1, 2, 5)$ are

$(-1-1, -2-2, 1-5)$

$= (-2, -4, -4)$

The direction ratios are proportional.

$$\frac{2}{-2} = \frac{4}{-4} = \frac{4}{-4}$$

Hence, the lines are mutually parallel.

Straight Line in Space Ex 28.2 Q4

The Cartesian equation of a line passing through (x_1, y_1, z_1) and with direction ratios (a_1, b_1, c_1)

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

The Cartesian equation of a line passing through $(-2, 4, -5)$ and parallel to the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is

$$\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$$

Straight Line in Space Ex 28.2 Q5

Given equations of lines are $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

Clearly,

$7 \times 1 + (-5) \times 2 + 1 \times 3$

$= 7 - 10 + 3$

$= 0$

$\therefore$ Lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

Straight Line in Space Ex 28.2 Q6

The direction ratios of a line joining the origin to the point $(2, 1, 1)$
are $(2-0, 1-0, 1-0) = (2, 1, 1)$

The direction ratios of a line joining $(3, 5, -1)$ and $(4, 3, -1)$
are $(4-3, 3-5, -1+1) = (1, -2, 0)$

Angle between the lines

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\cos\theta = \frac{2 \times 1 + 1 \times (-2) + 1 \times 0}{\sqrt{2^2 + 1^2 + 1^2}\sqrt{1^2 + (-2)^2 + 0^2}}$$

$$\cos\theta = \frac{0}{\sqrt{6}\sqrt{5}}$$

$$\cos\theta = 0$$

$$\therefore \theta = \frac{\pi}{2}$$

The lines are mutually perpendicular.

Straight Line in Space Ex 28.2 Q7

Vector equation of a line is

$\vec{r} = \vec{a} + \lambda\vec{b}$

The direction cosines of the x − axis are $(1, 0, 0)$. Equation of a line parallel to the x − axis and passing through the origin is

$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(1\hat{i} + 0\hat{j} + 0\hat{k})$

$\vec{r} = \lambda\hat{i}$

Straight Line in Space Ex 28.2 Q8(i)

We know that, If Q be the angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, then

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{\left|\vec{b_1}\right|.\left|\vec{b_2}\right|} \qquad ---(i)$$

Here, $\vec{r} = \left(4\hat{i} - \hat{j}\right) + \lambda\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)$

and, $\vec{r} = \left(\hat{i} - \hat{j} + 2\hat{k}\right) - \mu\left(2\hat{i} + 4\hat{j} - 4\hat{k}\right)$

$\Rightarrow$ $\vec{a_1} = 4\hat{i} - \hat{j}$, $\vec{b_1} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{a_2} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b_2} = 2\hat{i} + 4\hat{j} - 4\hat{k}$

$$\left|\vec{b_1}\right| = \sqrt{(1)^2 + (2)^2 + (-2)^2} = 3$$

$$\left|\vec{b_2}\right| = \sqrt{(2)^2 + (4)^2 + (-4)^2} = 6$$

Let θ be the angle between given lines. So using equation (i),

$$\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{\left|\vec{b_1}\right|.\left|\vec{b_2}\right|}$$

$$= \frac{\left(\hat{i} + 2\hat{j} - 2\hat{k}\right)\left(2\hat{i} + 4\hat{j} - 4\hat{k}\right)}{3.6}$$

$$= \frac{2 + 8 + 8}{18}$$

$$\cos\theta = 1$$

$$\theta = 0^\circ$$

Straight Line in Space Ex 28.2 Q8(ii)

We know that, angle between two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda\vec{b_2}$, is given by

$$\cos\theta = \frac{\vec{b_1}\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|} \qquad ---(i)$$

Given lines are,

$$\vec{r} = \left(3\hat{i} + 2\hat{j} - 4\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$$

$$\vec{r} = \left(5\hat{j} - 2\hat{k}\right) + \mu\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$$

$$\Rightarrow \qquad \vec{b_1} = \hat{i} + 2\hat{j} + 2\hat{k}, \quad \vec{b_2} = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$|\vec{b_1}| = \sqrt{(1)^2 + (2)^2 + (2)^2} = 3$$

$$|\vec{b_2}| = \sqrt{(3)^2 + (2)^2 + (6)^2} = 7$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{\vec{b_1}\vec{b_2}}{|\vec{b_1}|.|\vec{b_2}|}$$

$$= \frac{\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)}{3.7}$$

$$= \frac{3 + 4 + 12}{21}$$

$$= \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

Straight Line in Space Ex 28.2 Q8(iii)

