RD Sharma Solutions Class 12 Maths Chapter 22 Ex 22.3

Differential Equations Ex 22.3 Q1

$$y = be^{x} + ce^{2x} \quad \text{—(i)}$$

Differentiating both sides with respect to x,

$$\frac{dy}{dx} = be^{x} + 2ce^{2x} \quad \text{—(ii)}$$

Differentiating both sides with respect to x,

$$\frac{d^2y}{dx^2} = be^{x} + 4ce^{2x} \quad \text{—(iii)}$$

Now,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y$$

$$= be^{x} + 4ce^{2x} - 3\left(be^{x} + 2ce^{2x}\right) + 2\left(be^{x} + ce^{2x}\right)$$

$$= be^{x} + 4ce^{2x} - 3be^{x} - 6ce^{2x} + 2be^{x} + 2ce^{2x}$$

$$= 3be^{x} - 3be^{x} + 6ce^{2x} - 6ce^{2x}$$

$$= 0$$

So,

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$$

Differential Equations Ex 22.3 Q2

$$y = 4\sin 3x \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 4(3)\cos 3x$$

$$\frac{dy}{dx} = 12\cos 3x \qquad \text{—(ii)}$$

Differentiating it wtih respect to x,

$$\frac{d^2y}{dx^2} = -12(3)\sin 3x$$

$$\frac{d^2y}{dx^2} = -36\sin 3x \qquad \text{—(iii)}$$

Now,

$$\frac{d^2y}{dx^2} + 9y$$

$$= -36\sin 3x + 9(4\sin 3x)$$

$$= -36\sin 3x + 36\sin 3x$$

$$= 0$$

So, $y = 4\sin 3x$ is a solution of

$$\frac{d^2y}{dx^2} + 9y = 0$$

Differential Equations Ex 22.3 Q3

$$y = ae^{2x} + be^{-x} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2ae^{2x} - be^{-x} \qquad \text{—(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 4ae^{2x} + be^{-x} \qquad \text{—(ii)}$$

Now,

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y$$

$$= \left(4ae^{2x} + be^{-x}\right) - \left(2ae^{2x} - be^{-x}\right) - 2\left(ae^{2x} + be^{-x}\right)$$

$$= 4ae^{2x} + be^{-x} - 2ae^{2x} + be^{-x} - 2ae^{2x} - 2be^{-x}$$

$$= 4ae^{2x} - 4ae^{2x} + 2be^{-x} - 2be^{-x}$$

$$= 0$$

Differential Equations Ex 22.3 Q4

The given function is $y = A\cos x + B\sin x$ --------(i)

Differentiating both sides of eqn (i) w.r.t x, successively, we get

$$\frac{dy}{dx} = -A\sin x + B\cos x$$

$$\frac{d^2y}{dx^2} = -A\cos x - B\sin x$$

Substituting these values of $\frac{d^2y}{dx^2}$ and y in the given differential equation,

L.H.S = $(-A\cos x - B\sin x) + (A\cos x + B\sin x) = 0 =$ R.H.S

Therefore, the given function is a solution of the given differential equation.

Differential Equations Ex 22.3 Q5

$$y = A\cos 2x - B\sin 2x \qquad \text{—(i)}$$

Differentiating it with respect to *x*,

$$\frac{dy}{dx} = -2A\sin 2x - 2B\cos 2x$$

$$\frac{dy}{dx} = -2(A\sin 2x + B\cos 2x) \qquad \text{—(ii)}$$

Differentiating it with respect to *x*,

$$\frac{d^2y}{dx^2} = -2[2A\cos 2x - 2B\sin 2x]$$

$$= -4[A\cos 2x - B\sin 2x]$$

$$\frac{d^2y}{dx^2} = -4y$$

$$\frac{d^2y}{dx^2} + 4y = 0$$

Differential Equations Ex 22.3 Q6

$$y = Ae^{Bx} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = ABe^{Bx} \qquad \text{—(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = AB^2e^{Bx}$$

$$= \frac{\left(ABe^{Bx}\right)^2}{\left(Ae^{Bx}\right)}$$

$$\frac{d^2y}{dx^2} = \frac{1}{y}\left(\frac{dy}{dx}\right)^2$$

Differential Equations Ex 22.3 Q7

$$y = \frac{a}{x} + b \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -\frac{a}{x^2} \qquad \text{—(ii)}$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = \frac{2a}{x^3}$$

