RD Sharma Solutions Class 12 Maths Chapter 22 Ex 22.11

Differential Equations Ex 22.11 Q1

Let A be the surface area of balloon, so

$$\frac{dA}{dt} \propto t$$

$\Rightarrow$ $$\frac{dA}{dt} = \lambda t$$

$\Rightarrow$ $$\frac{d}{dt}\left(4\pi r^2\right) = \lambda t$$

$\Rightarrow$ $$8\pi r \frac{dr}{dt} = \lambda t$$

$\Rightarrow$ $$8\pi r\, dr = \lambda t$$

$\Rightarrow$ $$8\pi \int r\, dr = \lambda \int t\, dt$$

$\Rightarrow$ $$8\pi \frac{r^2}{2} = \frac{\lambda t^2}{2} + c$$

$\Rightarrow$ $$4\pi r^2 = \frac{\lambda t^2}{2} + c \;----(1)$$

Given $r = 1$ unit when $t = 0$, so

$$4\pi (1)^2 = 0 + c$$

$\Rightarrow$ $$4\pi = c$$

Using it is equation (i),

$$4\pi r^2 = \frac{\lambda t^2}{2} + 4\pi \;----(2)$$

Also, given $r = 2$ units when $t = 3$ sec.

$$4\pi (2)^2 = \frac{\lambda (3)^2}{2} + 4\pi$$

$\Rightarrow$ $$16\pi = \frac{9}{2}\lambda + 4\pi$$

$\Rightarrow$ $$\frac{9}{2}\lambda = 12\pi$$

$\Rightarrow$ $$\lambda = \frac{24}{9}\pi$$

$\Rightarrow$ $$\lambda = \frac{8}{3}\pi$$

Now,equation (2) becomes

$$4\pi r^2 = \frac{8\pi}{6}t^2 + 4\pi$$

$\Rightarrow$ $$4\pi\left(r^2 - 1\right) = \frac{4}{3}\pi t^2$$

$\Rightarrow$ $$r^2 - 1 = \frac{1}{3}t^2$$

$\Rightarrow$ $$r^2 = 1 + \frac{1}{3}t^2$$

$\therefore$ $$r = \sqrt{\left(1 + \frac{1}{3}t^2\right)}$$

Differential Equations Ex 22.11 Q2

Let the population after time t be P and initial population be P_0.

So,

$$\frac{dP}{dt} = 5\% \times P$$

$$\Rightarrow \quad \frac{dP}{dt} = \frac{P}{20}$$

$$\Rightarrow \quad 20\frac{dP}{P} = dt$$

$$\Rightarrow \quad 20\int\frac{dP}{P} = \int dt$$

$$\Rightarrow \quad 20\log|P| = t + c \text{ - - - - } (1)$$

Given $P = P_0$ when $t = 0$

$$20\log(P_0) = 0 + c$$

$$\Rightarrow \quad 20\log(P_0) = c$$

Now, equation (1) becomes

$$20\log(P) = t + 20\log(P_0)$$

$$\Rightarrow \quad 20\log\left(\frac{P}{P_0}\right) = t$$

Let time is t, when $P = 2P_0$, so,

$$20\log\left(\frac{2P}{P_0}\right) = t_1$$

$$\Rightarrow \quad 20\log 2 = t_1$$

Required time period $= 20\log 2$ years

Differential Equations Ex 22.11 Q3

Let P be the population at any time t and P_0 be the initial population.