We know that, angle between two lines $\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$, is given by

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|} \qquad ---(i)$$

Equation of given lines are,

$$\vec{r} = \lambda\left(\hat{i} + \hat{j} + 2\hat{k}\right) \text{ and}$$

$$\vec{r} = 2\hat{j} + \mu\left[\left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}\right]$$

$$\Rightarrow \qquad \overrightarrow{b_1} = \left(\hat{i} + \hat{j} + 2\hat{k}\right), \ \overrightarrow{b_2} = \left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{\overrightarrow{b_1}.\overrightarrow{b_2}}{\left|\overrightarrow{b_1}\right|\left|\overrightarrow{b_2}\right|}$$

$$= \frac{\left(\hat{i} + \hat{j} + 2\hat{k}\right)\left(\left(\sqrt{3} - 1\right)\hat{i} - \left(\sqrt{3} + 1\right)\hat{j} + 4\hat{k}\right)}{\sqrt{(1)^2 + (1)^2 + (2)^2}\ \sqrt{\left(\sqrt{3} - 1\right)^2 + \left(-\sqrt{3} - 1\right)^2 + (4)^2}}$$

$$= \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{\sqrt{6}.\sqrt{3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} + 16}}$$

$$= \frac{6}{\sqrt{6}.2\sqrt{6}}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Straight Line in Space Ex 28.2 Q9(i)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Here, given lines are,

$$\frac{x+4}{3}=\frac{y-1}{5}=\frac{z+3}{4} \quad \text{and} \quad \frac{x+1}{1}=\frac{y-4}{1}=\frac{z-5}{2}$$

$\Rightarrow \quad a_1=3,\ b_1=5,\ c_1=4,\ a_2=1,\ b_2=1,\ c_2=2$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta=\frac{(3)(1)+(5)(1)+(4)(2)}{\sqrt{(3)^2+(5)^2+(4)^2}\sqrt{(1)^2+(1)^2+(2)^2}}$$

$$=\frac{3+5+8}{\sqrt{50}\sqrt{6}}$$

$$=\frac{16}{10\sqrt{3}}$$

$$\cos\theta=\frac{8}{5\sqrt{3}}$$

$$\theta=\cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

Straight Line in Space Ex 28.2 Q9(ii)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \quad \text{and} \quad \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Given, equation of lines are,

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{-3} \quad \text{and} \quad \frac{x+3}{-1}=\frac{y-5}{8}=\frac{z-1}{4}$$

$\Rightarrow \quad a_1=2,\ b_1=3,\ c_1=-3,\ a_2=-1,\ b_2=8,\ c_2=4$

Let θ be the angle between two given lines, so using equation (i),

$$\cos\theta=\frac{(2)(-1)+(3)(8)+(-3)(4)}{\sqrt{(2)^2+(3)^2+(-3)^2}\sqrt{(-1)^2+(8)^2+(4)^2}}$$

$$=\frac{-2+24-12}{\sqrt{22}\sqrt{81}}$$

$$\cos\theta=\frac{10}{9\sqrt{22}}$$

$$\theta=\cos^{-1}\left(\frac{10}{9\sqrt{22}}\right)$$

Straight Line in Space Ex 28.2 Q9(iii)

We know that, angle between two lines

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \text{ and } \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

is given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Given lines are,

$$\frac{5 - x}{-2} = \frac{y + 3}{1} = \frac{1 - z}{3} \text{ and } \frac{x}{3} = \frac{1 - y}{-2} = \frac{z + 5}{-1}$$

$$\Rightarrow \quad \frac{x - 5}{2} = \frac{y + 3}{1} = \frac{z - 1}{-3} \text{ and } \frac{x}{3} = \frac{y - 1}{2} = \frac{z + 5}{-1}$$

$$\Rightarrow \quad a_1 = 2,\ b_1 = 1,\ c_1 = -3,\ a_2 = 3,\ b_2 = 2,\ c_2 = -1$$