$$= -\frac{2}{x}\left(-\frac{a}{x^2}\right)$$

$$\frac{d^2y}{dx^2} = -\frac{2}{x}\left(\frac{dy}{dx}\right)$$

$$\frac{d^2y}{dx^2} + \frac{2}{x}\left(\frac{dy}{dx}\right) = 0$$

Differential Equations Ex 22.3 Q8

$$y^2 = 4ax \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a$$

$$\frac{dy}{dx} = \frac{4a}{2y}$$

$$\frac{dy}{dx} = \frac{2a}{y} \qquad \text{—(ii)}$$

Now,

$$x\frac{dy}{dx} + a\frac{dy}{dx}$$

$$= 2\frac{xa}{y} + a\left(\frac{y}{2a}\right)$$

$$= \frac{4a^2x + ay^2}{2ay}$$

$$= \frac{ay^2 + ay^2}{2ay}$$

$$= y$$

So,

$$x\frac{dy}{dx} + a\frac{dx}{dy} = y$$

Differential Equations Ex 22.3 Q9

$$Ax^2+By^2=1$$

Differentiating it with respect to x,

$$2Ax+2By\frac{dy}{dx}=0$$

$$y\frac{dy}{dx}=\frac{-2Ax}{2B}$$

$$y\frac{dy}{dx}=-\frac{Ax}{B} \quad \text{---(i)}$$

Differentiating it with respect to x,

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=-\frac{A}{B}$$

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=\frac{y}{x}\frac{dy}{dx}$$

Using equation (i)

$$x\left\{y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right\}=y\frac{dy}{dx}$$

Differential Equations Ex 22.3 Q10

$$y=ax^3+bx^2+c$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=3ax^2+2bx$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2}=6ax+2b$$

Differentiating it with respect to x

$$\frac{d^3y}{dx^3}=6a$$

Differential Equations Ex 22.3 Q11

$$y=\frac{c-x}{1+cx} \quad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=\left[\frac{(1+cx)(-1)-(c-x)(c)}{(1+cx)}\right]$$

$$\frac{dy}{dx}=\left[\frac{-1-cx-c^2+cx}{(1+cx)^2}\right]$$

$$=\frac{-1-c^2}{(1+cx)^2}$$

$$\frac{dy}{dx}=\frac{-(1+c^2)}{(1+cx)^2} \quad \text{---(ii)}$$

Now,

$$\left(1+x^2\right)\frac{dy}{dx}+\left(1+y^2\right)$$

$$=\left(1+x^2\right)\left[\frac{-(1+c^2)}{(1+cx)^2}\right]+\left[1+\left(\frac{c-x}{1+cx}\right)^2\right]$$

$$=\frac{-(1+x^2)(1+c^2)}{(1+cx)^2}+\left[\frac{(1+cx)^2+(c-x)^2}{(1+cx)^2}\right]$$

$$=\frac{-1-x^2-c^2-x^2c^2+1+c^2x^2+2cx+c^2+x^2-2cx}{(1+cx)^2}$$

$$=\frac{0}{(1+cx)^2}$$

$$=0$$

So,

$$\left(1+x^2\right)\frac{dy}{dx}+\left(1+y^2\right)=0$$

Differential Equations Ex 22.3 Q12

$$y = e^x (A\cos x + B\sin x).....(i)$$

$$\frac{dy}{dx} = e^x (A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$
$$\frac{dy}{dx} = e^x \left[(A+B)\cos x - (A-B)\sin x\right]........(ii)$$

$$\frac{d^2y}{dx^2} = e^x\left[(A+B)\cos x - (A-B)\sin x\right] + e^x\left[-(A+B)\sin x - (A-B)\cos x\right]$$
$$\frac{d^2y}{dx^2} = 2e^x (B\cos x - A\sin x).........(iii)$$

Adding (i) and (iii) we get

$$y + \frac{1}{2}\frac{d^2y}{dx^2} = e^x\left[(A+B)\cos x - (A-B)\sin x\right]$$
$$2y + \frac{d^2y}{dx^2} = 2\frac{dy}{dx}$$
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

Hence $y = e^x(A\cos x + B\sin x)$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0.$$

Differential Equations Ex 22.3 Q13

$$y = cx + 2c^2 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{y}{dx} = c \qquad \text{—(ii)}$$

Now,

$$2\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y$$
$$= 2c^2 + xc - cx + 2c^2 \qquad \left[\text{Using equation (i) and(ii)}\right]$$
$$= 0$$

So,

$$2\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$$

Differential Equations Ex 22.3 Q14

$$y = -x - 1 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -1 \qquad \text{—(ii)}$$

