RD Sharma Solutions Class 12 Maths Chapter 24 Ex 24.2

Scalar or Dot Product Ex 24.2 Q1

Let $\vec{o}$, $\vec{a}$ and $\vec{b}$ be the position vector of the O, A and B.

P and Q are points of trisection of AB.

Position vector of point P $= \dfrac{2\vec{a} + \vec{b}}{3}$

Position vector of point Q $= \dfrac{\vec{a} + 2\vec{b}}{3}$

$$OP = \frac{2\vec{a} + \vec{b}}{3} - \vec{o} = \frac{2\vec{a} + \vec{b} - 3\vec{o}}{3} = \frac{2OA + OB}{3}$$

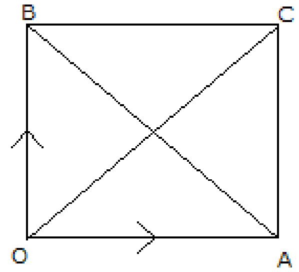
$$OQ = \frac{\vec{a} + 2\vec{b}}{3} - \vec{o} = \frac{\vec{a} + 2\vec{b} - 3\vec{o}}{3} = \frac{OA + 2OB}{3}$$

$$OP^2 + OQ^2 = \left(\frac{2OA + OB}{3}\right)^2 + \left(\frac{OA + 2OB}{3}\right)^2$$

$$= \frac{5\left(OA^2 + OB^2\right) + 8(OA)(OB)\cos 90^\circ}{9}$$

$$= \frac{5}{9}AB^2 \ldots\ldots\ldots\left[\because OA^2 + OB^2 = AB^2 \text{ and } \cos 90^\circ = 0\right]$$

Scalar or Dot Product Ex 24.2 Q2

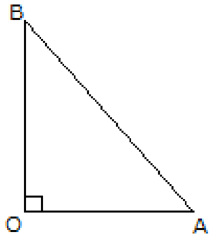


Let OACB be a quadrilateral such that its diagonal bisect each other at right angles.
We know that if the diagonals of a quadrilateral bisect each other then its a parallelogram.
$\therefore$ OACB is a parallelogram.
$\Rightarrow$ OA = BC and OB = AC.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the A and B.

AB and OC be the diagonals of quadrilateral which bisect each other at right angles.
$\therefore \overrightarrow{OC} \bullet \overrightarrow{AB} = 0$
$\Rightarrow \left(\vec{a}+\vec{b}\right) \bullet \left(\vec{b}-\vec{a}\right) = 0$
$\Rightarrow \left|\vec{b}\right|^2 = \left|\vec{a}\right|^2$
$\Rightarrow$ OB = OA
Simillarly we can show that
OA = OB = BC = CA
Hence OACB is a rhombus.

Scalar or Dot Product Ex 24.2 Q3



Let OAC be a right triangle, right angled at O.

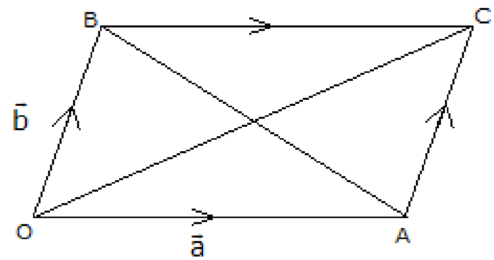
Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$
$\therefore \overrightarrow{OA} \bullet \overrightarrow{OB} = 0$
$\vec{a} \bullet \vec{b} = 0$

Now,
$\overrightarrow{AB}^2 = \left(\vec{b}-\vec{a}\right)^2 = \left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 - 2\vec{a}\bullet\vec{b} = \left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 - 0 = \left(\overrightarrow{OA}\right)^2 + \left(\overrightarrow{OB}\right)^2$

Hence proved.

Scalar or Dot Product Ex 24.2 Q4



Let OAC be a right triangle, right angled at O.

