RD Sharma Solutions Class 12 Maths Chapter 23 Ex 23.7

Algebra of Vectors Ex 23.7 Q1

Here, position vector of A = Position vector of $A = \vec{a} - 2\vec{b} + 3\vec{c}$

position vector of B = Position vector of $B = 2\vec{a} + 3\vec{b} - 4\vec{c}$

position vector of C = Position vector of $C = -7\vec{b} + 10\vec{c}$

$\overrightarrow{AB}$ = position vector of B − position vector of A

$$= \left(2\vec{a} + 3\vec{b} - 4\vec{c}\right) - \left(\vec{a} - 2\vec{b} + 3\vec{c}\right)$$

$$= 2\vec{a} + 3\vec{b} - 4\vec{c} - \vec{a} + 2\vec{b} - 3\vec{c}$$

$$\overrightarrow{AB} = \vec{a} + 5\vec{b} - 7\vec{c}$$

$\overrightarrow{BC}$ = position vector of C − position vector of B

$$= \left(-7\vec{b} + 10\vec{c}\right) - \left(2\vec{a} + 3\vec{b} - 4\vec{c}\right)$$

$$= -7\vec{b} + 10\vec{c} - 2\vec{a} - 3\vec{b} + 4\vec{c}$$

$$\overrightarrow{BC} = -2\vec{a} - 10\vec{b} + 14\vec{c}$$

From $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$$\overrightarrow{BC} = -2\left(\overrightarrow{AB}\right)$$

So, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are parallel but $\vec{B}$ is a common vector. Hence, A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q2(i)

Let the points be A, B, C

Position vector of $A = \vec{a}$

Position vector of $B = \vec{b}$

Position vector of $C = 3\vec{a} - 2\vec{b}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= \vec{b} - \vec{a}$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$= 3\vec{a} - 2\vec{b} - \vec{b}$

$= 3\vec{a} - 3\vec{b}$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$

Let $\overrightarrow{BC} = \lambda\left(\overrightarrow{AB}\right)$ [where λ is and scalar]

$3\vec{a} - 3\vec{b} = \lambda\left(\vec{b} - \vec{a}\right)$

$3\vec{a} - 3\vec{b} = \lambda\vec{b} - \lambda\vec{a}$

$3\vec{a} - 3\vec{b} = \lambda\vec{a} + \lambda\vec{b}$

Comparing the coefficients of LHS and RHS, we get

$-\lambda = 3$

$\lambda = 3$

$\lambda = -3$

Since the value of λ are different.

Therefore,

A, B, C are not collinear.

Algebra of Vectors Ex 23.7 Q2(ii)

Let the points be A, B, C

Position vector of $A = \vec{a} + \vec{b} + \vec{c}$
Position vector of $B = 4\vec{a} + 3\vec{b}$
Position vector of $C = 10\vec{a} + 7\vec{b} - 2\vec{c}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(4\vec{a} + 3\vec{b}\right) - \left(\vec{a} + \vec{b} + \vec{c}\right)$$
$$= 4\vec{a} + 3\vec{b} - \vec{a} - \vec{b} - \vec{c}$$
$$\overrightarrow{AB} = 3\vec{a} + 2\vec{b} - \vec{c}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(10\vec{a} + 7\vec{b} - 2\vec{c}\right) - \left(4\vec{a} + 3\vec{b}\right)$$
$$= 10\vec{a} + 7\vec{b} - 2\vec{c} - 4\vec{a} - 3\vec{b}$$
$$\overrightarrow{BC} = 6\vec{a} + 4\vec{b} - 2\vec{c}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$

$$\overrightarrow{BC} = 2\left(\overrightarrow{AB}\right)$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence, A,B,C are collinear.

Algebra of Vectors Ex 23.7 Q3

Let the points be A, B, C

Position vector of $A = \hat{i} + 2\hat{j} + 3\hat{k}$
Position vector of $B = 3\hat{i} + 4\hat{j} + 7\hat{k}$
Position vector of $C = -3\hat{i} - 2\hat{j} - 5\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right) - \left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$$
$$= 3\hat{i} + 4\hat{j} + 7\hat{k} - \hat{i} - 2\hat{j} - 3\hat{k}$$
$$\overrightarrow{AB} = 2\hat{i} + 2\hat{j} + 4\hat{k}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(-3\hat{i} - 2\hat{j} - 5\hat{k}\right) - \left(3\hat{i} + 4\hat{j} + 7\hat{k}\right)$$
$$= -3\hat{i} - 2\hat{j} - 5\hat{k} - 3\hat{i} - 4\hat{j} - 7\hat{k}$$
$$\overrightarrow{BC} = -6\hat{i} - 6\hat{j} - 12\hat{k}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$ we get

$$\overrightarrow{BC} = -3\left(\overrightarrow{AB}\right)$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence, A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q4

