RD Sharma Solutions Class 12 Maths Chapter 22 Ex 22.2

Differential Equations Ex 22.2 Q1

$$y^2 = (x-c)^3 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 3(x-c)^2$$

$$(x-c)^2 = \frac{2y}{3}\frac{dy}{dx}$$

$$(x-c)^2 = \left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{1}{2}}$$

Put the value of $(x-c)$ in equation (i),

$$y^2 = \left\{\left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{1}{2}}\right\}^3$$

$$y^2 = \left(\frac{2y}{3}\frac{dy}{dx}\right)^{\frac{3}{2}}$$

Squaring both the sides,

$$y^4 = \left(\frac{2y}{3}\frac{dy}{dx}\right)^3$$

$$y^4 = \frac{8y^3}{27}\left(\frac{dy}{dx}\right)^3$$

$$27y = 8\left(\frac{dy}{dx}\right)^3.$$

Differential Equations Ex 22.2 Q2

$$y = e^{mx} \quad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = me^{mx} \quad \text{—(ii)}$$

From equation (i),

$$y = e^{mx}$$

$$\log y = mx$$

$$m = \frac{\log y}{x}$$

Put the value of m and e^{mx} in equation (i),

$$\frac{dy}{dx} = \frac{\log y}{x} y$$

$$x\frac{dy}{dx} = y\log y$$

Differential Equations Ex 22.2 Q3(i)

$$y^2 = 4ax \quad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a \quad \text{—(ii)}$$

Put the value of a from equation (i) in (ii),

$$2y\frac{dy}{dx} = 4\left(\frac{y^2}{4x}\right)$$

$$2y\frac{dy}{dx} = \frac{y^2}{x}$$

$$2x\frac{dy}{dx} = y$$

Differential Equations Ex 22.2 Q3(ii)

$$y = cx + 2c^2 + c^3 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = c \qquad \text{—(ii)}$$

Put the value of c from equation (ii) in (i),

$$y = \left(\frac{dy}{dx}\right)x + 2\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^3$$

Differential Equations Ex 22.2 Q3(iii)

$$xy = a^2 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$x\frac{dy}{dx} + y(1) = 0$$

$$x\frac{dy}{dx} + y = 0$$

Differential Equations Ex 22.2 Q3(iv)

$$y = ax^2 + bx + c$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2ax + b$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 2a$$

Again, differentiating it with respect to x,

$$\frac{d^3y}{dx^3} = 0$$

Differential Equations Ex 22.2 Q4

$$y = Ae^{2x} + Be^{-2x} \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 2Ae^{2x} - 2Be^{-2x}$$

Again, differentiating it with respect to x,

$$\frac{d^2y}{dx^2} = 4Ae^{2x} + 4Be^{-2x}$$

$$= 4\left(Ae^{2x} + Be^{-2x}\right)$$

$$\frac{d^2y}{dx^2} = 4y \qquad \text{[Using equation (i)]}$$

Differential Equations Ex 22.2 Q5

$$x = A\cos nt + B\sin nt$$

Differentiating with respect to t,

$$\frac{dx}{dt} = -An\sin nt + nB\cos nt$$

Again, differentiating with respect to t,

$$\frac{d^2x}{dt^2} = -An^2\cos nt - n^2B\sin t$$

$$= -n^2\left(A\cos nt + B\sin nt\right)$$

$$\frac{d^2x}{dt^2} = -n^2x \qquad \text{[Using equation (i)]}$$

$$\frac{d^2x}{dt^2} + n^2x = 0$$

Differential Equations Ex 22.2 Q6

$$y^2 = a\left(b - x^2\right)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = a(-2x) \qquad \text{—(i)}$$

Again, differentiating it with respect to x,

$$2\left[y\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{dy}{dx}\right] = -2a$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -\left(\frac{2y}{-2x}\frac{dy}{dx}\right)$$

Using equatoin (i)

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = \frac{y}{x}\frac{dy}{dx}$$

