RD Sharma Solutions Class 12 Maths Chapter 23 Ex 23.2

Algebra of Vectors Ex 23.2 Q1

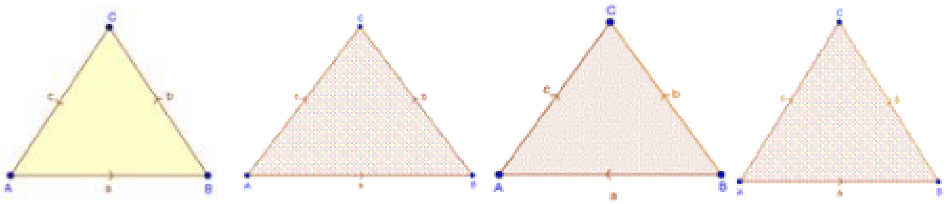
Given that, P, Q, R are collinear.

It also given that, $\overrightarrow{PQ} = \vec{a}$ and $\overrightarrow{QR} = \vec{b}$

$$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$$
$$= \vec{a} + \vec{b}$$

$$\overrightarrow{PR} = \vec{a} + \vec{b}$$

Algebra of Vectors Ex 23.2 Q2



Given that, $\vec{a}, \vec{b}$, and $\vec{c}$ are three sides of a triangle.

$\vec{a} + \vec{b} + \vec{c}$

$= \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

$= \overrightarrow{AC} + \overrightarrow{CA}$ $\left[\text{Since } \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\right]$

$= \overrightarrow{AC} - \overrightarrow{AC}$ $\left[\text{Since } \overrightarrow{CA} = \overrightarrow{AC}\right]$

$= \vec{0}$

So, $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

Triangle law says that, if vectors are represented in magnitude and direction by the two sides of triangle taken is same order, then their sum is represented by the third side taken in reverse order.

Thus,

$\vec{a} + \vec{b} = \vec{c}$

or

$\vec{a} + \vec{c} = \vec{b}$

$\vec{b} + \vec{c} = \vec{a}$

Algebra of Vectors Ex 23.2 Q3

Here, it is given that $\vec{a}$ and $\vec{b}$ are two non-collinear vectors having the same initial point.

Let $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{AD}$, So we can draw a parallelogram ABCD as above.

By the properties of parallelogram

$\overrightarrow{BC} = \vec{b}$ and $\overrightarrow{DC} = \vec{a}$

In ΔABC,

Using triangle law,

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\vec{a} + \vec{b} = \overrightarrow{AC}$ - -(i)

In ΔABD,

Using triangle law,

$\overrightarrow{AD} + \overrightarrow{DB} = \overrightarrow{AB}$

$\vec{b} + \overrightarrow{DB} = \vec{a}$

$\overrightarrow{DB} = \vec{a} - \vec{b}$ - -(ii)

From equation (i) and (ii), we get that

$\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are diagonals of a parallelogram whose adjacent sides are $\vec{a}$ and $\vec{b}$

Algebra of Vectors Ex 23.2 Q4

Given that m is a scalar and $\vec{a}$ is a vector such that

$m\vec{a} = \vec{0}$

$$m\left(a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right) = 0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k} \qquad \left[\text{since let } \vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}\right]$$

$$ma_1\hat{i} + mb_1\hat{j} + mc_1\hat{k} = 0 \times \hat{i} + 0 \times \hat{j} + 0 \times \hat{k}$$

Comparing the coefficients of $\hat{i}, \hat{j}, \hat{k}$ of LHS and RHS,

$ma_1 = 0 \Rightarrow m = 0$ or $a_1 = 0$ (i)

$mb_1 = 0 \Rightarrow m = 0$ or $b_1 = 0$ (ii)

$mc_1 = 0 \Rightarrow m = 0$ or $c_1 = 0$ (iii)

From (i), (ii) and (iii)

$m = 0$ or $a_1 = b_1 = c_1 = 0$

$\Rightarrow m = 0$ or $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{j} = 0$

$\Rightarrow m = 0$ or $\vec{a} = 0$

Algebra of Vectors Ex 23.2 Q5

(i)