Let θ be the angle between given lines, so using equation (i),

$$\cos\theta = \frac{(2)(3) + (1)(2) + (-3)(-1)}{\sqrt{(2)^2 + (1)^2 + (-3)^2}\sqrt{(3)^2 + (2)^2 + (-1)^2}}$$

$$= \frac{6 + 2 + 3}{\sqrt{14}\sqrt{14}}$$

$$\cos\theta = \frac{11}{14}$$

$$\theta = \cos^{-1}\left(\frac{11}{14}\right)$$

Straight Line in Space Ex 28.2 Q9(iv)

We know that, angle between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

Equation of given lines are,

$$\frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{2y-3}{3}=\frac{z-5}{2}$$

$$\Rightarrow \quad \frac{x-2}{3}=\frac{y+3}{-2}, z=5 \text{ and } \frac{x+1}{1}=\frac{\frac{y-3}{3}}{\frac{3}{2}}=\frac{z-5}{2}$$

$$\Rightarrow \quad a_1=3,\ b_1=-2,\ c_1=0,\ a_2=1,\ b_2=\frac{3}{2},\ c_2=2$$

Let θ be the angle between given lines, so from equation (i),

$$\cos\theta=\frac{(3)(1)+(-2)\left(\frac{3}{2}\right)+(0)(2)}{\sqrt{(3)^2+(-2)^2+(0)^2}\sqrt{(1)^2+\left(\frac{3}{2}\right)^2+(2)^2}}$$

$$=\frac{3-3+0}{\sqrt{38}\sqrt{\frac{29}{4}}}$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

Straight Line in Space Ex 28.2 Q9(v)

$$\frac{x-5}{1}=\frac{2y+6}{-2}=\frac{z-3}{1} \text{ and } \frac{x-2}{3}=\frac{y+1}{4}=\frac{z-6}{5}$$

$\hat{a}=\hat{i}-2\hat{j}+\hat{k}, \hat{b}=3\hat{i}+4\hat{j}+5\hat{k}$ are the vectors parallel to above lines

$\therefore$ angle between $\hat{a}$ and $\hat{b} \rightarrow \cos\theta=\dfrac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}$

$$\cos\theta=\frac{(\hat{i}-2\hat{j}+\hat{k})\cdot(3\hat{i}+4\hat{j}+5\hat{k})}{|\hat{i}-2\hat{j}+\hat{k}||\hat{i}-2\hat{j}+\hat{k}|}=\frac{3-8+5}{|\hat{i}-2\hat{j}+\hat{k}||\hat{i}-2\hat{j}+\hat{k}|}=0$$

$$\cos\theta=0 \rightarrow \theta=90^\circ$$

Straight Line in Space Ex 28.2 Q9(vi)

$$\frac{x-2}{2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{y-4}{2}=\frac{z-5}{4}$$

$\hat{a}=2\hat{i}+7\hat{j}-3\hat{k}, \hat{b}=-1\hat{i}+4\hat{j}+4\hat{k}$ are the vectors parallel to above lines

$\therefore$ angle between $\hat{a}$ and $\hat{b} \to \cos\theta=\dfrac{\hat{a}\cdot\hat{b}}{|\hat{a}||\hat{b}|}$

$$\cos\theta=\frac{\left(2\hat{i}+7\hat{j}-3\hat{k}\right)\cdot\left(-1\hat{i}+2\hat{j}+4\hat{k}\right)}{\left|\left(2\hat{i}+7\hat{j}-3\hat{k}\right)\right|\left|\left(-1\hat{i}+2\hat{j}+4\hat{k}\right)\right|}=0$$

$\cos\theta=0 \to \theta=90^\circ$

Straight Line in Space Ex 28.2 Q10(i)

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}} \qquad ---(i)$$

Here, $a_1=5, b_1=-12, c_1=13$

$a_2=-3, b_2=4, c_2=5$

Let θ be the required angle, so using equation (i),

$$\cos\theta=\frac{(5)(-3)+(-12)(4)+(13)(5)}{\sqrt{(5)^2+(-12)^2+(13)^2}\sqrt{(-3)^2+(4)^2+(5)^2}}$$

$$=\frac{-15-48+65}{\sqrt{169\times 2}\sqrt{25\times 2}}$$

$$=\frac{2}{65\times 2}$$

$$\cos\theta=\frac{1}{65}$$

$$\theta=\cos^{-1}\left(\frac{1}{65}\right)$$

Straight Line in Space Ex 28.2 Q10(ii)