Let the points be A, B, C

Position vector of $A = 10\hat{i} + 3\hat{j}$

Position vector of $B = 12\hat{i} - 5\hat{j}$

Position vector of $C = a\hat{i} + 11\hat{j}$

Given that, A,B,C are collinear

$\Rightarrow \overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear

$\Rightarrow \overrightarrow{AB} = \lambda\left(\overrightarrow{BC}\right)$ (Where λ is same scalar)

$\Rightarrow$ Position vector of B – Position vector of $A = \lambda -$ (Position vector of C – Position vector of B)

$$\Rightarrow \left(12\hat{i} - 5\hat{j}\right) - \left(10\hat{i} + 3\hat{j}\right) = \lambda\left[\left(a\hat{i} + 11\hat{j}\right) - \left(12\hat{i} - 5\hat{j}\right)\right]$$

$$\Rightarrow 12\hat{i} - 5\hat{j} - 10\hat{i} - 3\hat{j} = \lambda\left(a\hat{i} + 11\hat{j} - 12\hat{i} + 5\hat{j}\right)$$

$$\Rightarrow 2\hat{i} - 8\hat{j} = \left(\lambda a - 12\lambda\right)\hat{i} = \left(11\lambda + 5\lambda\right)\hat{j}$$

Comparing the coefficients of LHS and RHS, we get

$\lambda a - 12\lambda = 2$ (i)

$-8 = 11\lambda + 5\lambda$ (ii)

$-8 = 16\lambda$

$\lambda = \frac{-8}{16}$

$\lambda = -\frac{1}{2}$

Put the value of λ in equation (i),

$$\lambda a - 12\lambda = 2$$

$$\left(-\frac{1}{2}\right)a - 12\left(-\frac{1}{2}\right) = 2$$

$$-\frac{1}{2}a + \frac{12}{2} = 2$$

$$-\frac{1}{2}a + 6 = 2$$

$$-\frac{1}{2}a = 2 - 6$$

$$-\frac{1}{2}a = -4$$

$$a = (-4) \times (-2)$$

$$a = 8$$

Algebra of Vectors Ex 23.7 Q5

Let A, B, C be the points then

Position vector of $A = \vec{a} + \vec{b}$
Position vector of $B = \vec{a} - \vec{b}$
Position vector of $C = \vec{a} + \lambda\vec{b}$

$\overrightarrow{AB}$ = Position vector of B − Position vector of A

$$= \left(\vec{a} - \vec{b}\right) - \left(\vec{a} + \vec{b}\right)$$
$$= \vec{a} - \vec{b} - \vec{a} - \vec{b}$$
$$\overrightarrow{AB} = -2\vec{b}$$

$\overrightarrow{BC}$ = Position vector of C − Position vector of B

$$= \left(\vec{a} + \lambda\vec{b}\right) - \left(\vec{a} - \vec{b}\right)$$
$$= \vec{a} + \lambda\vec{b} - \vec{a} + \vec{b}$$
$$= \lambda\vec{b} + \vec{b}$$
$$\overrightarrow{BC} = \left(\lambda + 1\right)\vec{b}$$

Using $\overrightarrow{AB}$ and $\overrightarrow{BC}$, we get

$$\overrightarrow{AB} = \left[\frac{(\lambda + 1)}{-2}\right]\left(\overrightarrow{BC}\right)$$

Let $\left(\frac{\lambda + 1}{-2}\right) = \mu$

Since λ is a real number. So,
μ is also a real no.

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$, but $\vec{B}$ is a common vector. Hence,
A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q6

Here, $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OB} + \overrightarrow{OC}$

$$\overrightarrow{OA} - \overrightarrow{BO} = \overrightarrow{BO} - \overrightarrow{CO}$$
$$\overrightarrow{AB} = \overrightarrow{BC}$$

So, $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but $\vec{B}$ is a common vector. Hence,
A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q7

Let the given points be A and B

Position vector of $A = 2\hat{i} - 3\hat{j} + 4\hat{k}$

Position vector of $B = -4\hat{i} + 6\hat{j} - 8\hat{k}$

Let O be the initial point having postion vector

$$0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$$

$\overrightarrow{OA}$ = Position vector of A – Position vector of O

$$= \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) - \left(0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}\right)$$

$$= 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$\overrightarrow{OB}$ = Position vector of B – Position vector of O

$$= \left(-4\hat{i} + 6\hat{j} - 8\hat{k}\right) - \left(0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}\right)$$

$$\overrightarrow{OB} = -4\hat{i} + 6\hat{j} - 8\hat{k}$$

Using OA and OB, we get

$$\overrightarrow{OB} = -2\left(\overrightarrow{OA}\right)$$

Therefore, $\overrightarrow{OA}$ is parallel to $\overrightarrow{OB}$ but O is the common point to them. Hence, A and B are collinear.