Let $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

$\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

Given that, $a = -b$

$a_1\hat{i} + b_1\hat{j} + c_1\hat{k} = -a_2\hat{i} - b_2\hat{j} - c_2\hat{k}$

Comparing the coefficients of i, j, k in LHS and RHS,

$a_1 = -a_2$ (1)

$b_1 = -b_2$ (2)

$c_1 = -c_2$ (3)

$\left|\vec{a}\right| = \sqrt{a_1^2 + b_1^2 + c_1^2}$

Using (1), (2)and (3),

$\left|\vec{a}\right| = \sqrt{(-a_2)^2 + (-b_2)^2 + (-c_2)^2}$

$\left|\vec{a}\right| = \sqrt{a_2^2 + b_2^2 + c_2^2}$

$\therefore \left|\vec{a}\right| = \left|\vec{b}\right|$

(ii)

Given a and b are two vectors such that $\left|\vec{a}\right| = \left|\vec{b}\right|$

It means magnitude of vector $\vec{a}$ is equal to the magnitude of vector $\vec{b}$, but we cannot conclude anything about the direction of the vector.

So,it is false that

$\left|\vec{a}\right| = \left|\vec{b}\right| \Rightarrow \vec{a} = \pm\vec{b}$

(iii)

Given for any vector $\vec{a}$ and $\vec{b}$

$\left|\vec{a}\right| = \left|\vec{b}\right|$

It means magnitude of the vector $\vec{a}$ and $\vec{b}$ are equal but we cannot say any thing about the direction of the vector $\vec{a}$ and $\vec{b}$. And we know that $\vec{a} = \vec{b}$ means magnitude and same direction. So, it is false.

Algebra of Vectors Ex 23.2 Q6

Here it is given that $ABCD$ is a quadrilateral.

In ΔADC, using triangle law, we get
$\overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{CA}$ --(i)

In ΔABC, using triangle law, we get
$\overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{BA}$ --(ii)

Put value of $\overrightarrow{CA}$ in equation (ii),
$\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA}$

Adding $\overrightarrow{BA}$ on both the sides,
$\overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = \overrightarrow{BA} + \overrightarrow{BA}$

$\therefore\ \overrightarrow{BA} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} = 2\overrightarrow{BA}$

Algebra of Vectors Ex 23.2 Q7

(i)
Given that $ABCDE$ is a pentagon.

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$
$= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$
$= \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA}$ $\left[\text{Using triangle law in } \Delta ABC,\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\right]$
$= \left(\overrightarrow{AC} + \overrightarrow{CD}\right) + \overrightarrow{DE} + \overrightarrow{EA}$
$= \left(\overrightarrow{AD}\right) + \overrightarrow{DE} + \overrightarrow{EA}$ $\left[\text{Using triangle law in } \Delta ACD,\ \overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD}\right]$
$= \overrightarrow{AD} + \overrightarrow{DA}$
$= \overrightarrow{AD} - (-\overrightarrow{AD})$
$= \vec{0}$

$\therefore\ \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE} + \overrightarrow{EA} = \vec{0}$

(ii)
It is given that ABCDE is a pentagon, So

$\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$
$= \left(\overrightarrow{AB} + \overrightarrow{BC}\right) + \overrightarrow{AE} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC}$
$= \overrightarrow{AC} + \overrightarrow{DC} + \left(\overrightarrow{AE} + \overrightarrow{ED}\right) + \overrightarrow{AC}$ $\left[\text{Using triangle law in } \Delta ABC,\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}\right]$
$= \overrightarrow{AC} + \overrightarrow{DC} + \left(\overrightarrow{AD}\right) + \overrightarrow{AC}$
$= \overrightarrow{AC} + \overrightarrow{DC} - \overrightarrow{DA} + \overrightarrow{AC}$
$= \overrightarrow{AC} + \overrightarrow{DC} + \overrightarrow{AD} + \overrightarrow{AC}$
$= \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{DC} + \overrightarrow{AC}$
$= \overrightarrow{AC} + \overrightarrow{AC} + \overrightarrow{AC}$
$= 3\overrightarrow{AC}$

So,
$\overrightarrow{AB} + \overrightarrow{AE} + \overrightarrow{BC} + \overrightarrow{DC} + \overrightarrow{ED} + \overrightarrow{AC} = 3\overrightarrow{AC}$

Algebra of Vectors Ex 23.2 Q8

Let O be the centre of a regular octagon, we know that the centre of a regular octagon bisects all the diagonals passing through it.

