RD Sharma Solutions Class 12 Maths Chapter 22 Ex 22.5

Differential Equations Ex 22.5 Q1

$$\frac{dy}{dx} = x^2 + x - \frac{1}{x}, \ x \neq 0$$

$$\int dy = \int \left(x^2 + x - \frac{1}{x}\right) dx$$

$$y = \frac{x^3}{3} + \frac{x^2}{2} - \log|x| + c, \ x \neq 0$$

Differential Equations Ex 22.5 Q2

$$\frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}, \ x \neq 0$$

$$\int dy = \int \left(x^5 + x^2 - \frac{2}{x}\right) dx$$

$$y = \frac{x^6}{6} + \frac{x^3}{3} - 2\log|x| + c, \ x \neq 0$$

Differential Equations Ex 22.5 Q3

$$\frac{dy}{dx} + 2x = e^{3x}$$

$$\frac{dy}{dx} = e^{3x} - 2x$$

$$\int dy = \int \left(e^{3x} - 2x\right) dx$$

$$y = \frac{e^{3x}}{3} - \frac{2x^2}{2} + c$$

$$y = \frac{e^{3x}}{3} - x^2 + c$$

$$y + x^2 = \frac{1}{3}e^{3x} + c$$

Differential Equations Ex 22.5 Q4

$$\left(x^2 + 1\right)\frac{dy}{dx} = 1$$

$$\int dy = \int \frac{dx}{x^2 + 1}$$

$$y = \tan^{-1} x + c$$

Differential Equations Ex 22.5 Q6

$$(x+2)\frac{dy}{dx} = x^2+3x+7$$

$$dy = \left(\frac{x^2+3x+7}{x+2}\right)dx$$

$$dy = \left(x+1+\frac{5}{x+2}\right)dx$$

$$\int dy = \int\left(x+1+\frac{5}{x+2}\right)dx$$

$$y = \frac{x^2}{2}+x+5\log|x+2|+c$$

$$x \neq -2$$

Differential Equations Ex 22.5 Q7

$$\frac{dy}{dx} = \tan^{-1}x$$

$$dy = \tan^{-1}x dx$$

$$\int dy = \int \tan^{-1}x dx$$

$$y = \tan^{-1}x \times \int 1 dx - \int\left(\frac{1}{1+x^2}\int dx\right)dx + c$$

Using integration by parts

$$y = x\tan^{-1}x - \int\frac{x}{1+x^2}dx + c$$

$$y = x\tan^{-1}x - \frac{1}{2}\int\frac{2x}{1+x^2}dx + c$$

$$y = x\tan^{-1}x - \frac{1}{2}\log\left|1+x^2\right| + c$$

Differential Equations Ex 22.5 Q8

$$\frac{dy}{dx} = \log x$$

$$\Rightarrow dy = \log x \times dx$$

$$\Rightarrow \int dy = \int \log x dx$$

$$\Rightarrow y = \log x \times \int 1 dx - \int\left(\frac{1}{x}\int 1 dx\right)dx + C \quad [\textit{Using integration by parts}]$$

$$\Rightarrow y = x\log x - \int dx + C$$

$$\Rightarrow y = x\log x - x + C$$

$$\Rightarrow y = x(\log x - 1) + C, \textit{ where } x \in (0, \infty)$$

Differential Equations Ex 22.5 Q9

$$\frac{1}{x}\frac{dy}{dx} = \tan^{-1} x$$

$$dy = x\tan^{-1} x dx$$

$$\int dy = \int x\tan^{-1} x dx$$

$$y = \tan^{-1} x \int x dx - \int \left(\frac{1}{1+x^2} \int x dx \right) dx + c$$

Using integration by parts

$$y = \frac{x^2}{2}\tan^{-1} x - \int \frac{x^2}{2(1+x^2)} dx + c$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2} dx + c$$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \left(1 - \frac{1}{x^2+1}\right) dx + c$$