So

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \quad \frac{dP}{dt} = \lambda P$$

$$\Rightarrow \quad \frac{dP}{dt} = \lambda\, dt$$

$$\Rightarrow \quad \int \frac{dP}{dt} = \lambda \int dt +$$

$$\Rightarrow \quad \log P = \lambda t + c \; --- (1)$$

Here, $P = P_0$ t when $t = 0$,

$$\log(P_0) = 0 + c$$

$$\Rightarrow \quad c = \log(P_0)$$

Now, equation (1) becomes

$$\log(P) = \lambda t + \log(P_0)$$

$$\Rightarrow \quad \log\left(\frac{P}{P_0}\right) = \lambda t \; --- (2)$$

Given $P = 2P_0$ when $t = 25$

$$\log\left(\frac{2P_0}{P_0}\right) = 25\lambda$$

$$\Rightarrow \quad \log 2 = 25\lambda$$

$$\Rightarrow \quad \lambda = \frac{\log 2}{25}$$

Now equation (2) becomes

$$\log\left(\frac{P}{P_0}\right) = \left(\frac{\log 2}{25}\right)t$$

let t_1 be the time to become population 500000 from 100000, so,

$$\log\left(\frac{500000}{100000}\right) = \frac{\log 2}{25} t_1$$

$$\Rightarrow \quad t_1 = \frac{25 \log 5}{\log 2}$$

$$\Rightarrow \quad = \frac{25(1.609)}{(0.6931)} = 58$$

Required time $= 58$ years

Differential Equations Ex 22.11 Q4

Let C be the count of bacteria at any time t.
It is given that

$$\frac{dC}{dt} \propto C$$

$$\Rightarrow \frac{dC}{dt} = \lambda C, \text{ where } \lambda \text{ is a constant of proportionality}$$

$$\Rightarrow \frac{dC}{C} = \lambda dt$$

$$\Rightarrow \int \frac{dC}{C} = \lambda \int dt$$

$$\Rightarrow \log C = \lambda t + \log K....(1)$$

Initially, at $t = 0$, $C = 100000$

Thus, we have,

$$\log 100000 = \lambda \times 0 + \log K....(2)$$

$$\Rightarrow \log 100000 = \log K....(3)$$

At $t = 2$, $C = 100000 + 100000 \times \frac{10}{100} = 110000$

Thus, from (1), *we have,*

$$\log 110000 = \lambda \times 2 + \log K....(4)$$

Subtracting equation (2) *from* (4), *we have,*

$$\log 110000 - \log 100000 = 2\lambda$$

$$\Rightarrow \log 11 \times 10000 - \log 10 \times 10000 = 2\lambda$$

$$\Rightarrow \log \frac{11 \times 10000}{10 \times 10000} = 2\lambda$$

$$\Rightarrow \log \frac{11}{10} = 2\lambda$$

$$\Rightarrow \lambda = \frac{1}{2} \log \frac{11}{10} ....(5)$$

We need to find the time 't' in which the count reaches 200000.

Substituting the values of λ *and* K *from equations* (3) *and* (5) *in equation* (1), *we have*

$$\log 200000 = \frac{1}{2} \log \frac{11}{10} t + \log 100000$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log 200000 - \log 100000$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log \frac{200000}{100000}$$

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} t = \log 2$$

$$\Rightarrow t = \frac{2 \log 2}{\log \frac{11}{10}} \text{ hours}$$

Differential Equations Ex 22.11 Q5

Given that, interest is compounded 6% per annum. Let P be principal

$$\frac{dP}{dt} = \frac{Pr}{100}$$

$$\frac{dP}{dt} = \frac{r}{100}dt$$

$$\int \frac{dP}{P} = \int \frac{r}{100}dt$$

$$\log P = \frac{rt}{100} + c - - - (1)$$

Let P_0 be the initial principal at $t = 0$,

$$\log(P_0) = 0 + c$$

$$c = \log(P_0)$$

Put value of C is equation (1)

$$\log(P) = \frac{rt}{100} + \log(P_0)$$

$$\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$$

Case I:

Here, $P_0 = 1000, t = 10$ years and $r = 6$

$$\log\left(\frac{P}{1000}\right) = \frac{6 \times 10}{100}$$

$$\log P - \log 1000 = 0.6$$

$$\log P = \log e^{0.6} + \log 1000$$

$$= \log\left(e^{0.6} + 1000\right)$$

$$= \log(1.822 + 1000)$$

$$\log P = \log 1822$$

so,

$$P = \text{Rs}1822$$

Rs 1000 will be Rs 1822 after 10 years

Differential Equations Ex 22.11 Q6

Let A be the amount of bacteria present at time t and A_0 be the initial amount of bacteria. Here,