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q7

$$y^2 - 2ay + x^2 = a^2 \qquad \text{—(i)}$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} - 2a\frac{dy}{dx} + 2x = 0$$

$$y\frac{dy}{dx} + x = a\frac{dy}{dx}$$

$$a = \frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}$$

Put the value of a in equation (i),

$$y^2 - 2\left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right]y + x^2 = \left[\frac{y\frac{dy}{dx} + x}{\frac{dy}{dx}}\right]^2$$

Put $\frac{dy}{dx} = y'$

$$y^2 - 2\left(\frac{yy' + x}{y'}\right)y + x^2 = \left(\frac{yy' + x}{y'}\right)^2$$

$$\frac{y'y^2 - 2y'y^2 - 2xy + y'x^2}{y'} = \frac{y'^2y^2 + x^2 + 2xyy'}{y'^2}$$

$$y'^2y^2 - 2y'^2y^2 - 2xyy' + y'^2x^2 - y'^2y^2 - x^2 - 2xyy' = 0$$

$$-4xyy' + y'^2x^2 - x^2 - 2y'^2y^2 = 0$$

$$y'^2\left(x^2 - 2y^2\right) - 4xyy' - x^2 = 0$$

Differential Equations Ex 22.2 Q8

$$(x-a)^2+(y-b)^2=r^2 \qquad \text{—(i)}$$

Differentiating with respect to x,

$$2(x-a)+2(y-b)\frac{dy}{dx}=0$$

$$(x-a)+(y-b)\frac{dy}{dx}=0 \qquad \text{—(ii)}$$

Differentiating with respect to x,

$$1+(y-b)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)=0$$

$$1+(y-b)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$$

$$(y-b)=-\left\{\frac{\left(\frac{dy}{dx}\right)^2+1}{\frac{d^2y}{dx^2}}\right\} \qquad \text{—(iii)}$$

Put $(y-b)$ in equation (ii),

$$(x-a)-\left\{\frac{\left(\frac{dy}{dx}\right)^2+1}{\frac{d^2y}{dx^2}}\right\}\frac{dy}{dx}=$$

$$(x-a)\left(\frac{d^2y}{dx^2}\right)-\left(\frac{dy}{dx}\right)^3-\left(\frac{dy}{dx}\right)=0$$

$$(x-a)\frac{d^2y}{dx^2}=\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3$$

$$(x-a)=\frac{\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}} \qquad \text{—(iv)}$$

Put the value of $(x-a)$ and $(y-b)$ from equation (iii) and (iv) in equation (i),

$$\left\{\frac{\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}}\right\}^2+\left\{\frac{\left(\frac{dy}{dx}\right)^2+1}{\frac{d^2y}{dx^2}}\right\}^2=r^2$$

Put $\frac{dy}{dx}=y'$ and $\frac{d^2y}{dx^2}=y''$

$$\left(y'+y'^3\right)^2+\left(y'^2+1\right)^2=r^2y''^2$$

$$y'^2\left(1+y'^2\right)^2+\left(1+y'^2\right)^2=r^2y''^2$$

Differential Equations Ex 22.2 Q9

We know that, equation of a circle with centre at (h,k) and radius r is given by,

$$\left(x-h^2\right)+(y-k)^2=r^2 \qquad \text{---(i)}$$

Here, centre lies, on y-axis, so $h=0$

$$\Rightarrow \quad x^2+(y-k)^2=r^2 \qquad \text{---(ii)}$$

Also, given that, circle is passing through origin, so

$$0+k^2=r^2$$
$$k^2=r^2$$

So, equation (ii) becomes,

$$x^2+(y-k)^2=k^2$$
$$x^2+y^2-2yk=0$$
$$2yk=x^2+y^2$$
$$k=\frac{x^2+y^2}{2y}$$

Differentiating with respect to x,

$$0=\frac{2y\left(2x+2y\frac{dy}{dx}\right)-\left(x^2+y^2\right)2\frac{dy}{dx}}{(2y)^2}$$
$$0=4xy+4y^2\frac{dy}{dx}-2x^2\frac{dy}{dx}-2y^2\frac{dy}{dx}$$
$$0=2y^2\frac{dy}{dx}-2x^2\frac{dy}{dx}+4xy$$
$$x^2\frac{dy}{dx}-y^2\frac{dy}{dx}=2xy$$
$$\left(x^2-y^2\right)\frac{dy}{dx}=2xy$$

