RD Sharma Solutions Class 12 Maths Chapter 22 Ex 22.8

Differential Equations Ex 22.8 Q1

$$\frac{dy}{dx} = (x + y + 1)^2$$

Let $x + y + 1 = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

So,

$$\frac{dv}{dx} - 1 = v^2$$

$$\frac{dv}{dx} = v^2 + 1$$

$$\int \frac{1}{v^2 + 1} = \int dx$$

$$\tan^{-1}(v) = x + c$$

$$\tan^{-1}(x + y + 1) = x + c$$

Differential Equations Ex 22.8 Q2

$$\frac{dy}{dx} \times \cos(x-y) = 1$$

Let $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = 1 - \frac{dv}{dx}$$

So,

$$\left(1 - \frac{dv}{dx}\right)\cos v = 1$$

$$1 - \frac{dv}{dx} = \sec v$$

$$1 - \sec v = \frac{dv}{dx}$$

$$dx = \frac{dv}{1 - \sec v}$$

$$dx = \frac{\cos v}{1 - \cos v} dv$$

$$\int dx = \int \frac{\cos^2\frac{v}{2} - \sin^2\frac{v}{2}}{2\sin^2\frac{v}{2}} dv$$

$$\int dx = \int \frac{1}{2}\cot\left(\frac{v}{2}\right) dv - \frac{1}{2} dv$$

$$2\int dx = \int \cot^2\left(\frac{v}{2}\right) dv - \int dv$$

$$2\int dx = \int\left(\operatorname{cosec}^2\frac{v}{2} - 1\right) dv - \int dv$$

$$2x = -2\cot\left(\frac{v}{2}\right) dv - v - v + c_1$$

$$2(x + v) = -2\cot\frac{v}{2} + c_1$$

$$x + x - y = -\cot\left(\frac{x-y}{2}\right) + c$$

$$c + y = \cot\left(\frac{x-y}{2}\right)$$

Differential Equations Ex 22.8 Q3

$$\frac{dy}{dx} = \frac{(x-y)+3}{2(x-y)+5}$$

Let $x - y = v$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

So,

$$1 - \frac{dv}{dx} = \frac{v+3}{2v+5}$$

$$\frac{dv}{dx} = 1 - \frac{v+3}{2v+5}$$

$$= \frac{2v+5-v-3}{2v+5}$$

$$\frac{dv}{dx} = \frac{v+2}{2v+5}$$

$$\frac{2v+5}{v+2}dv = dx$$

$$\frac{(2v+4)+1}{v+2}dv = dx$$

$$\int\left(2+\frac{1}{v+2}\right)dv = \int dx$$

$$2v + \log|v+2| = x + c$$

$$2(x-y) + \log|x-y+2| = x + c$$

Differential Equations Ex 22.8 Q4

$$\frac{dy}{dx} = (x+y)^2$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

So,

$$\frac{dv}{dx} - 1 = v^2$$

$$\frac{dv}{dx} = 1 + v^2$$

$$\int \frac{1}{1+v^2}dv = \int dx$$

$$\tan^{-1} v = x + c$$

$$\tan^{-1}(x+y) = x + c$$

$$x + y = \tan(x+c)$$

Differential Equations Ex 22.8 Q5

$$(x+y)^2\frac{dy}{dx}=1$$

Let $x+y=v$

$$1+\frac{dy}{dx}=\frac{dv}{dx}$$

$$\frac{dy}{dx}=\frac{dv}{dx}-1$$

So,

$$v^2\left(\frac{dv}{dx}-1\right)=1$$

$$\frac{dv}{dx}=\frac{1}{v^2}+1$$

$$\frac{dv}{dx}=\frac{v^2+1}{v^2}$$

$$\frac{v^2}{v^2+1}dv=dx$$

$$\int\frac{v^2+1-1}{v^2+1}dv=\int dx$$

$$\int\left(1-\frac{1}{v^2+1}\right)dv=\int dx$$

$$v-\tan^{-1}(v)=x+c$$

$$x+y-\tan^{-1}(x+y)=x+c$$

$$y-\tan^{-1}(x+y)=c$$

Differential Equations Ex 22.8 Q6

$$\cos^2(x-2y)=1-\frac{2dy}{dx}$$

Let $x-2y=v$

$$1-\frac{2dy}{dx}=\frac{dv}{dx}$$

So,

$$\cos^2 v=\frac{dv}{dx}$$

$$\int dx=\int\sec^2 v\,dv$$

$$x=\tan v+c$$

$$x=\tan(x-2y)+c$$

Differential Equations Ex 22.8 Q7

The given differential equation can be written as

$$\frac{dy}{dx} = \frac{1}{\cos(x+y)}$$

Let $x + y = u$. Then,

$$1 + \frac{dy}{dx} = \frac{du}{dx} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} - 1$$