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = \lambda A$$

$$\int \frac{dA}{A} = \int \lambda dt$$

$$\log A = \lambda t + c --- (1)$$

When $t = 0$, $A = A_0$

$$\log(A_0) = 0 + c$$

$$c = \log A_0$$

Using equation (1),

$$\log A = \lambda t + \log A_0$$

$$\log\left(\frac{A}{A_0}\right) = \lambda t ----- (2)$$

Given, bacteria triples is 5 hours, so $A = 3A_0$, when $t = 5$

so, $$\log\left(\frac{3A_0}{A_0}\right) = 5\lambda$$

$$\log 3 = 5\lambda$$

$$\lambda = \frac{\log 3}{5}$$

Putting the value of λ in equation (2)

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5} t$$

Case I: let A_1 be the number of bacteria present 10 hours, os

$$\log\left(\frac{A_1}{A_0}\right) = \frac{\log 3}{5} \times 10$$

$$\log\left(\frac{A_1}{A_0}\right) = 2\log 3$$

$$\log\left(\frac{A_1}{A_0}\right) = 2(1.0986)$$

$$\log\left(\frac{A_1}{A_0}\right) = 2.1972$$

$$A_1 = A_0 e^{2.1972}$$

$$A_1 = A_0 9$$

thus

There will be 9 times the bateria present is 10 hours.

Case II: let t_1 be the time necessary for the bacteria to be 10 times, os

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 3}{5} \times t$$

$$\log\left(\frac{10A_0}{A_0}\right) = \frac{\log 3}{5} \times t_1$$

$$5 \log 10 = \log 3 t_1$$

$$5\frac{\log 10}{\log 3} = t_1$$

Required time is $\frac{5\log 10}{\log 3}$ hours

Differential Equations Ex 22.11 Q7

Let P be the population of the city at any time t.
It is given that

$$\frac{dP}{dt} \propto P$$

$$\Rightarrow \frac{dP}{dt} = \lambda P, \text{ where } \lambda \text{ is a constant of proportionality}$$

$$\Rightarrow \frac{dP}{P} = \lambda dt$$

$$\Rightarrow \int \frac{dP}{P} = \lambda \int dt$$

$$\Rightarrow \log P = \lambda t + \log K \text{....(1)}$$

Initially, at $t = 1990,\ P = 200000$

Thus, we have,

$$\log 200000 = \lambda \times 1990 + \log K \text{....(2)}$$

At $t = 2000,\ P = 250000$

Thus, from (1), *we have,*

$$\log 250000 = \lambda \times 2000 + \log K \text{....(3)}$$

Subtracting equation (2) *from* (3), *we have,*

$$\log 250000 - \log 200000 = 10\lambda$$

$$\Rightarrow \log \frac{4}{5} = 10\lambda$$

$$\Rightarrow \lambda = \frac{1}{10} \log \frac{4}{5} \text{....(4)}$$

Substituting the value of λ *from equation* (4) *in equation* (1), *we have*

$$\log 200000 = 1990 \times \frac{1}{10} \log \frac{4}{5} + \log K$$

$$\Rightarrow \log K = \log 200000 - 199 \times \log \frac{4}{5} \text{....(5)}$$

Substituting the value of λ, $\log K$ *and* $t = 2010$ *in equation* (1), *we have*

$$\log P = \left\{ \frac{1}{10} \log \frac{4}{5} \right\} 2010 + \log 200000 - 199 \times \log \frac{4}{5}$$

$$\Rightarrow \log P = \log \left\{ \frac{4}{5} \right\}^{201} + \log \left(200000 \times \left(\frac{5}{4} \right)^{199} \right)$$

$$\Rightarrow P = \left\{ \frac{4}{5} \right\}^{201} \times 200000 \times \left(\frac{5}{4} \right)^{199}$$

$$\Rightarrow P = \left(\frac{5}{4} \right)^{2} \times 200000 = \frac{25}{16} \times 200000 = 312500$$