Taking O as origin let $\vec{a}$ and $\vec{b}$ be the position vector of the $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$

$\therefore \overrightarrow{OA} \bullet \overrightarrow{OB} = 0$

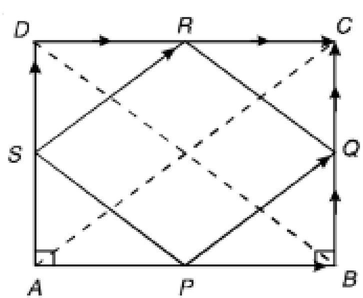
$\vec{a} \bullet \vec{b} = 0$

Now,

$\overrightarrow{AB}^2 = \left(\vec{b}-\vec{a}\right)^2 = \left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 - 2\vec{a} \bullet \vec{b} = \left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 - 0 = \left(\overrightarrow{OA}\right)^2 + \left(\overrightarrow{OB}\right)^2$

Hence proved.

Scalar or Dot Product Ex 24.2 Q5



ABCD be a rectangle.
Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively.
Now,

$\overrightarrow{PQ} = \overrightarrow{PB} + \overrightarrow{BQ} = \frac{1}{2}\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \frac{1}{2}\overrightarrow{AC}$........(i)

$\overrightarrow{SR} = \overrightarrow{SD} + \overrightarrow{DR} = \frac{1}{2}\left(\overrightarrow{AD} + \overrightarrow{DC}\right) = \frac{1}{2}\overrightarrow{AC}$........(ii)

From (i) and (ii), we have

$\overrightarrow{PQ} = \overrightarrow{SR}$ i.e. sides PQ and SR are equal and parallel.
$\therefore$ PQRS is a parallelogram.

$(PQ)^2 = \overrightarrow{PQ} \bullet \overrightarrow{PQ} = \left(\overrightarrow{PB} + \overrightarrow{BQ}\right) \bullet \left(\overrightarrow{PB} + \overrightarrow{BQ}\right) = |PB|^2 + |BQ|^2$........(iii)

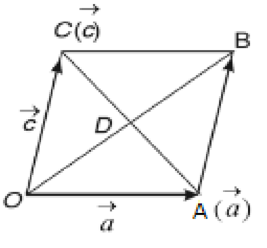
$(PS)^2 = \overrightarrow{PS} \bullet \overrightarrow{PS} = \left(\overrightarrow{PA} + \overrightarrow{PS}\right) \bullet \left(\overrightarrow{PA} + \overrightarrow{PS}\right) = |PA|^2 + |AS|^2 = |PB|^2 + |BQ|^2$........(iv)

From (iii) and (iv) we get,

$(PQ)^2 = (PQ)^2$ i. e. PQ = PS

$\Rightarrow$ The adjacent sides of PQRS are equal.
$\therefore$ PQRS is a rhombus.

Scalar or Dot Product Ex 24.2 Q6



Let OABC be a rhombus, whose diagonals OB and AC intersect at point D.
Let O be the origin.
Let the position vector of A and C be $\vec{a}$ and $\vec{c}$ respectively then,

$\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OC} = \vec{c}$

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OA} + \overrightarrow{OC} = \vec{a} + \vec{c}$............$\left[\because \overrightarrow{AB} = \overrightarrow{OC}\right]$

Position vector of mid-point of $\overrightarrow{OB} = \frac{1}{2}\left(\vec{a} + \vec{c}\right)$

Position vector of mid-point of $\overrightarrow{AC} = \frac{1}{2}\left(\vec{a} + \vec{c}\right)$

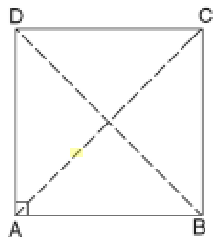
$\therefore$ Midpoints of OB and AC coincide.
$\therefore$ Diagonal OB and AC bisect each other.