We know that, angle (θ) between lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Here, $a_1 = 2,\ b_1 = 2,\ c_1 = 1$
$a_2 = 4,\ b_2 = 1,\ c_2 = 8$

Let θ be required angle, so using equation (i),

$$\cos\theta = \frac{(2)(4) + (2)(1) + (1)(8)}{\sqrt{(2)^2 + (2)^2 + (1)^2}\sqrt{(4)^2 + (1)^2 + (8)^2}}$$

$$= \frac{8+2+8}{3.9}$$

$$= \frac{18}{27}$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Straight Line in Space Ex 28.2 Q10(iii)

We know that, angle (θ) between two lines

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \text{ and } \frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

is given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad ---(i)$$

Here, $a_1 = 1,\ b_1 = 2,\ c_1 = -2$
$a_2 = -2,\ b_2 = 2,\ c_2 = 1$

Let θ be the required angle, so using equation (i),

$$\cos\theta = \frac{(1)(-2) + (2)(2) + (-2)(1)}{\sqrt{(1)^2 + (2)^2 + (-2)^2}\sqrt{(-2)^2 + (2)^2 + (1)^2}}$$

$$= \frac{-2+4-2}{3.3}$$

$$= \frac{0}{9}$$

$$\cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

Straight Line in Space Ex 28.2 Q10(vi)

a, b, c and $b-c,\ c-a,\ a-b$ are direction ratios

these are the vectors with above direction ratios

$\hat{x} = a\hat{i} + b\hat{j} + c\hat{k}, \hat{y} = (b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}$

are the vectors parallel to two given lines

$\therefore$ angle between the lines with above

direction ratios are $\hat{x}$ and $\hat{y} \rightarrow \cos\theta = \dfrac{\hat{x}\cdot\hat{y}}{|\hat{x}||\hat{y}|}$

$$\cos\theta = \frac{(a\hat{i} + b\hat{j} + c\hat{k})\cdot((b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k})}{|(a\hat{i} + b\hat{j} + c\hat{k})||(b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}|}$$

$$= \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}$$

$$= \frac{ab - ac + bc - ba + ca - cb}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}} = 0$$

$\cos\theta = 0 \rightarrow \theta = 90°$

Straight Line in Space Ex 28.2 Q11

We know that, angle (θ) between two lines

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1} \quad \text{and} \quad \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

is given by,

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Here, Direction ratios of first line is 2, 2, 1

$\Rightarrow \quad a_1 = 2,\ b_1 = 2,\ c_1 = 1$

Direction ratios of the line joining $(3,1,4)$ and $(7,2,12)$ is given by

$= (7-3), (2-1), (12-4)$

$= 4, 1, 8$

$\Rightarrow \quad a_2 = 4,\ b_2 = 1,\ c_2 = 8$

Let θ be the required angle, so using equation (i),

$$\cos\theta = \frac{(2)(4) + (2)(1) + (1)(8)}{\sqrt{(2)^2 + (2)^2 + (1)^2}\sqrt{(4)^2 + (1)^2 + (8)^2}}$$

$$= \frac{8 + 2 + 8}{3.9}$$

$$\cos\theta = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Straight Line in Space Ex 28.2 Q12

We know that equation of a line passing through (x_1, y_1, z_1) and direction ratios are a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (1, 2, -4)$
and required line is parallel to the given line

$$\frac{x-3}{4} = \frac{y-5}{2} = \frac{z+1}{3}$$

$\Rightarrow$ Direction ratios of the required line are proportional to 4, 2, 3
$\Rightarrow$ $a = 4\lambda,\ b = 2\lambda,\ c = 3\lambda$

So, required equation of the line is

$$\Rightarrow \frac{x-1}{4\lambda} = \frac{y-2}{2\lambda} = \frac{z+4}{3\lambda}$$

$$\Rightarrow \frac{x-1}{4} = \frac{y-2}{2} = \frac{z+4}{3}$$

Straight Line in Space Ex 28.2 Q13

We know that, equation of a line passing through (x_1, y_1, z_1) and direction ratios are a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(i)$$