Differential Equations Ex 22.11 Q8

Given,

$$C'(x) = \frac{dC}{dx} = 2 + 0.15x$$

$$dC = (2 + 0.15x)\,dx$$

$$\int dC = \int (2 + 0.15x)\,dx$$

$$C = 2x + \frac{0.15x^2}{2} + \lambda \;----(1)$$

Given $C = 100$ when $x = 0$, so

$$100 = 0 + 0 + \lambda$$

$$\lambda = 100$$

Put the value of λ in equation (1) total cost function is

$$C(x) = 2x + \frac{0.15x^2}{2} + 100$$

$$C(x) = 2x + 0.075x^2 + 100$$

Differential Equations Ex 22.11 Q9

Let P be principal at any time t at the rate of $r\%$ per annum, so

$$\frac{dP}{dt} = \frac{\Pr}{100}$$

$$\frac{dP}{P} = \frac{r}{100}dt$$

$$\int \frac{dP}{P} = \frac{r}{100}\int dt$$

$$\log P = \frac{rt}{100} + c \text{ - - - } (1)$$

Let P_0 be the initial amount, so

$$\log(P_0) = 0 + c$$

$$c = \log(P_0)$$

Put the value of C in equation (1),

$$\log P = \frac{rt}{100} + \log P_0$$

$$\log P - \log P_0 = \frac{rt}{100}$$

$$\log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$$

For $t = 1, r = 8\%$

$$\log\left(\frac{P}{P_0}\right) = \frac{8 \times 1}{100}$$

$$\log\frac{P}{P_0} = 0.08$$

$$\frac{P}{P_0} = e^{0.08}$$

$$\frac{P}{P_0} = 1.0833$$

$$\frac{P}{P_0} - 1 = 1.0833 - 1$$

$$\frac{P - P_0}{P_0} = 0.0833$$

percentage increase in amount in one year

$$= 0.0833 \times 100$$

$$= 8.33\%$$

Required percentage = 8.33%

Differential Equations Ex 22.11 Q10

Here,

$$L\frac{di}{dt}+Ri=E$$

$$\frac{di}{dt}+\frac{R}{L}i=\frac{E}{L}$$

It is a linear differential equation. Compound it with $\frac{\text{dy}}{\text{dx}}+Py=Q$

$$P=\frac{R}{L},\ Q=\frac{E}{L}$$

$$I.F.=e^{\int P dt}$$

$$=e^{\int\frac{P}{L}dt}$$

$$I.F.==e^{\left(\frac{R}{L}\right)t}$$

Solution of the equation is given by

$$i\left(I.F.\right)=\int Q\left(I.F.\right)dt+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\int\frac{E}{L}\left(e^{\left(\frac{R}{L}\right)t}\right)dt+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\frac{E}{L}\times\frac{L}{R}\left(e^{\left(\frac{R}{L}\right)t}\right)+c$$

$$i\left(e^{\left(\frac{R}{L}\right)t}\right)=\frac{E}{L}\left(e^{\left(\frac{R}{L}\right)t}\right)+c$$

$$i=\left(\frac{E}{L}\right)+c\left(e^{\left(\frac{R}{L}\right)t}\right)\text{-----}(1)$$