So,

$$(y - x)dy - (y^2 - x^2)dx$$

$$= \left[(y - 2)\frac{dy}{dx} - (y^2 - x^2)\right]dx$$

$$= \left[(-x - 1 - x)(-1) - \{(-x - 1)^2 - x^2\}\right]$$

Using equation (i) and (ii),

$$= \left[x + 1 + x - (x^2 + 1 + 2x - x^2)\right]dx$$

$$= [2x + 1 - 2x - 1]dx$$

$$= 0$$

So,

$$(y - x)dy - (y^2 - x^2)dx = 0$$

Differential Equations Ex 22.3 Q15

$$y^2 = 4a(x+a) \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a(1)$$

$$\frac{dy}{dx} = \frac{2a}{y} \qquad \text{—(ii)}$$

Now,

$$y\left\{1-\left(\frac{dy}{dx}\right)^2\right\}$$

$$= \left[y^2\left\{1-\left(\frac{dy}{dx}\right)^2\right\}\right]\frac{1}{y}$$

$$= \left[4a(x+a) - 4a(x+a)\left(\frac{2a}{y}\right)^2\right]\frac{1}{y}$$

Using equation (i) and (ii)

$$= \left[4ax + 4a^2 - \frac{16a3x}{y^2} - \frac{16a^4}{y^2}\right]\frac{1}{y}$$

$$= \frac{4a}{y^3}\left[xy^2 + ay^2 - 4a^2x - 4a^3\right]$$

$$= \frac{4a}{y^3}\left[y^2(a+x) - 4a^2(x+a)\right]$$

$$= \frac{4a}{y^3}(a+x)\left(y^2 - 4a^2\right)$$

$$= \frac{4a}{y^3}\left(\frac{y^2}{4a}\right)\left(y^2 - 4a^2\right)$$

Using equation (i) and (ii)

$$= \frac{1}{y}\left(y^2 - 4a^2\right)$$

$$= \frac{1}{y}\left[4ax + 4a^2 - 4a\right]$$

$$= \frac{1}{y}(4ax)$$

$$= 2x\left(\frac{2a}{y}\right)$$

$$= 2x\frac{dy}{dx}$$

So,

$$y\left\{1-\left(\frac{dy}{dx}\right)^2\right\} = 2x\frac{dy}{dx}$$

Differential Equations Ex 22.3 Q16

$$y = ce^{\tan^{-1} x}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = ce^{\tan^{-1} x} \times \left(\frac{1}{1+x^2}\right)$$

$$\left(1+x^2\right)\frac{dy}{dx} = ce^{\tan^{-1} x}$$

$$\left(1+x^2\right)\frac{dy}{dx} = y$$

Again, differentiating it with respecet to x,

$$2x\frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = \frac{dy}{dx}$$

$$2x\frac{dy}{dx} - \frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = 0$$

$$(2x-1)\frac{dy}{dx} + \left(1+x^2\right)\frac{d^2y}{dx^2} = 0$$

Differential Equations Ex 22.3 Q17

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$$

$$y = e^{m\cos^{-1}x}$$

$$\frac{dy}{dx} = \frac{me^{m\cos^{-1}x}}{-\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-my}{\sqrt{1-x^2}} \text{........(i)}$$

$$\frac{d^2y}{dx^2} = \frac{\sqrt{(1-x^2)}.\left(-m\frac{dy}{dx}\right) - (-my)\frac{(-2x)}{2\sqrt{(1-x^2)}}}{(1-x^2)} \text{ [From (i)]}$$

$$\frac{d^2y}{dx^2} = \frac{(-m)(-my) - x\frac{dy}{dx}}{(1-x^2)} \text{[From (i)]}$$

$$(1-x^2)\frac{d^2y}{dx^2} = m^2y - x\frac{dy}{dx}$$

$$(1-x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$$

Hence Proved

Differential Equations Ex 22.3 Q18

$$y=\log\left(x+\sqrt{x^2+a^2}\right)^2$$

Differentiating it with respect to x,

$$\frac{dy}{dx}\frac{1}{\left(x+\sqrt{a^2+x^2}\right)^2}\times 2\left(x+\sqrt{x^2+a^2}\right)\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$=\frac{2}{\left(x+\sqrt{a^2+x^2}\right)}\times\left(1+\frac{1}{2\sqrt{x^2+a^2}}(2x)\right)$$

$$=\frac{2}{\left(x+\sqrt{a^2+x^2}\right)}\left(\frac{\sqrt{x^2+a^2}+x}{2\sqrt{x^2+a^2}}\right)$$

$$\frac{dy}{dx}=\frac{1}{\sqrt{a^2+x^2}}$$

$$\sqrt{a^2+x^2}\frac{dy}{dx}=1 \qquad \text{---(i)}$$

Again, differentiating it with respecet to x,

$$\sqrt{1-x^2}\frac{d^2y}{dx^2}+\frac{1}{2\sqrt{1-x^2}}(-2x)\frac{dy}{dx}=-m\frac{dy}{dx}$$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2}-\frac{x}{\sqrt{1-x^2}}\frac{dy}{dx}-m\left(\frac{-e^{m\cos^{-1}x}m}{\sqrt{1-x^2}}\right)=0$$