$$y = \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x + c$$

$$y = \frac{1}{2}\left(x^2+1\right)\tan^{-1} x - \frac{1}{2}x + c$$

Differential Equations Ex 22.5 Q10

$$\frac{dy}{dx} = \cos^3 x \sin^2 x + x\sqrt{2x+1}$$

$$dy = \left(\cos^3 x \sin^2 x + x\sqrt{2x+1}\right) dx$$

$$\int dy = \int \cos^3 x \sin^2 x dx + \int x\sqrt{2x+1} dx$$

$$y = I_1 + I_2 \qquad ---(i)$$

$$I_1 = \int \cos^3 x \sin^2 x dx$$

$$= \int \cos^2 x \times \cos x \times \sin^2 x \times dx$$

$$I_1 = \int \left(1 - \sin^2 x\right) \sin^2 x \cos x dx$$

Put $\sin x = t$

$$\cos x dx = dt$$

$$I_1 = \int \left(1 - t^2\right) t^2 dt$$

$$= \int \left(t^2 - t^4\right) dt$$

$$= \frac{t^3}{3} - \frac{t^5}{5} + c_1$$

$$I_1 = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + c_1$$

And,

$$I_2 = \int x\sqrt{2x+1} dx$$

Put $2x + 1 = v^2$

$$2dx = 2vdv$$

$$I_2 = \int \left(\frac{v^2 - 1}{2}\right) v \times v dv$$

$$= \frac{1}{2}\int \left(v^4 - v^2\right) dv$$

$$= \frac{1}{2}\left(\frac{v^5}{5} - \frac{v^3}{3}\right) + c_2$$

$$I_2 = \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + c_2$$

Put the I_1 and I_2 in equation (i),

$$y = I_1 + I_2$$

$$y = \frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}} + c$$

As $c = c_1 + c_2$

Differential Equations Ex 22.5 Q11

$$(\sin x + cox) dy + (\cos x - \sin x) dx = 0$$

$$(\sin x + \cos x) dy = (\sin x - \cos x) dx$$

$$dy = \frac{(\sin x - \cos x)}{\sin x + \cos x} dx$$

$$\int dy = -\int \left(\frac{\cos x - \sin x}{\sin x + \cos x}\right) dx$$

Put $\sin x + \cos x = t$

$$(\cos x - \sin x) dx = dt$$

$$\int dy = -\int \frac{1}{t} dt$$

$$y = -\log|t| + c$$

$$y + \log|\sin x + \cos x| = c$$

Differential Equations Ex 22.5 Q12

$$\frac{dy}{dx} - x\sin^2 x = \frac{1}{x\log x}$$

$$\frac{dy}{dx} = \frac{1}{x \times \log x} + x\sin^2 x$$

$$dy = \left(\frac{1}{x\log x} + x\sin^2 x\right)dx$$

$$\int dy = \int \frac{1}{x\log x}dx + \int x\sin^2 x dx$$

$$y = I_1 + I_2 \qquad \text{---(i)}$$

$$I_1 = \int \frac{1}{x\log x}dx$$

Let $\log x = t$

$$\frac{1}{x}dx = dt$$

$$I_1 = \int \frac{dt}{t}$$

$$= \log|t| + c_1$$

$$I_1 = \log|\log x| + c_1$$

$$I_2 = \int x\sin^2 x dx$$

$$= \int x\frac{(1-\cos 2x)}{2}dx$$

$$= \frac{1}{2}\int (x - x\cos 2x)dx$$

$$= \frac{1}{2}\int x dx - \frac{1}{2}\int x\cos 2x dx$$

$$= \frac{1}{2}\left(\frac{x^2}{2}\right) - \frac{1}{2}\left[x\int \cos 2x dx - \int\left(1 \times \int \cos 2x dx\right)dx\right] + c_2$$

$$= \frac{x^2}{4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} - \int \frac{\sin 2x}{2}dx\right] + c_2$$

$$= \frac{x^2}{4} - \frac{1}{2}\left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right] + c_2$$

$$I_2 = \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c_2$$

Put the value of I_1 and I_2 in equation (i),

$$y = I_1 + I_2$$

$$y = \log|\log x| + \frac{x^2}{4} - \frac{x\sin 2x}{4} - \frac{\cos 2x}{8} + c \text{ as } c_1 + c_2 = c$$

Differential Equations Ex 22.5 Q13

$$\frac{dy}{dx} = x^5 \tan^{-1}\left(x^3\right)$$
$$dy = x^5 \tan^{-1}\left(x^3\right)dx$$
$$\int dy = \int x^5 \tan^{-1}\left(x^3\right)dx$$

Put $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\Rightarrow \quad x^2 dx = \frac{dt}{3}$

So,

$$\int dy = \frac{1}{3}\left[\tan^{-1} t \int t dt - \int\left(\frac{1}{1+t^2} \times \int t dt\right)\right]dt + c$$