$\overrightarrow{OB} \bullet \overrightarrow{AC} = \left(\vec{a} + \vec{c}\right) \bullet \left(\vec{c} - \vec{a}\right) = \left(\vec{c} + \vec{a}\right) \bullet \left(\vec{c} - \vec{a}\right) = \left|\vec{c}\right|^2 - \left|\vec{a}\right|^2 = \overrightarrow{OC} - \overrightarrow{OA} = 0$

$\left[\because \text{OC and OA are sides of the rhombus}\right]$

$\Rightarrow \overrightarrow{OB} \perp \overrightarrow{AC}$

Scalar or Dot Product Ex 24.2 Q7



Let ABCD be a rectangle.

Take A as origin.

Let position vectors of point B, D be $\vec{a}$ and $\vec{b}$ respectively.

By parallelogram law,

$\overrightarrow{AC} = \vec{a} + \vec{b}$ and $\overrightarrow{BD} = \vec{a} - \vec{b}$

As ABCD is a rectangle, AB ⊥ AD

$\Rightarrow \vec{a} \bullet \vec{b} = 0$.........(i)

Now, diagonals AC and BD are perpendicular iff $\overrightarrow{AC} \bullet \overrightarrow{BD} = 0$

$\Rightarrow \left(\vec{a} + \vec{b}\right)\left(\vec{a} - \vec{b}\right) = 0$

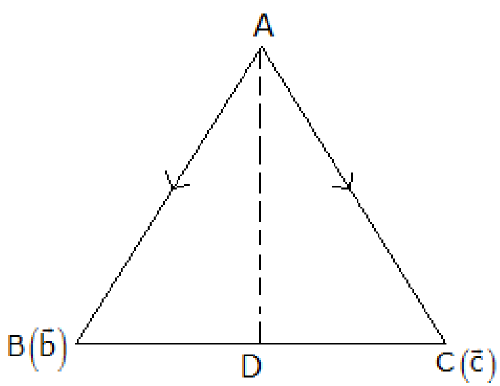
$\Rightarrow \left(\vec{a}\right)^2 - \left(\vec{b}\right)^2 = 0$

$\Rightarrow \left|\overrightarrow{AB}\right|^2 = \left|\overrightarrow{AD}\right|^2$

$\Rightarrow |AB| = |AD|$

Hence ABCD is a square.

Scalar or Dot Product Ex 24.2 Q8



Take A as origin, let the position vectors of B and C are $\vec{b}$ and $\vec{c}$ respectively.

Position vector of D = $\frac{\vec{b}+\vec{c}}{2}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$.

$$\overrightarrow{AD} = \frac{\vec{b}+\vec{c}}{2} - \vec{0} = \frac{\vec{b}+\vec{c}}{2}$$

Consider, $2\left(AD^2 + CD^2\right)$

$$= 2\left[\left(\frac{\vec{b}+\vec{c}}{2}\right)^2 + \left(\frac{\vec{b}+\vec{c}}{2} - \vec{c}\right)^2\right]$$

$$= 2\left[\left(\frac{\vec{b}+\vec{c}}{2}\right)^2 + \left(\frac{\vec{b}-\vec{c}}{2}\right)^2\right]$$

$$= \frac{1}{2}\left[\left(\vec{b}+\vec{c}\right)^2 + \left(\vec{b}-\vec{c}\right)^2\right]$$

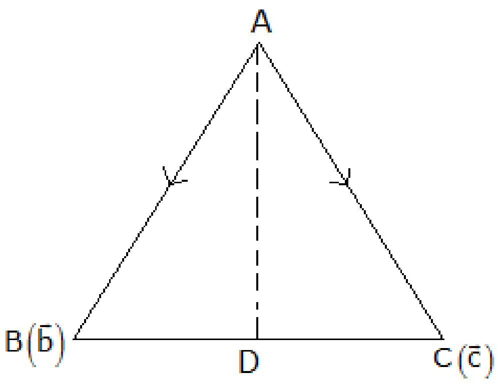
$$= \left(\vec{b}\right)^2 + \left(\vec{c}\right)^2$$

$$= \left(\overrightarrow{AB}\right)^2 + \left(\overrightarrow{AC}\right)^2$$

$$= AB^2 + AC^2$$

Hence proved.