Differential Equations Ex 22.2 Q10

Equation of circle with centre (h,k) and radius r is given by

$$(x-h)^2+(y-k)^2=r^2 \qquad \text{---(i)}$$

Here, centre lie on x-axis, so

$$k=0$$

$$\Rightarrow \quad (x-h)^2+y^2=r^2 \qquad \text{---(ii)}$$

Also, given that, circle is passing through $(0,0)$, so,

$$h^2=r^2$$

So, equation (ii) becomes,

$$(x-h)^2+y^2=h^2$$

$$x^2+h^2-2xh+y^2=h^2$$

$$x^2-2xh+y^2=0$$

$$2xh=x^2+y^2$$

$$h=\frac{x^2+y^2}{2x}$$

Differentiating it with respect to x,

$$0=\frac{\left(2x+2y\frac{dy}{dx}\right)2x-\left(x^2+y^2\right)2}{(2x)^2}$$

$$\left(2x+2y\frac{dy}{dx}\right)2x-\left(x^2+y^2\right)^2=0$$

$$2x^2+2yx\frac{dy}{dx}-x^2-y^2=0$$

$$\left(x^2-y^2\right)+2xy\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q11

Let A be the surface area of rain drain, V be its volume, and r be the radius of rain drop.

Given,

$$\frac{dV}{dt}\propto A$$

$$\frac{dV}{dt}=-kA \qquad [\text{negative because } V \text{ decreases with increase in } t]$$

where k is the constant of proportionality.

So,

$$\frac{d}{dt}\left(\frac{4\pi}{3}r^3\right)=-k\left(4\pi r^2\right)$$

$$4\pi r^2\frac{dr}{dt}=-k\left(4\pi r^2\right)$$

$$\frac{dr}{dt}=-k$$

Differential Equations Ex 22.2 Q12

Equation of parabolas with lotus rectum '$(4a)$' and whose area is parallel to x axes and vertex at (h,k) is given by,

$$(y-k)^2 = 4a(x-h)$$

Differentiating with respect to x,

$$2(y-k)y_1 = 4a(1)$$

$$(y-k)y_1 = 2a \qquad \text{—(i)}$$

Differentiating with respect to x,

$$(y-k)y_2 + (y_1)(y_1) = 0$$

$$(y-k)y_2 + (y_1)^2 = 0$$

$$\left(\frac{2a}{y_1}\right)^{y_2} + (y_1)^2 = 0$$

Using equation (i)

$$2ay_2 + (y_1)^3 = 0$$

Differential Equations Ex 22.2 Q13

$$y = 2(x^2-1) + ce^{-x^2} \qquad \text{—(i)}$$

Differentiating it in equation (i),

$$\frac{dy}{dx} = 4x - 2cxe^{-x^2} \qquad \text{—(ii)}$$

Now,

$$\frac{dy}{dx} + 2xy$$

$$= 4x - 2cxe^{-x^2} + 2x\left[2(x^2-1) + ce^{-x^2}\right]$$

$$= 4x - 2cxe^{-x^2} + 4x^3 - 4x + 2xce^{-x^2}$$

$$= 4x^3$$

So,

$$\frac{dy}{dx} + 2xy = 4x^3$$

Which is given equation, so

$y = 2(x^2+1) + ce^{-x^2}$ is the solution of the equation.