Putting $x + y = u$ and $\frac{dy}{dx} = \frac{du}{dx} - 1$ the given differential equation, we get

$$\Rightarrow \quad \frac{du}{dx} - 1 = \frac{1}{\cos u}$$

$$\Rightarrow \quad \frac{du}{dx} = \frac{1+\cos u}{\cos u}$$

$$\Rightarrow \quad \frac{\cos u}{1+\cos u} du = dx$$

$$\Rightarrow \quad \frac{\cos u(1-\cos u)}{1-\cos^2 u} du = dx$$

$$\Rightarrow \quad \left(\cot u \operatorname{cosec} u - \cot^2 u\right) du = dx$$

$$\Rightarrow \quad \left(\cot u \operatorname{cosec} u - \operatorname{cosec}^2 u + 1\right) du = dx$$

$$\Rightarrow \quad -\operatorname{cosec} u + \cot u + u = x + C$$

$$\Rightarrow \quad -\operatorname{cosec}(x+y) + \cot(x+y) + x + y = x + C$$

$$\Rightarrow \quad -\operatorname{cosec}(x+y) + \cot(x+y) + y = C$$

$$\Rightarrow \quad -\frac{1-\cos(x+y)}{\sin(x+y)} + y = C$$

$$\Rightarrow \quad -\tan\left(\frac{x+y}{2}\right) + y = C$$

We have,

$y(0) = 0$ i.e. $y = 0$ when $x = 0$

Putting $x = 0$ and $y = 0$ in (i), we get $C = 0$.

Putting $C = 0$ in (i), we get

$$-\tan\left(\frac{x+y}{2}\right) + y = 0 \Rightarrow y = \tan\left(\frac{x+y}{2}\right),$$ which is the required solution.

Differential Equations Ex 22.8 Q8

$$\frac{dy}{dx} = \tan(x+y)$$

Let $x + y = v$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dv}{dx} - 1 = \tan v$$

$$\frac{dv}{dx} = 1 + \tan v$$

$$\frac{1}{1+\tan v} dv = dx$$

$$\frac{\cos v}{\cos v + \sin v} dv = dx$$

$$\left(\frac{2\cos v}{\cos v + \sin v}\right) dv = 2dx$$

$$\left(\frac{\cos v + \sin v + \cos v - \sin v}{\cos v + \sin v}\right) dv = 2dx$$

$$\int dv + \int \left(\frac{\cos v - \sin v}{\cos v + \sin v}\right) dv = 2\int dx$$

$$v + \log\left|\cos v + \sin v\right| = 2x + c$$

$$x + y + \log\left|\cos(x+y) + \sin(x+y)\right| = 2x + c$$

$$y - x + \log\left|\cos(x+y) + \sin(x+y)\right| = c$$

Differential Equations Ex 22.8 Q9

$$2v - v\frac{dv}{dx} = \frac{dv}{dx}$$

$$\Rightarrow 2v = v\frac{dv}{dx} + \frac{dv}{dx}$$

$$\Rightarrow 2v = (v+1)\frac{dv}{dx}$$

$$\Rightarrow \frac{(v+1)}{v} dv = 2dx$$

$$(x+y)(dx-dy)=dx+dy$$

$$(x+y)\left(1-\frac{dy}{dx}\right)=1+\frac{dy}{dx}$$

Let $x+y=v$

$$1+\frac{dy}{dx}=\frac{dv}{dx}$$

So,

$$v\left(1-\left(\frac{dv}{dx}-1\right)\right)=\frac{dv}{dx}$$

$$v\left(2-\frac{dv}{dx}\right)=\frac{dv}{dx}$$

$$2v-v\frac{dv}{dx}=\frac{dv}{dx}$$

$$\Rightarrow 2v=v\frac{dv}{dx}+\frac{dv}{dx}$$

$$\Rightarrow 2v=(v+1)\frac{dv}{dx}$$

$$\Rightarrow \frac{(v+1)}{v}dv=2dx$$

$$\int\left(1+\frac{1}{v}\right)dv=2\int dx$$

$$v+\log|v|=2x+c$$

$$x+y+\log|x+y|=2x+c$$

$$y-x+\log|x+y|=c$$

Differential Equations Ex 22.8 Q10

$$(x+y+1)\frac{dy}{dx}=1$$

Let $x+y=v$

$$1+\frac{dy}{dx}=\frac{dv}{dx}$$

$$\frac{dy}{dx}=\frac{dv}{dx}-1$$

So,

$$(v+1)\left(\frac{dv}{dx}-1\right)=1$$

$$(v+1)\frac{dv}{dx}-(v+1)=1$$

$$(1+v)\frac{dv}{dx}=1+1+v$$

$$\frac{v+1dv}{2+v}=dx$$

$$\int\left(1-\frac{1}{v+2}\right)dv=\int dx$$

$$v-\log|v+2|=x+\log c$$

$$x+y-\log|x+y+2|=x+\log c$$

$$y=\log c|x+y+2|$$

$$e^{y}=c(x+y+2)$$

$$ke^{y}=x+y+2 \qquad [k=1/c]$$

$$x=ke^{y}-y-2$$

Differential Equations Ex 22.8 Q11

$$\frac{dy}{dx} + 1 = e^{x+y}$$

Let $x+y = v$

$$1+\frac{dy}{dx} = \frac{dv}{dx}$$

$\therefore$ Given differential equation becomes,

$$\frac{dv}{dx} = e^{v}$$

$$\frac{1}{e^{v}}dv = dx$$

Integrating on both the sides we get

$$-e^{-v} = x + C$$

$$\therefore -e^{-(x+y)} = x + C$$