Scalar or Dot Product Ex 24.2 Q9



Take A as origin, let the position vectors of B and C are $\vec{b}$ and $\vec{c}$ respectively.

Position vector of D = $\frac{\vec{b}+\vec{c}}{2}$, $\overrightarrow{AB} = \vec{b}$ and $\overrightarrow{AC} = \vec{c}$.

$$\overrightarrow{AD} = \frac{\vec{b}+\vec{c}}{2} - \vec{0} = \frac{\vec{b}+\vec{c}}{2}$$

AD is perpendicular to BC

$$\Rightarrow \overrightarrow{AD} \bullet \overrightarrow{BC} = 0$$

$$\Rightarrow \left(\frac{\vec{b}+\vec{c}}{2}\right) \bullet \left(\vec{c} - \vec{b}\right) = 0$$

$$\Rightarrow \left(\vec{b}+\vec{c}\right) \bullet \left(\vec{c} - \vec{b}\right) = 0$$

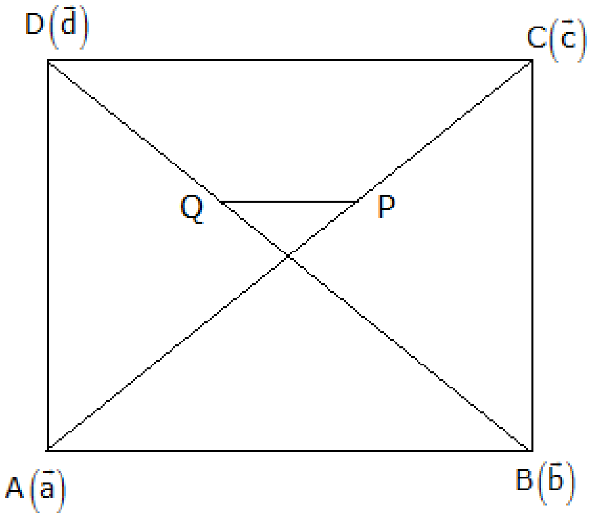
$$\Rightarrow \left|\vec{c}\right|^2 = \left|\vec{b}\right|^2$$

$$\Rightarrow \left|\vec{c}\right| = \left|\vec{b}\right|$$

$$\Rightarrow AC = AB$$

Hence ΔABC is an isoscales triangle.

Scalar or Dot Product Ex 24.2 Q10



Take O as origin, let the position vectors of A, B C and D are $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ respectively.

Position vector of P = $\dfrac{\vec{a}+\vec{c}}{2}$

Position vector of Q = $\dfrac{\vec{a}+\vec{d}}{2}$

LHS = $AB^2 + BC^2 + CD^2 + DA^2$

$= \left(\vec{b}-\vec{a}\right)^2 + \left(\vec{c}-\vec{b}\right)^2 + \left(\vec{d}-\vec{c}\right)^2 + \left(\vec{d}-\vec{a}\right)^2$

$= 2\left[\left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 + \left(\vec{c}\right)^2 + \left(\vec{d}\right)^2 - \vec{a}\vec{b}\cos\theta_1 - \vec{b}\vec{c}\cos\theta_2 - \vec{d}\vec{c}\cos\theta_3 - \vec{c}\vec{a}\cos\theta_4\right]$

RHS = $AC^2 + BD^2 + 4PQ^2$

$= \left(\vec{c}-\vec{a}\right)^2 + \left(\vec{d}-\vec{b}\right)^2 + 4\left(\dfrac{\vec{a}+\vec{d}}{2} - \dfrac{\vec{a}+\vec{c}}{2}\right)^2$

$= 2\left[\left(\vec{a}\right)^2 + \left(\vec{b}\right)^2 + \left(\vec{c}\right)^2 + \left(\vec{d}\right)^2 - \vec{a}\vec{b}\cos\theta_1 - \vec{b}\vec{c}\cos\theta_2 - \vec{d}\vec{c}\cos\theta_3 - \vec{c}\vec{a}\cos\theta_4\right]$

= LHS

Hence proved.