Differential Equations Ex 22.2 Q14

$$y = (\sin^{-1}x)^2 + A\cos^{-1}x + B$$

$$\frac{dy}{dx} = 2\sin^{-1}x \times \left(\frac{1}{\sqrt{1-x^2}}\right) + A \times \left(\frac{-1}{\sqrt{1-x^2}}\right) + 0$$

$$\sqrt{1-x^2}\frac{dy}{dx} = 2\sin^{-1}x - A$$

$$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{1}{\sqrt{1-x^2}}\right)(-2x) = 2 \times \left(\frac{1}{\sqrt{1-x^2}}\right) - 0$$

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 2 = 0$$

Note: Answer given in the book is incorrect.

Differential Equations Ex 22.2 Q15(i)

Consider the given equation.,

$(2x+a)^2+y^2=a^2....(1)$

Differentiating the above equation with respect to x, we have,

$$2(2x+a)+2y\frac{dy}{dx}=0$$

$$\Rightarrow (2x+a)+y\frac{dy}{dx}=0$$

$$\Rightarrow 2x+a=-y\frac{dy}{dx}$$

$$\Rightarrow a=-2x-y\frac{dy}{dx}$$

Substituting the value of a in equation (1), we have

$$\left(2x-2x-y\frac{dy}{dx}\right)^2+y^2=\left(-2x-y\frac{dy}{dx}\right)^2$$

$$\Rightarrow \left(y\frac{dy}{dx}\right)^2+y^2=\left(4x^2+y^2\left(\frac{dy}{dx}\right)^2+4xy\frac{dy}{dx}\right)$$

$$\Rightarrow y^2=4x^2+4xy\frac{dy}{dx}$$

$$\Rightarrow y^2-4x^2-4xy\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q15(ii)

$$(2x-a)^2-y^2=a^2$$

$$4x^2+a^2-ax-y^2=a^2$$

$$4x^2-4ax-y^2=0$$

$$4ax=4x^2-y^2$$

$$a=\frac{4x^2-y^2}{4x}$$

Differentiating it with respect to *x*,

$$0=\left[\frac{4x\left(8x-2y\frac{dy}{dx}\right)-4\left(4x^2-y^2\right)}{(4x)^2}\right]$$

$$32x^2-8xy\frac{dy}{dx}-16x^2+4y^2=0$$

$$16x^2-8xy\frac{dy}{dx}+4y^2=0$$

$$4x^2+y^2=2xy\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q15(iii)

Consider the given equation,

$$(x-a)^2+2y^2=a^2 \ldots.(1)$$

Differentiating the above equation with respect to x, we have

$$2(x-a)+4y\frac{dy}{dx}=0$$

$$\Rightarrow (x-a)+2y\frac{dy}{dx}=0$$

$$\Rightarrow (x-a)=-2y\frac{dy}{dx}$$

$$\Rightarrow a=x+2y\frac{dy}{dx}$$

Substituting the value of a in equation (1), we have

$$\left(x-x+2y\frac{dy}{dx}\right)^2+2y^2=\left(x+2y\frac{dy}{dx}\right)^2$$

$$\Rightarrow 4y^2\left(\frac{dy}{dx}\right)^2+2y^2=x^2+4y^2\left(\frac{dy}{dx}\right)^2+4xy\frac{dy}{dx}$$

$$\Rightarrow 2y^2-x^2=4xy\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q16(i)

$$x^2+y^2=a^2$$

Differentiating it with respect to *x*,

$$2x+2y\frac{dy}{dx}=0$$

$$x+y\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q16(ii)

$$x^2-y^2=a^2$$

Differentiating it with respect to *x*,

$$2x-2y\frac{dy}{dx}=0$$

$$x-y\frac{dy}{dx}=0$$

Differential Equations Ex 22.2 Q16(iii)

$$y^2=4ax$$

$$\frac{y^2}{x}=4a$$

Differentiating it with respect to *x*,

$$\left[\frac{x \times 2y\frac{dy}{dx}-y^2(1)}{x^2}\right]=0$$

$$2xy\frac{dy}{dx}-y^2=0$$

$$2x\frac{dy}{dx}-y=0$$

Differential Equations Ex 22.2 Q16(iv)