Using integration by parts

$$y = \frac{1}{3}\left[\frac{t^2}{2} + \tan^{-1} t - \int \frac{t^2}{2\left(t^2+1\right)}dt\right] + c$$
$$= \frac{1}{6}t^2 \tan^{-1} t - \frac{1}{6}\int\left(\frac{t^2}{t^2+1}\right)dt + c$$
$$y = \frac{1}{6}t^2 \tan^{-1} t - \frac{1}{6}\int\left(1 - \frac{1}{t^2+1}\right)dt + c$$
$$= \frac{1}{6}t^2 \tan^{-1} t - \frac{1}{6}t + \frac{1}{6}\tan^{-1} t + c$$
$$y = \frac{1}{6}\left(t^2+1\right)\tan^{-1} t - \frac{1}{6}t + c$$
$$y = \frac{1}{6}\left[\left(t^2+1\right)\tan^{-1} t - t\right] + c$$

So,

$$y = \frac{1}{6}\left[\left(x^6+1\right)\tan^{-1}\left(x^3\right) - x^3\right] + c$$

Differential Equations Ex 22.5 Q14

$$\sin^4 x \frac{dy}{dx} = \cos x$$
$$dy = \frac{\cos x}{\sin^4 x}dx$$
$$\int dy = \int \frac{\cos x}{\sin^4 x}dx$$

Put $\sin x = t$

$$\cos x dx = dt$$
$$\int dy = \int \frac{dt}{t^4}$$
$$y = \frac{1}{-3t^3} + c$$
$$y = -\frac{1}{3\sin^3 x} + c$$
$$y = -\frac{1}{3}\cos ec^3 x + c$$

Differential Equations Ex 22.5 Q15

$$\cos x\frac{dy}{dx}-\cos 2x=\cos 3x$$
$$\cos x\frac{dy}{dx}=\cos 3x+\cos 2x$$
$$\frac{dy}{dx}=\frac{4\cos^3 x-3\cos x+2\cos^2 x-1}{\cos x}$$
$$\frac{dy}{dx}=\frac{4\cos^3 x}{\cos x}-\frac{3\cos x}{\cos x}+\frac{2\cos^2 x}{\cos x}-\frac{1}{\cos x}$$
$$\frac{dy}{dx}=4\cos^2 x-3+2\cos x-\sec x$$
$$\frac{dy}{dx}=4\left(\frac{\cos 2x+1}{2}\right)-3+2\cos x-\sec x$$
$$dy=\left(2\cos 2x+2-3+2\cos x-\sec x\right)dx$$
$$\int dy=\int\left(2\cos 2x-1+2\cos x-\sec x\right)dx$$
$$y=\sin 2x-x+2\sin x-\log\left|\sec x+\tan x\right|+c$$

Differential Equations Ex 22.5 Q16

$$\sqrt{1-x^4}dy=xdx$$
$$dy=\frac{xdx}{\sqrt{1-x^4}}$$
$$\int dy=\int\frac{xdx}{\sqrt{1-x^4}}$$

Let $x^2=t$

$$2xdx=dt$$

$\Rightarrow$ $xdx=\frac{dt}{2}$

$$\int dy=\int\frac{dt}{2\sqrt{1-t^2}}$$
$$y=\frac{1}{2}\sin^{-1}(t)+c$$
$$y=\frac{1}{2}\sin^{-1}\left(x^2\right)+c$$

Differential Equations Ex 22.5 Q17

$$\sqrt{a+x}dy+xdx=0$$
$$\sqrt{a+x}dy=-xdx$$
$$dy=\frac{-x}{\sqrt{a+x}}dx$$
$$\int dy=-\int\frac{x}{\sqrt{a+x}}dx$$

Put $a+x=t^2$

$$dx=2tdt$$
$$\int dy=-\int\left(\frac{t^2-a}{t}\right)2tdt$$
$$\int dy=2\int\left(a-t^2\right)dt$$
$$y=2\left(at-\frac{t^3}{3}\right)+c$$
$$y+\frac{2}{3}t^3-2at=c$$
$$y+\frac{2}{3}(a+x)^{\frac{3}{2}}-2a\sqrt{a+x}=c$$

Differential Equations Ex 22.5 Q18

$$\left(1+x^2\right)\frac{dy}{dx}-x=2\tan^{-1}x$$
$$\left(1+x^2\right)\frac{dy}{dx}=2\tan^{-1}x+x$$
$$dy=\left(\frac{2\tan^{-1}x+x}{1+x^2}\right)dx$$
$$\int dy=\int\left(\frac{2\tan^{-1}x+x}{1+x^2}\right)dx$$
$$y=\int(2t+\tan t)dt \qquad \left[\tan^{-1}x=t\right]$$
$$=\frac{1}{2}\log\left|1+x^2\right|+\left(\tan^{-1}x\right)^2+c$$