Using equation (i),

$$\sqrt{a^2+x^2}\frac{d^2y}{dx^2}+\frac{2x}{2\sqrt{a^2+x^2}}\frac{dy}{dx}=0$$

$$\left(a^2+x^2\right)\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$$

Differential Equations Ex 22.3 Q19

$$y=2\left(x^2-1\right)+ce^{-x^2} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=2(2x)+ce^{-x^2}(-2x)$$

$$\frac{dy}{dx}=4x-2cxe^{-x^2} \qquad \text{—(ii)}$$

Now,

$$\frac{dy}{dx}+2xy$$

$$=4x-2cxe^{-x^2}+2x\left[2\left(x^2-1\right)+ce^{-x^2}\right]$$

Using equation (i) and (ii),

$$=4x-2cxe^{-x^2}+2x\left(2x^2-2+ce^{-x^2}\right)$$

$$=4x-2cxe^{-x^2}+4x^3-4x+2xce^{-x^2}$$

$$=4x^3$$

So,

$$\frac{dy}{dx}+2xy=4x^3$$

Differential Equations Ex 22.3 Q20

$$y = e^{-x} + ax + b$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = -e^{-x} + a$$

Differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = e^{-x}$$

$$\frac{1}{e^{-x}}\frac{d^2y}{dx^2} = 1$$

$$e^{x}\frac{d^2y}{dx^2} = 1$$

Differential Equations Ex 22.3 Q21(i)

$$y = ax \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = a$$

$$= \frac{ax}{x} \qquad [\because x \in R - \{0\}]$$

$$\frac{dy}{dx} = \frac{y}{x} \qquad [\text{Using equation (i)}]$$

$$x\frac{dy}{dx} = y$$

So, $y = ax$ is the solution of the given equation.

Differential Equations Ex 22.3 Q21(ii)

$$y = \pm\sqrt{a^2 - x^2}$$

Squaring both the sides,

$$y^2 = \left(a^2 - x^2\right)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = -2x$$

$$y\frac{dy}{dx} = -x$$

$$x + y\frac{dy}{dx} = 0$$

So,

$y = \pm\sqrt{a^2 - x^2}$ is the solution of the given equation.

Differential Equations Ex 22.3 Q21(iii)

$$y = \frac{a}{x+a}$$

$$\frac{dy}{dx} = \frac{a}{(x+a)^2} \times (-1) = -\frac{a}{(x+a)^2}$$

Consider,

$$x\frac{dy}{dx} + y = -\frac{ax}{(x+a)^2} + \frac{a}{x+a} = \frac{-ax+ax+a^2}{(x+a)^2} = \frac{a^2}{(x+a)^2} = y^2$$

$$x\frac{dy}{dx} + y = y^2$$

Hence $y = \frac{a}{x+a}$ is the solution of the differential equation $x\frac{dy}{dx} + y = y^2$.

Differential Equations Ex 22.3 Q21(iv)

$$y = ax + b + \frac{1}{2x}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = a - \frac{1}{2x^2}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 0 - \frac{(-2)}{2x^3}$$

$$\frac{d^2y}{dx^2} = \frac{1}{x^3}$$

$$x^3\frac{d^2y}{dx^2} = 1$$

So,

$y = ax + b + \frac{1}{2x}$ **is the solution of the given equation.**

Differential Equations Ex 22.3 Q21(v)

$$y = \frac{1}{4}(x \pm a)^2$$

Case I:

$$y = \frac{1}{4}(x + a)^2$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{4} 2(x + a)$$

$$\frac{dy}{dx} = \frac{1}{2}(x + a)$$

Squaring both sides,

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(x + a)^2$$

$$\left(\frac{dy}{dx}\right)^2 = y \quad \text{[Using equation (i)]}$$

So,

$y = \frac{1}{4}(x + a)$ **is the solution of the given equation.**

Case II:

$$y = \frac{1}{4}(x - a)^2 \quad \text{---(ii)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = \frac{1}{4} 2(x - a)$$

$$\frac{dy}{dx} = \frac{1}{2}(x - a)$$

Squaring both the sides,

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{4}(x - a)^2$$

$$\left(\frac{dy}{dx}\right)^2 = y^2 \quad \text{[Using equation (ii)]}$$

So,

$y = \frac{1}{4}(x - a)$ **is the solution of the given equation.**