$$x^2+(y-b)^2=1 \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$2x+2(y-b)\frac{dy}{dx}=0$$

$$x+(y-b)\frac{dy}{dx}=0$$

$$(y-b)\frac{dy}{dx}=-x$$

$$(y-b)=\frac{-x}{\frac{dy}{dx}}$$

Put the value of $(y-b)$ is equation (i)

$$x^2\left(\frac{-x}{\frac{dy}{dx}}\right)^2=1$$

$$x^2\left(\frac{dy}{dx}\right)^2+x^2=\left(\frac{dy}{dx}\right)^2$$

$$x^2\left\{\left(\frac{dy}{dx}\right)^2+1\right\}=\left(\frac{dy}{dx}\right)^2$$

Differential Equations Ex 22.2 Q16(v)

$$(x-a)^2-y^2=1 \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$2(x-a)-2y\frac{dy}{dx}=0$$

$$(x-a)-y\frac{dy}{dx}=0$$

$$(x-a)=y\frac{dy}{dx}$$

Put the value of $(x-a)$ is equation (i)

$$\left(y\frac{dy}{dx}\right)^2-y^2=1$$

$$y^2\left(\frac{dy}{dx}\right)^2-y^2=1$$

Differential Equations Ex 22.2 Q16(vi)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{b^2x^2 - a^2y^2}{a^2b^2} = 1$$

$$b^2x^2 - a^2y^2 = a^2b^2$$

Differentiating it with respect to x,

$$2xb^2 - 2a^2y\frac{dy}{dx} = 0$$

$$xb^2 - ya^2\frac{dy}{dx} = 0 \qquad \text{---(i)}$$

Again, differentiating it with respect to x,

$$b^2 - a^2\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)\right) = 0$$

$$b^2 = a^2\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right)$$

Put the value of b^2 in equation (i)

$$xb^2 - ya^2\frac{dy}{dx} = 0$$

$$xa^2\left(y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right) - ya^2\frac{dy}{dx} = 0$$

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

$$x\left\{y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right\} = y\frac{dy}{dx}$$

Differential Equations Ex 22.2 Q16(vii)

$$y^2 = 4a(x - b)$$

Differentiating it with respect to x,

$$2y\frac{dy}{dx} = 4a$$

Again, differentiating it with respect to x,

$$2\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right)\right] = 0$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$$

Differential Equations Ex 22.2 Q16(viii)

$$y = ax^3$$

Differentiating it with respect to x,

$$\frac{dy}{dx} = 3ax^2$$

$$= 3\left(\frac{y}{x^3}\right)x^2$$

Using equation (i)

$$\frac{dy}{dx} = \frac{3y}{x}$$

$$x\frac{dy}{dx} = 3y$$

Differential Equations Ex 22.2 Q16(ix)

$$x^2+y^2=ax^3$$
$$\frac{x^2+y^2}{x^3}=a$$

Differentiating it with respect to x,

$$\left[\frac{(x^3)\left(2x+2y\frac{dy}{dx}\right)-(x^2+y^2)(3x^2)}{(x^3)^2}\right]=0$$

$$2x^4+2x^3y\frac{dy}{dx}-3x^4-3x^2y^2=0$$

$$2x^3y\frac{dy}{dx}-x^4-3x^2y^2=0$$

$$2x^3y\frac{dy}{dx}=x^4+3x^2y^2$$

$$2x^3y\frac{dy}{dx}=x^2\left(x^2+3y^2\right)$$

$$2xy\frac{dy}{dx}=\left(x^2+3y^2\right)$$

Differential Equations Ex 22.2 Q16(x)

$$y=e^{ax} \qquad \text{---(i)}$$

Differentiating it with respect to x,

$$\frac{dy}{dx}=ae^{ax}$$

$$\frac{dy}{dx}=ay \qquad \text{---(ii)}$$

From equation (i),

$$y=e^{ax}$$
$$\log y=ax$$
$$a=\frac{\log y}{x}$$

Put the value of a in equation (ii),

$$\frac{dy}{dx}=\left(\frac{\log y}{x}\right)y$$

$$x\frac{dy}{dx}=y\log y$$

Differential Equations Ex 22.2 Q17

We know that the equation of said family of ellipses is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{---------(i)}$$