Algebra of Vectors Ex 23.7 Q8

Here, $A = (m, -1)$

$B = (2, 1)$

$C = (4, 5)$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(2\hat{i} + \hat{j}\right) - \left(m\hat{i} - \hat{j}\right)$$

$$= 2\hat{i} + \hat{j} - m\hat{i} + \hat{j}$$

$$= (2 - m)\hat{i} + 2\hat{j}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(4\hat{i} + 5\hat{j}\right) - \left(2\hat{i} + \hat{j}\right)$$

$$= 4\hat{i} + 5\hat{j} - 2\hat{i} - \hat{j}$$

$$\overrightarrow{BC} = 2\hat{i} + 4\hat{j}$$

A, B, C are collinear. So, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are collinear.

So, $\overrightarrow{AB} = \lambda\left(\overrightarrow{BC}\right)$

$(2 - m)\hat{i} + 2\hat{j} = \lambda\left(2\hat{i} + 4\hat{j}\right)$, for λ scalar

$(2 - m)\hat{i} + 2\hat{j} = 2\lambda i + 4\lambda\hat{j}$

Comparing the coefficient of LHS and RHS.

$$2 - m = 2\lambda$$

$$\frac{2 - m}{2} = \lambda \qquad \text{(i)}$$

$$2 = 4\lambda$$
$$\frac{2}{4} = \lambda$$
$$\frac{1}{2} = \lambda \qquad \text{(ii)}$$

Using (i) and (ii)

$$\frac{2-m}{2} = \frac{1}{2}$$
$$4 - 2m = 2$$
$$-2m = 2$$
$$-2m = 2 - 4$$
$$-2m = -2$$
$$m = \frac{-2}{-2}$$
$$m = 1$$

$\therefore m = 1$

Algebra of Vectors Ex 23.7 Q9

Here, let $A = (3, 4)$
$B = (-5, 16)$
$C = (5, 1)$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$$= \left(-5\hat{i} + 16\hat{j}\right) - \left(3\hat{i} + 4\hat{j}\right)$$
$$= -5\hat{i} + 16\hat{j} - 3\hat{i} - 4\hat{j}$$
$$\overrightarrow{AB} = -8\hat{i} + 12\hat{j}$$

$\overrightarrow{BC}$ = Position vector of C – Position vector of B

$$= \left(5\hat{i} + \hat{j}\right) - \left(-5\hat{i} + 16\hat{j}\right)$$
$$= 5\hat{i} + \hat{j} + 5\hat{i} - 16\hat{j}$$
$$\overrightarrow{BC} = 10\hat{i} - 15\hat{j}$$

So, $4\left(\overrightarrow{AB}\right) = -5\left(\overrightarrow{BC}\right)$

$\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$ but B is a common point.

Hence, A, B, C are collinear.

Algebra of Vectors Ex 23.7 Q10

Here, it is given that vectors

$a = 2\hat{i} - 3\hat{j}$ and $b = -6\hat{i} + m\hat{j}$ are collinear.

So, $a = \lambda b$, for a scalar λ

$2\hat{i} - 3\hat{j} = \lambda\left(-6\hat{i} + m\hat{j}\right)$

$2\hat{i} - 3\hat{j} = -6\lambda\hat{i} + \lambda m\hat{j}$

Comparing the coefficients of LHS and RHS,

$2 = -6\lambda$

$\lambda = \frac{2}{-6}$

$\lambda = \frac{-1}{3}$ (i)

$-3 = \lambda m$

$\lambda = \frac{-3}{m}$ (ii)

From (i) and (ii),

$\frac{-1}{3} = \frac{-3}{m}$

$m = 3 \times 3$

$= 9$

$\therefore m = 9$

Algebra of Vectors Ex 23.7 Q11

The given points are A (1, -2, -8), B (5, 0, -2), and C (11, 3, 7).