Here, $(x_1, y_1, z_1) = (-1, 2, 1)$
and required line is parallel to the given line

$$\frac{2x-1}{4} = \frac{3y+5}{2} = \frac{2-z}{3}$$

$$\Rightarrow \frac{x - \frac{1}{2}}{2} = \frac{y + \frac{5}{3}}{\frac{2}{3}} = \frac{z-2}{-3}$$

$\Rightarrow$ Direction ratios of the required line are proportional to $2, \frac{2}{3}, -3$

$\Rightarrow$ $a = 2\lambda,\ b = \frac{2}{3}\lambda,\ c = -3\lambda$

So, required equation of the line using equation (i),

$$\frac{x+1}{2\lambda} = \frac{y-2}{\frac{2}{3}\lambda} = \frac{z-1}{-3\lambda}$$

$$\Rightarrow \frac{x+1}{2} = \frac{y-2}{\frac{2}{3}} = \frac{z-1}{-3}$$

Straight Line in Space Ex 28.2 Q14

We know that equation of a line passing through the point $\bar{a}$ and is the direction of vector $\bar{b}$ is

$$\vec{r} = \bar{a} + \lambda\bar{b} \qquad ---(i)$$

Here, $\bar{a} = 2\hat{i} - \hat{j} + 3\hat{k}$

and given that the required line is parallel to

$$\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

$$\Rightarrow \qquad \bar{b} = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\mu$$

So, required equation of the line using equation (i) is

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\mu$$

$$\vec{r} = \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + \lambda'\left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$$

where λ' is a scalar such that $\lambda' = \lambda.\mu$

Straight Line in Space Ex 28.2 Q15

We know that, equation of a line passing through (x_1, y_1, z_1) with direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

So, equation of required line passing through $(2, 1, 3)$ is

$$\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c} \qquad ---(1)$$

Given that line $\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ is perpendicular to line (i), so

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$
$$(a)(1) + (b)(2) + (c)(3) = 0$$
$$a + 2b + 3c = 0 \qquad ---(2)$$

And line $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ is perpendicular to line (i), so

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$
$$(a)(-3) + (b)(2) + (c)(5) = 0$$
$$-3a + 2b + 5c = 0 \qquad ---(3)$$

Solving equation (2) and (3) by cross multiplication,

$$\frac{a}{(2)(5)-(2)(3)} = \frac{b}{(-3)(3)-(1)(5)} = \frac{c}{(1)(2)-(-3)(2)}$$

$$\Rightarrow \quad \frac{a}{10-6} = \frac{b}{-9-5} = \frac{c}{2+6}$$

$$\Rightarrow \quad \frac{a}{4} = \frac{b}{-14} = \frac{c}{8}$$

$$\Rightarrow \quad \frac{a}{2} = \frac{b}{-7} = \frac{c}{4} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = 2\lambda,\ b = -7\lambda,\ c = 4\lambda$$

Using a, b, c in equation (i),

$$\frac{x-2}{2\lambda} = \frac{y-1}{-7\lambda} = \frac{z-3}{4\lambda}$$

$$\Rightarrow \quad \frac{x-2}{2} = \frac{y-1}{-7} = \frac{z-3}{4}$$

Straight Line in Space Ex 28.2 Q16

We know that equation of a line passing through a point with position vector $\vec{\alpha}$ and perpendiculat to $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \lambda \vec{b_2}$ is given by

$$\vec{r} = \vec{\alpha} + \lambda\left(\vec{b_1} \times \vec{b_2}\right) \qquad ---(i)$$

Here, $\vec{\alpha} = \left(\hat{i} + \hat{j} - 3\hat{k}\right)$

and required line is perpendicular to

$$\vec{r} = \hat{i} + \lambda\left(2\hat{i} + \hat{j} - 3\hat{k}\right) \text{ and}$$

$$\vec{r} = \left(2\hat{i} + \hat{j} - \hat{k}\right) + \mu\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$$\Rightarrow \qquad \vec{b_1} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right), \ \vec{b_2} = \hat{i} + \hat{j} + \hat{k}$$

Now,

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}\left(1+3\right) - \hat{j}\left(2+3\right) + \hat{k}\left(2-1\right)$$

$$\vec{b_1} \times \vec{b_2} = 4\hat{i} - 5\hat{j} + \hat{k}$$

Using equation, required equation of line is

$$\vec{r} = \vec{\alpha} + \lambda\left(\vec{b_1} \times \vec{b_2}\right)$$

$$\vec{r} = \left(\hat{i} + \hat{j} - 3\hat{k}\right) + \lambda\left(4\hat{i} - 5\hat{j} + \hat{k}\right)$$