Initiatially there was no current, so put $i=0, t=0$

$$0=\frac{F}{R}+ce^0$$

$$0=\frac{F}{R}+c$$

$$c=-\frac{F}{R}$$

Using Equation (1)

$$i=\frac{F}{R}-\frac{F}{R}e^{\left(-\frac{R}{L}\right)t}$$

$$i=\frac{F}{R}\left(1-e^{\left(-\frac{R}{L}\right)t}\right)$$

Differential Equations Ex 22.11 Q11

Let A be the quantity of mass at any time t, so

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dP} = -\lambda A$$

$$\frac{dA}{A} = -\lambda dt$$

$$\int \frac{dA}{A} = -\lambda \int dt$$

$$\log A = -\lambda t + c - - - (1)$$

Let initial quantity of mass be A_0, so

$$\log A_0 = -\lambda (0) + c$$

$$\log (A_0) = c$$

Now, equation (1) becames,

$$\log A = -\lambda t + \log A_0$$

$$\log \left(\frac{A}{A_0}\right) = -\lambda t$$

Let t_1 be the required time to half the mass, so $A = \frac{1}{2} A_0$,

Now, $$\log \left(\frac{A}{A_0}\right) = -\lambda t$$

$$\log \left(\frac{A}{2A}\right) = -\lambda t$$

$$-\log 2 = -\lambda t$$

$$\frac{1}{\lambda} \log 2 = t$$

Required time is $\frac{1}{\lambda}$ log2 units where λ is constant of proportionality.

Differential Equations Ex 22.11 Q12

Let A be the quantity of radius at any time t, so

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dp} = -\lambda A$$

$$\frac{dA}{A} = -\lambda t$$

$$\int \frac{dA}{A} = -\lambda \int dt$$

$$\log A = -\lambda t + c - - - (1)$$

Let A_0 be the initial amount of radius percentage , so

$$\log A_0 = -\lambda (0) + c$$

$$c = \log (A_0)$$

Using, equation (1),

$$\log\ A = -\lambda t + \log A_0$$

$$\log\left(\frac{A}{A_0}\right) = -\lambda t - - - - - (2)$$

Given,its half-life is 1590 years, so

$$\log\left(\frac{\frac{1}{2}A_0}{A_0}\right) = -\lambda (1590)$$

$$\log\left(\frac{1}{2}\right) = -\lambda (1590)$$

$$-\log 2 = -\lambda (1590)$$

$$\log 2 = \lambda (1590)$$

$$\frac{\log 2}{1590} = \lambda$$

Now, equation (1) becomes

$$\log\left(\frac{A}{A_0}\right) = -\frac{\log 2}{1590} t$$

Differential Equations Ex 22.11 Q13

Slope of tangent at point $(x, y) = -\frac{x}{y}$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$y\,dy = -x dx$$