Differential Equations Ex 22.5 Q19

$$\frac{dy}{dx}=x\log x$$
$$dy=x\log x\,dx$$
$$\int dy=\int x\log x\,dx$$
$$y=\log|x|\int x\,dx-\int\left(\frac{1}{x}\int x\,dx\right)dx+c$$

Using integration by parts

$$=\frac{x^2}{2}\log|x|-\int\frac{x^2}{2x}dx+c$$
$$=\frac{x^2}{2}\log|x|-\frac{1}{2}\int x\,dx+c$$
$$y=\frac{x^2}{2}\log|x|-\frac{x^2}{4}+c$$

Differential Equations Ex 22.5 Q20

$$\frac{dy}{dx} = xe^x - \frac{5}{2} + \cos^2 x$$

$$dy = \left(xe^x - \frac{5}{2} + \cos^2 x\right)dx$$

$$\int dy = \int xe^x dx - \frac{5}{2}\int dx + \int \cos^2 x dx$$

$$\int dy = \int xe^x dx - \frac{5}{2}\int dx + \int\left(\frac{1+\cos 2x}{2}\right)dx$$

$$= \int xe^x - \frac{5}{2}\int dx + \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x dx$$

$$\int dy = \int xe^x - 2\int dx + \frac{1}{2}\int \cos 2x dx$$

$$y = \left[x \times \int e^x dx - \int \left\{1 \times \int e^x dx\right\} dx\right] - 2x + \frac{1}{2}\frac{\sin 2x}{2} + c$$

Using integration by parts

$$y = xe^x - e^x - 2x + \frac{1}{4}\sin 2x + c$$

$$y = xe^x - e^x - 2x + \frac{1}{4}\sin 2x + c$$

Differential Equations Ex 22.5 Q21

The given differential equation is:

$$\left(x^3 + x^2 + x + 1\right)\frac{dy}{dx} = 2x^2 + x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + x}{\left(x^3 + x^2 + x + 1\right)}$$

$$\Rightarrow dy = \frac{2x^2 + x}{(x+1)\left(x^2+1\right)}dx$$

Integrating both sides, we get:

$$\int dy = \int \frac{2x^2 + x}{(x+1)\left(x^2+1\right)}dx \qquad ...(1)$$

$$\text{Let } \frac{2x^2 + x}{(x+1)\left(x^2+1\right)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}. \qquad ...(2)$$

$$\Rightarrow \frac{2x^2 + x}{(x+1)\left(x^2+1\right)} = \frac{Ax^2 + A + (Bx+C)(x+1)}{(x+1)\left(x^2+1\right)}$$

$$\Rightarrow 2x^2 + x = Ax^2 + A + Bx^2 + Bx + Cx + C$$

$$\Rightarrow 2x^2 + x = (A+B)x^2 + (B+C)x + (A+C)$$

Comparing the coefficients of x^2 and x, we get:

$A+B=2$

$B+C=1$

$A+C=0$

Solving these equations, we get:

$A=\frac{1}{2}$, $B=\frac{3}{2}$ and $C=\frac{-1}{2}$

Substituting the values of A, B, and C in equation (2), we get:

$$\frac{2x^2+x}{(x+1)(x^2+1)}=\frac{1}{2}\cdot\frac{1}{(x+1)}+\frac{1}{2}\frac{(3x-1)}{(x^2+1)}$$

Therefore, equation (1) becomes:

$$\int dy=\frac{1}{2}\int\frac{1}{x+1}dx+\frac{1}{2}\int\frac{3x-1}{x^2+1}dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{2}\int\frac{x}{x^2+1}dx-\frac{1}{2}\int\frac{1}{x^2+1}dx$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\cdot\int\frac{2x}{x^2+1}dx-\frac{1}{2}\tan^{-1}x+C$$
$$\Rightarrow y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+C$$
$$\Rightarrow y=\frac{1}{4}\left[2\log(x+1)+3\log(x^2+1)\right]-\frac{1}{2}\tan^{-1}x+C$$
$$\Rightarrow y=\frac{1}{4}\left[(x+1)^2(x^2+1)^3\right]-\frac{1}{2}\tan^{-1}x+C \quad ...(3)$$

Now, $y=1$ when $x=0$.