$$\therefore \overrightarrow{AB} = (5-1)\hat{i} + (0+2)\hat{j} + (-2+8)\hat{k} = 4\hat{i} + 2\hat{j} + 6\hat{k}$$
$$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7+2)\hat{k} = 6\hat{i} + 3\hat{j} + 9\hat{k}$$
$$\overrightarrow{AC} = (11-1)\hat{i} + (3+2)\hat{j} + (7+8)\hat{k} = 10\hat{i} + 5\hat{j} + 15\hat{k}$$
$$\left|\overrightarrow{AB}\right| = \sqrt{4^2 + 2^2 + 6^2} = \sqrt{16+4+36} = \sqrt{56} = 2\sqrt{14}$$
$$\left|\overrightarrow{BC}\right| = \sqrt{6^2 + 3^2 + 9^2} = \sqrt{36+9+81} = \sqrt{126} = 3\sqrt{14}$$
$$\left|\overrightarrow{AC}\right| = \sqrt{10^2 + 5^2 + 15^2} = \sqrt{100+25+225} = \sqrt{350} = 5\sqrt{14}$$
$$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$$

Thus, the given points A, B, and C are collinear.

Now, let point B divide AC in the ratio $\lambda : 1$. Then, we have:

$$\overrightarrow{OB} = \frac{\lambda\overrightarrow{OC} + \overrightarrow{OA}}{(\lambda+1)}$$
$$\Rightarrow 5\hat{i} - 2\hat{k} = \frac{\lambda\left(11\hat{i} + 3\hat{j} + 7\hat{k}\right) + \left(\hat{i} - 2\hat{j} - 8\hat{k}\right)}{\lambda+1}$$
$$\Rightarrow (\lambda+1)\left(5\hat{i} - 2\hat{k}\right) = 11\lambda\hat{i} + 3\lambda\hat{j} + 7\lambda\hat{k} + \hat{i} - 2\hat{j} - 8\hat{k}$$
$$\Rightarrow 5(\lambda+1)\hat{i} - 2(\lambda+1)\hat{k} = (11\lambda+1)\hat{i} + (3\lambda-2)\hat{j} + (7\lambda-8)\hat{k}$$

On equating the corresponding components, we get:

$$5(\lambda+1) = 11\lambda + 1$$
$$\Rightarrow 5\lambda + 5 = 11\lambda + 1$$
$$\Rightarrow 6\lambda = 4$$
$$\Rightarrow \lambda = \frac{4}{6} = \frac{2}{3}$$

Hence, point B divides AC in the ratio $2:3$.

Algebra of Vectors Ex 23.7 Q12

We have

$\overrightarrow{AP}$ = Position vector of P − Position vector of A

$\Rightarrow \overrightarrow{AP} = \hat{i} + 2\hat{j} + 3\hat{k} - (-2\hat{i} + 3\hat{j} + 5\hat{k}) = 3\hat{i} - \hat{j} - 2\hat{k}$

$\overrightarrow{PB}$ = Position vector of B − Position vector of P

$\Rightarrow \overrightarrow{PB} = 7\hat{i} - \hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k}) = 6\hat{i} - 2\hat{j} - 4\hat{k}$

Clearly, $\overrightarrow{PB} = 2\overrightarrow{AP}$

so vectors $\overrightarrow{AP}$ and $\overrightarrow{PB}$ are collinear.

But P is a point common to $\overrightarrow{AP}$ and $\overrightarrow{PB}$.

Hence P, A, B are collinear points.

Similarly, $\overrightarrow{CP} = \hat{i} + 2\hat{j} + 3\hat{k} - (-3\hat{i} - 2\hat{j} - 5\hat{k}) = 4\hat{i} + 4\hat{j} + 8\hat{k}$

and $\overrightarrow{PD} = 3\hat{i} + 4\hat{j} + 7\hat{k} - (\hat{i} + 2\hat{j} + 3\hat{k}) = 2\hat{i} + 2\hat{j} + 4\hat{k}$

So vectors $\overrightarrow{CP}$ and $\overrightarrow{PD}$ are collinear.

But P is a common point to $\overrightarrow{CP}$ and $\overrightarrow{CD}$.

Hence, C, P, D are collinear points.

Thus, A, B, C, D and P are points such that A, P, B and C, P, D are two sets of collinear points. Hence AB and CD intersect at the point P

Algebra of Vectors Ex 23.7 Q13

Points (λ, - 10, 3), (1 -1, 3) and (3, 5, 3) are collinear.

∴ (λ, - 10, 3) = x(1 -1, 3) + y(3, 5, 3) for some scalars x and y.

⇒ λ = x + 3y, -10 = -x + 5y and 3 = 3x + 3y

Solving -10 = -x + 5y and 3 = 3x + 3y for x and y we get,

$x = \frac{5}{2}$ and $y = -\frac{3}{2}$

Now,

λ = x + 3y

$\Rightarrow \lambda = \frac{5}{2} + 3\left(-\frac{3}{2}\right) = -2$