DIfferentiating (i) w.r.t. x , we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\cdot\frac{dy}{dx}=0$$

$$\Rightarrow \qquad \frac{y}{x}\left(\frac{dy}{dx}\right)=\frac{-b^2}{a^2} \qquad \text{---------(ii)}$$

DIfferentiating (ii) w.r.t. x , we get

$$\frac{y}{x}\left(\frac{d^2y}{dx^2}\right)+\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)\frac{dy}{dx}=0$$

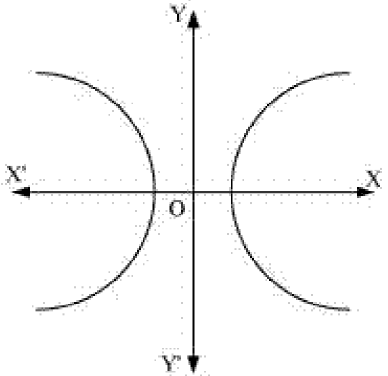
$$\Rightarrow \qquad xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

which is the required differential equation.

Differential Equations Ex 22.2 Q18

The equation of the family of hyperbolas with the centre at origin and foci along the x-axis is:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad ...(1)$$



Differentiating both sides of equation (1) with respect to x, we get:

$$\frac{2x}{a^2}-\frac{2yy'}{b^2}=0$$

$$\Rightarrow \frac{x}{a^2}-\frac{yy'}{b^2}=0 \qquad ...(2)$$

Again, differentiating both sides with respect to x, we get:

$$\frac{1}{a^2}-\frac{y'\cdot y'+yy''}{b^2}=0$$

$$\Rightarrow \frac{1}{a^2}=\frac{1}{b^2}\left((y')^2+yy''\right)$$

Substituting the value of $\frac{1}{a^2}$ in equation (2), we get:

$$\frac{x}{b^2}\left((y')^2+yy''\right)-\frac{yy'}{b^2}=0$$

$$\Rightarrow x(y')^2+xyy''-yy'=0$$

$$\Rightarrow xyy''+x(y')^2-yy'=0$$

This is the required differential equation.

Differential Equations Ex 22.2 Q19

Let C denote the family of circles in the second quadrant and touching the coordinate axes.

Let $(-a, a)$ be the coordinate of the centre of any member of this family.

Equation representing the family C is

$$(x+a)^2 + (y-a)^2 = a^2 \qquad \text{---------(i)}$$

or $x^2 + y^2 + 2ax - 2ay + a^2 = 0 \qquad$ ---------(ii)

Differentiating eqn (ii) w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} + 2a - 2a\frac{dy}{dx} = 0$$

$$\Rightarrow x + y\frac{dy}{dx} = a\left(\frac{dy}{dx} - 1\right)$$

$$\Rightarrow \qquad a = \frac{x + yy'}{y' - 1}$$

Substituting the value of a in (ii), we get

$$\left[x + \frac{x + yy'}{y' - 1}\right]^2 + \left[y - \frac{x + yy'}{y' - 1}\right]^2 = \left[\frac{x + yy'}{y' - 1}\right]^2$$

$$\Rightarrow \quad \left[xy' - x + x + yy'\right]^2 + \left[yy' - y - x - yy'\right]^2 = \left[x + yy'\right]^2$$

$$\Rightarrow \quad (x+y)^2 y'' + (x+y)^2 = \left[x + yy'\right]^2$$

$$\Rightarrow \quad (x+y)^2\left[\left(y'\right)^2 + 1\right] = \left[x + yy'\right]^2$$

which is the differential equation representing the given family of circles.