$$\Rightarrow 1=\frac{1}{4}\log(1)-\frac{1}{2}\tan^{-1}0+C$$
$$\Rightarrow 1=\frac{1}{4}\times 0-\frac{1}{2}\times 0+C$$
$$\Rightarrow C=1$$

Substituting C = 1 in equation (3), we get:

$$y=\frac{1}{4}\left[\log(x+1)^2(x^2+1)^3\right]-\frac{1}{2}\tan^{-1}x+1$$

Differential Equations Ex 22.5 Q22

$$\sin\left(\frac{dy}{dx}\right) = k, \quad y(0) = 1$$

$$\frac{dy}{dx} = \sin^{-1} k$$

$$dy = \sin^{-1} k dx$$

$$\int dy = \int \sin^{-1} k dx$$

$$y = x \sin^{-1} k + c \qquad \text{---(i)}$$

Put $x = 0, y = 1$

$$1 = 0 + c$$

$$1 = c$$

Put $c = 1$ in equation (i),

$$y = x \sin^{-1} k + 1$$

$$y - 1 = x \sin^{-1} k$$

Differential Equations Ex 22.5 Q23

$$e^{\frac{dy}{dx}} = x + 1, \quad y(0) = 3$$

$$\frac{dy}{dx} = \log(x+1)$$

$$dy = \log(x+1) dx$$

$$\int dy = \int \log(x+1) dx$$

$$y = \log(x+1) \int 1 \times dx - \int \left(\frac{1}{x+1} \times \int 1 \times dx\right) dx + c$$

Using integration by parts

$$y = x \log(x+1) - \int \left(\frac{x}{x+1}\right) dx + c$$

$$= x \log(x+1) - \int \left(1 - \frac{1}{x+1}\right) dx + c$$

$$= x \log(x+1) - x + \log(x+1) + c$$

$$y = (x+1) \log(x+1) - x + c \qquad \text{---(i)}$$

Put $y = 3, x = 0$

$$3 = 0 + c$$

$\Rightarrow$ $c = 3$

Using equation (i),

$$y = (x+1) \log(x+1) - x + 3$$

Differential Equations Ex 22.5 Q24

$$c'(x) = 2 + 0.15x, \quad c(0) = 100$$

$$c'(x) dx = (2 + 0.15x) dx$$

$$\int c'(x) dx = \int 2dx + 0.15 \int x dx$$

$$c(x) = 2x + 0.15 \frac{x^2}{2} + c \qquad \text{---(i)}$$

Put $x = 0, c(x) = 100$

$$100 = 2(0) + 0 + c$$

$$100 = c$$

Put $c = 100$ in equation (i),

$$c(x) = 2x + (0.15) \frac{x^2}{2} + 100$$

Differential Equations Ex 22.5 Q25

$$x\frac{dy}{dx} + 1 = 0, \quad y(-1) = 0$$

$$x\frac{dy}{dx} = -1$$

$$dy = -\frac{dx}{x}$$

$$\int dy = -\int \frac{dx}{x}$$

$$y = -\log|x| + c \qquad \text{---(i)}$$

Put $x = -1$ and $y = 0$

$$0 = 0 + c$$

$$c = 0$$

Put $c = 0$ in equation (i),

$$y = -\log|x|, x < 0$$

Differential Equations Ex 22.5 Q26

$$x(x^2 - 1)\frac{dy}{dx} = 1, y(2) = 0$$

$$\frac{dy}{dx} = \frac{1}{x(x^2 - 1)}$$

$$dy = \frac{1}{x(x^2 - 1)}dx$$

$$\int dy = \int\left(\frac{1}{x(x^2 - 1)}\right)dx$$

$$y = \frac{1}{2}\int\frac{1}{x-1}dx - \int\frac{1}{x}dx + \frac{1}{2}\int\frac{1}{x+1}dx$$

$$= \frac{1}{2}\log|x-1| - \log|x| + \frac{1}{2}\log|x+1| + c$$

Putting $x = 2, y = 0$, we have

$$y = \frac{1}{2}\log|x-1| - \log|x| + \frac{1}{2}\log|x+1| + c$$

$$0 = \frac{1}{2}\log|2-1| - \log|2| + \frac{1}{2}\log|2+1| + c$$

$$c = \log|2| - \frac{1}{2}\log|3|$$

Putting the value of c, we have

$$y = \frac{1}{2}\log|x-1| - \log|x| + \frac{1}{2}\log|x+1| + c$$

$$= \log\frac{4}{3}\left(\frac{x^2 - 1}{x^2}\right)$$