RD Sharma Solutions Class 12 Maths Chapter 23 Ex 23.3

Algebra of Vectors Ex 23.3 Q1

Point R divides the line joining the two points P and Q in the ratio 1:2 internally.

$$\text{Position vector of point R} = \frac{1\left(\vec{a}-2\vec{b}\right)+2\left(2\vec{a}+\vec{b}\right)}{1+2} = \frac{5\vec{a}}{3}$$

Point R divides the line joining the two points P and Q in the ratio 1:2 externally.

$$\text{Position vector of point R} = \frac{1\left(\vec{a}-2\vec{b}\right)-2\left(2\vec{a}+\vec{b}\right)}{1-2} = \frac{-3\vec{a}-4\vec{b}}{-1} = 3\vec{a}+4\vec{b}$$

Algebra of Vectors Ex 23.3 Q2

Here itis given that $\vec{a},\vec{b},\vec{c}$ and $\vec{d}$ be the position vectors of the four distinct points A,B,C,D such that

$\vec{b}-\vec{a}=\vec{c}-\vec{d}$

Given that,

$\vec{b}-\vec{a}=\vec{c}-\vec{d}$

$\overrightarrow{AB}=\overrightarrow{DC}$

So, AB is parallel and equal to DC (in magnitude).

Hence,

$ABCD$ *is* a parallelogram.

Algebra of Vectors Ex 23.3 Q3

Here, it is given that $\vec{a}, \vec{b}$ are position vector of A and B.

Let C be a point in AB produced such that $AC = 3AB$.

It is clear that point C divides the line AB in ratio 3 : 2 externally.
So position vector point C is given by

$$\vec{c} = \frac{m\vec{b} - n\vec{a}}{m - n}$$

$$= \frac{3\vec{b} - 2\vec{a}}{3 - 2}$$

$$\vec{c} = 3\vec{b} - 2\vec{a}$$

Again, let D be a point in BA produced such that $BD = 2BA$.

Let $\vec{d}$ be the position vector of D. It is clear that point D divides the line AB in 1 : 2 externally. So position vector of D is given by

$$\vec{d} = \frac{m\vec{a} - n\vec{b}}{m - n}$$

$$= \frac{2\vec{a} - \vec{b}}{2 - 1}$$

$$\vec{d} = 2\vec{a} - \vec{b}$$

$$\vec{c} = 3\vec{b} - 2\vec{a}$$

$$\vec{d} = 2\vec{a} - \vec{b}$$

Algebra of Vectors Ex 23.3 Q4

We have given that

$$3\vec{a} - 2\vec{b} + 5\vec{c} - 6\vec{d} = \vec{0}$$

$$3\vec{a} + 5\vec{c} = 2\vec{b} + 6\vec{d} \qquad \text{(i)}$$

Sum of the coefficients on both the sides of the equation (i) is 8, so divide equation (i) by 8 on both the sides,

$$\frac{3\vec{a} + 5\vec{c}}{8} = \frac{2\vec{b} + 6\vec{d}}{8}$$

$$\frac{3\vec{a} + 5\vec{c}}{3 + 5} = \frac{2\vec{b} + 6\vec{d}}{2 + 6}$$

It shows that position vector of a point p dividing AC in the ratio 3 : 5, is same as that of a point dividing BD in the ratio of 2 : 6.

Hence, point P is common to AC and BD. Therefore, P is the point of intersection of AC and BD.

So, A, B, C and D are coplanar.

Position vector of point P is given by

$$\frac{3\vec{a} + 5\vec{c}}{8} \quad \text{or} \quad \frac{2\vec{b} + 6\vec{d}}{8}$$

Algebra of Vectors Ex 23.3 Q5

We have given that

$5\vec{p} - 2\vec{q} + 6\vec{r} - 9\vec{s} = \vec{0}$

Where $\vec{p}, \vec{q}, \vec{r}$ and $\vec{s}$ are the position vectors of point P, Q, R and S.

$5\vec{p} + 6\vec{r} = 2\vec{q} + 9\vec{s}$ (i)

Sum of the coefficients on both the sides of the equation (i) is 11. So divide equation (i) by 11 on both the sides.

$$\frac{5\vec{p} + 6\vec{r}}{11} = \frac{2\vec{q} + 9\vec{s}}{11}$$

$$\frac{5\vec{p} + 6\vec{r}}{5 + 6} = \frac{2\vec{q} + 9\vec{s}}{2 + 9}$$

It shows that position vector of a point A dividing PR in the ratio of $6:5$ and QS in the ratio of $9:2$. Thus, A is the common point to PR and QS and it is also point of intersection of PQ and QS.

So,

P, Q, R and S are coplanar

Position vector of point A is given by

$$\frac{5p + 6q}{11} \quad \text{or} \quad \frac{2\vec{q} + 9\vec{s}}{11}$$

Algebra of Vectors Ex 23.3 Q6

Let ABC be a triangle.

Let the position vectors of A, B and C with respect to some origin, O be $\vec{a}, \vec{b}$ and $\vec{c}$ respectively.

Let D be the point on BC where the bisector of $\angle A$ meets.

Let $\vec{d}$ position vector of D which divides BC internally in the ratio β and γ, where $\beta = |\overrightarrow{AC}|$ and $\gamma = |\overrightarrow{AB}|$

Thus, $\beta = |\vec{c} - \vec{a}|$ and $\gamma = |\vec{b} - \vec{a}|$

Thus, by section formula, the position vector of D is given by

$$\overrightarrow{OD} = \frac{\beta\vec{b} + \gamma\vec{c}}{\beta + \gamma}$$

Let $\alpha = |\vec{b} - \vec{c}|$

Incentre is the concurrent point of angle bisectors.

Thus, Incentre divides the line AD in the ratio $\alpha : \beta + \gamma$

Thus, the position vector of incentre is

$$\text{equal to } \frac{\alpha\vec{a} + \frac{\beta\vec{b} + \gamma\vec{c}}{(\beta + \gamma)} \times (\beta + \gamma)}{\alpha + \beta + \gamma} = \frac{\alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}}{\alpha + \beta + \gamma}$$