Straight Line in Space Ex 28.2 Q17

We know that equation of a line passing through (x_1, y_1, z_1) and direction ratios as a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ----(1)$$

So, equation of a line passing through $(1, -1, 1)$ is

$$\frac{x - 1}{a} = \frac{y + 1}{b} = \frac{z - 1}{c} \qquad ---(2)$$

Now, Directions ratios of the line joining $A(4, 3, 2)$ and $B(1, -1, 0)$

$= (1 - 4), (-1 - 3), (0 - 2)$

$\Rightarrow$ Direction ratios of line $AB = -3, -4, -2$

and, Directions ratios of the line joining $C(1, 2, -1)$ and $D(2, 1, 1)$

$= (2 - 1), (1 - 2), (1 + 1)$

$\Rightarrow$ Direction ratios of line $CD = 1, -1, 2$

Given that, line AB is perpendicular to line (2), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(-3) + (b)(-4) + (c)(-2) = 0$$
$$-3a + 4b - 2c = 0$$
$$3a + 4b + 2c = 0 \qquad ---(3)$$

and, line CD is also perpendicular to line (2), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$
$$(a)(1) + (b)(-1) + (c)(2) = 0$$
$$a - b + 2c = 0 \qquad ---(4)$$

Solving equation (3) and (4) using cross multiplication,

$$\frac{a}{(4)(2) - (-1)(2)} = \frac{b}{(1)(2) - (3)(2)} = \frac{c}{(3)(-1) - (4)(1)}$$

$$\Rightarrow \quad \frac{a}{8 + 2} = \frac{b}{2 - 6} = \frac{c}{-3 - 4}$$

$$\Rightarrow \quad \frac{a}{10} = \frac{b}{-4} = \frac{c}{-7} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad a = 10\lambda,\ b = -4\lambda,\ c = -7\lambda$$

Straight Line in Space Ex 28.2 Q18

We know that equation of a line passing through a point (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

So, equation of required line passing through $(1, 2, -4)$ is

$$\frac{x - 1}{a} = \frac{y - 2}{b} = \frac{z + 4}{c} \qquad ---(1)$$

Given that, line $\frac{x - 8}{8} = \frac{y + 9}{-16} = \frac{z - 10}{7}$ is perpendicular to line (1), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$\Rightarrow \quad (a)(8) + (b)(-16) + (c)(7) = 0$

$\Rightarrow \quad 8a - 16b + 7c = 0 \qquad ----(2)$

also, line $\frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5}$ is perpendicular to line (1), so

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$\Rightarrow \quad (3)(a) + (8)(b) + (-5)(c) = 0$

$\Rightarrow \quad 3a + 8b - 5c = 0 \qquad ----(3)$

Solving equation (2) and (3) by cross-multiplication,

$$\frac{a}{(-16)(-5) - (8)(7)} = \frac{b}{(3)(7) - (8)(-5)} = \frac{c}{(8)(8) - (3)(-16)}$$

$\Rightarrow \quad \frac{a}{80 - 56} = \frac{b}{21 + 40} = \frac{c}{64 + 48}$

$\Rightarrow \quad \frac{a}{24} = \frac{b}{61} = \frac{c}{112} = \lambda$ (Say)

$\Rightarrow \quad a = 24\lambda,\ b = 61\lambda,\ c = 112\lambda$

Put a, b, c in equation (1) to get required equation of the line, so

$$\frac{x - 1}{24\lambda} = \frac{y - 2}{61\lambda} = \frac{z + 4}{112\lambda}$$

$\Rightarrow \quad \frac{x - 1}{24} = \frac{y - 2}{61} = \frac{z + 4}{112}$

Straight Line in Space Ex 28.2 Q19

Equation of lines are,

$$\frac{x - 5}{7} = \frac{y + 2}{-5} = \frac{z}{1}$$

and, $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$

Now, $a_1a_2 + b_1b_2 + c_1c_2$

$= (7)(1) + (-5)(2) + (1)(3)$

$= 7 - 10 + 3$

$= 0$

So, given lines are perpendicular.