Thus,

$\overrightarrow{OA} = -\overrightarrow{OE}$ (i)

$\overrightarrow{OB} = -\overrightarrow{OF}$ (ii)

$\overrightarrow{OC} = -\overrightarrow{OG}$ (iii)

$\overrightarrow{OD} = -\overrightarrow{OH}$ (iv)

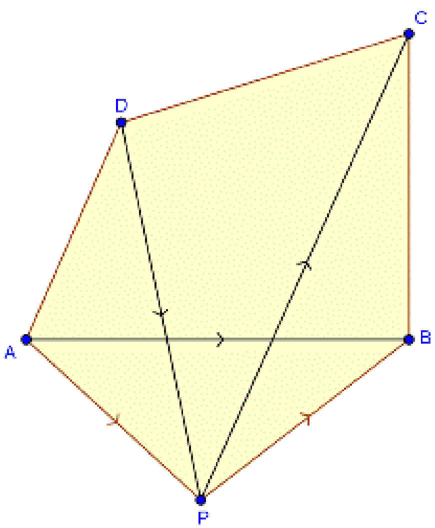
Adding equation (i), (ii), and (iv),

$$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = -\overrightarrow{OE} - \overrightarrow{OF} - \overrightarrow{OG} - \overrightarrow{OH}$$

$$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + = -\left(\overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH}\right)$$

$$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} + \overrightarrow{OE} + \overrightarrow{OF} + \overrightarrow{OG} + \overrightarrow{OH} = \vec{0}$$

Algebra of Vectors Ex 23.2 Q9



Given,

$$\overrightarrow{AP} + \overrightarrow{PB} + \overrightarrow{PD} = \overrightarrow{PC}$$

$$\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{PC} - \overrightarrow{PD}$$

$$\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{PC} + \overrightarrow{DP} \quad \left[\text{Since } \overrightarrow{DP} = -\overrightarrow{PD}\right]$$

$$\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{DP} + \overrightarrow{PC}$$

$$\overrightarrow{AB} = \overrightarrow{DC} \quad \left[\begin{array}{l}\text{Using triangle law in } \Delta APB,\ \overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{AB} \\ \text{Using triangle law in } \Delta DPC,\ \overrightarrow{DP} + \overrightarrow{PC} = \overrightarrow{DC}\end{array}\right]$$

Therefore, AB is parallel to DC and equal is magnitude.

Hence, $ABCD$ is a parallelogram.

Algebra of Vectors Ex 23.2 Q10

We need to show that

$$\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AO}$$

We know that centre O of the hexagon bisects the diagonal $\overrightarrow{AD}$

$$\therefore \qquad \overrightarrow{AO}=\frac{1}{2}\overrightarrow{AD};\ \overrightarrow{BO}=-\overrightarrow{EO};\ \overrightarrow{CO}=-\overrightarrow{FO}$$

Now

$\overrightarrow{AB}+\overrightarrow{BO}=\overrightarrow{AO}$

$\overrightarrow{AC}+\overrightarrow{CO}=\overrightarrow{AO}$

$\overrightarrow{AD}+\overrightarrow{DO}=\overrightarrow{AO}$

$\overrightarrow{AE}+\overrightarrow{EO}=\overrightarrow{AO}$

$\overrightarrow{AF}+\overrightarrow{FO}=\overrightarrow{AO}$

Adding these equations we get

$$\left(\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}\right)+\left(\overrightarrow{BO}+\overrightarrow{CO}+\overrightarrow{DO}+\overrightarrow{EO}+\overrightarrow{FO}\right)$$
$$=5\overrightarrow{AO}$$

$$\Rightarrow \qquad \left(\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}\right)+\overrightarrow{DO}=5\overrightarrow{AO}$$

But $\quad \overrightarrow{DO}=-\overrightarrow{AO}$

$$\therefore \qquad \overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=6\overrightarrow{AO}.$$