Straight Line in Space Ex 28.2 Q20

We know that, equation of a line passing through the point (x_1, y_1, z_1) and direction ratios a, b, c is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad ---(1)$$

So, equation of line passing through $(2, -1, -1)$ is

$$\frac{x - 2}{a} = \frac{y + 1}{b} = \frac{z + 1}{c} \qquad ---(2)$$

Line (2) is parallel to given line,

$$6x - 2 = 3y + 1 = 2z - 2$$

$$\Rightarrow \quad \frac{6x - 2}{6} = \frac{3y + 1}{6} = \frac{2z - 2}{6}$$

$$\Rightarrow \quad \frac{x - \frac{1}{3}}{1} = \frac{y + \frac{1}{2}}{2} = \frac{z - \frac{1}{3}}{3}$$

So, $a = \lambda$, $b = 2\lambda$, $c = 3\lambda$

Using a, b, c in equation (2) to get required equation of line,

$$\frac{x - 2}{\lambda} = \frac{y + 1}{2\lambda} = \frac{z + 1}{3\lambda}$$

$$\Rightarrow \quad \frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z + 1}{3} = \lambda \text{ (Say)}$$

$$\Rightarrow \quad x = \lambda + 2,\ y = 2\lambda - 1,\ z = 3\lambda - 1$$

So,

$$x\hat{i} + y\hat{j} + z\hat{k} = (\lambda + 2)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 1)\hat{k}$$

$$\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$

Straight Line in Space Ex 28.2 Q21

The direction of ratios of the lines, $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$, are -3, $2k$, 2 and $3k$, 1, -5 respectively.
It is known that two lines with direction ratios, a_1, b_1, c_1 and a_2, b_2, c_2, are perpendicular, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore -3(3k) + 2k \times 1 + 2(-5) = 0$$

$$\Rightarrow -9k + 2k - 10 = 0$$

$$\Rightarrow 7k = -10$$

$$\Rightarrow k = \frac{-10}{7}$$

Therefore, for $k = -\frac{10}{7}$, the given lines are perpendicular to each other.

Straight Line in Space Ex 28.2 Q22

The coordinates of A, B, C, and D are (1, 2, 3), (4, 5, 7), (−4, 3, −6), and (2, 9, 2) respectively.
The direction ratios of AB are (4 − 1) = 3, (5 − 2) = 3, and (7 − 3) = 4
The direction ratios of CD are (2 −(− 4)) = 6, (9 − 3) = 6, and (2 −(−6)) = 8

It can be seen that, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$

Therefore, AB is parallel to CD.
Thus, the angle between AB and CD is either 0° or 180°.

Straight Line in Space Ex 28.2 Q23

Given equation of line are,

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1} \text{ and}$$

$$\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$$

$$\Rightarrow \quad \frac{x-5}{5\lambda+2} = \frac{y-2}{-5} = \frac{z-1}{1} \quad ---(1)$$

$$\text{and,} \quad \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3} \quad ---(2)$$

Given that line (1) and (2) are perpendicular,

So, $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(5\lambda+2)(1) + (-5)(2\lambda) + (1)(3) = 0$$

$$5\lambda + 2 - 10\lambda + 3 = 0$$

$$-5\lambda + 5 = 0$$

$$\lambda = \frac{5}{5}$$

$$\lambda = 1$$

Straight Line in Space Ex 28.2 Q24

The direction ratios of the line are

$$\frac{x+2}{2} = \frac{2y-7}{6} = \frac{5-z}{6}$$

2,6,6

The direction cosines of the line are

$$l = \frac{2}{\sqrt{2^2+6^2+6^2}} = \frac{2}{\sqrt{76}}$$

$$m = \frac{6}{\sqrt{2^2+6^2+6^2}} = \frac{6}{\sqrt{76}}$$

$$n = \frac{6}{\sqrt{2^2+6^2+6^2}} = \frac{6}{\sqrt{76}}$$

$$\left(\frac{2}{\sqrt{76}}, \frac{6}{\sqrt{76}}, \frac{6}{\sqrt{76}}\right)$$

∴ Vector equation of the line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\vec{r} = (-\vec{i} + 2\vec{j} + 3\vec{k}) + \lambda(2\vec{i} + 6\vec{j} + 6\vec{k})$$