$$\int y\,dt = -\int x\,dx$$

$$\frac{y^2}{2} + \frac{x^2}{2} = c_1$$

$$x^2 + y^2 = c - - - - - (1)$$

Given, curve is passing through $(3, -4)$, so

$$(3)^2 + (-4)^2 = c$$

$$9 + 16 = c$$

$$c = 25$$

So, using equation (1),

$$x^2 + y^2 = 25$$

$$x^2 + y^2 = 25$$

Differential Equations Ex 22.11 Q14

$$y - x\frac{dy}{dx} = y^2 + \frac{dy}{dx}$$

$$\frac{dy}{dx} + x\frac{dy}{dx} = y - y^2$$

$$(1+x)\frac{dy}{dx} = y - y^2$$

$$\frac{dy}{y-y^2} = \frac{dx}{1+x}$$

$$\frac{dy}{y(1-y)} = \frac{dx}{1+x}$$

$$\int\left(\frac{1}{y} + \frac{1}{1-y}\right)dx = \int\frac{dx}{1+x}$$

$$\log|y| - \log|1-y = \log|1+x\|+\log|c|$$

$$\frac{y}{1-y} = c(1+x)$$

$$y = (1-y)c(1+x) - - - -(1)$$

It is passing through $(2,2)$ so,

$$2 = (1-2)c(1+2)$$

$$2 = -3c$$

$$c = -\frac{2}{3}$$

Now,equation (1) becomes,

$$y = -\frac{2}{3}(1-y)(1+x)$$

$$3y = -2(1+x-y-xy)$$

$$3y + 2 + 2x - 2y - 2xy = 0$$

$$y + 2x - 2xy + 2 = 0$$

$$2xy - 2x - 2 - y = 0$$

Chapter 22 Differential Equations Ex 22.11 Q15

It is passing through $\left(1, \frac{\pi}{4}\right)$, so,

$$\tan\left(\frac{\pi}{4}\right) = -\log|1| + c$$

$$1 = 0 + c$$

$$c = 1$$

Now, equation (1) becomes

$$\tan\left(\frac{y}{x}\right) = -\log|x| + 1$$

Therefore,

$$\tan\left(\frac{y}{x}\right) = \log\left|\frac{e}{x}\right|$$

Differential Equations Ex 22.11 Q16

Let $P(x,y)$ be the point of contact of tangent and curve $y = f(x)$. and It cuts axes at A and B so, equatin of tangent at $P(x,y)$

$$Y - y = \frac{dy}{dx}(X - x)$$

Put $X = 0$

$$Y - y = \frac{dy}{dx}(-x)$$

$$Y = y - x\frac{dy}{dx}$$

So, coordinate of $A = \left(0, y - x\frac{dy}{dx}\right)$

Put $Y = 0$,

$$0 - y = \frac{dy}{dx}(X - x)$$

$$-y\frac{dx}{dy} = X - x$$

$$X = x - y\frac{dx}{dy}$$

Coordinate of $B = \left(x - y\frac{dx}{dy}, 0\right)$

Given, (intercept on x - axis) = 4(ordinate)

$$x - y\frac{dx}{dy} = 4y$$

$$y\frac{dx}{dy} + 4y = x$$

$$\frac{dx}{dy} + 4 = \frac{x}{y}$$

$$\frac{dx}{dy} - \frac{x}{y} = -4$$

It is a linear different equation. Comparing it with $\frac{dx}{dy} + Px = Q$

$P = -\frac{1}{y}$, $Q = -4$

$$I.F. = e^{\int p\,dy}$$

$$= e^{-\int \frac{1}{y}dy}$$

$$= e^{-\log y}$$

$$= \frac{1}{y}$$

Solution of the equation is given by,

$$x(I.F.) = \int Q(I.F.)dy + \log c$$

$$x\left(\frac{1}{y}\right) = \int(-4)\left(\frac{1}{y}\right)dy + \log c$$

$$\frac{x}{y} = -4\log y + \log c$$

$$e^{\frac{x}{y}} = \frac{c}{y^4}$$

Differential Equations Ex 22.11 Q17

Slope at any point = $y + 2x$

$$\frac{dy}{dx} = y + 2x$$

$$\frac{dy}{dx} - y = 2x$$

It is a linear differential equation. comparing it with $\frac{dy}{dx} + Py = Q$

$P = -1, Q = 2x$

$$I.F. = e^{\int P dx}$$

$$= e^{\int(-1)dx}$$

$$= e^{-x}$$

Solution of the equation is given by

$$(I.F.) = \int Q\,(I.F.)\,dx + c$$

$$y\left(e^{-x}\right) = \int (2x)\left(e^{-x}\right)dx + c$$

$$y\left(e^{-x}\right) = 2\int xe^{-x}dx + c$$

$$y\left(e^{-x}\right) = 2\left[x\left(-e^{-x}\right) + \int 1e^{-x}dx\right] + c$$

$$y\left(e^{-x}\right) = -2xe^{-x} - 2e^{-x} + c$$

$$y = -2x - 2 + ce^{x}$$

$$y + 2(x+1) = ce^{x} \quad --- (1)$$

It is passing through origin,

$$0 + 2(0+1) = ce^{0}$$

$$2 = c$$

Now, equation (1) becomes,

$$y + 2(x+1) = 2e^{x}$$

Differential Equations Ex 22.11 Q18

Given, tangent makes on angle $\tan^{-1}(2x+3y)$ with x-axis,

Slope of tangent $= \tan\theta$

$$\frac{dy}{dx} = \tan\left(\tan^{-1}(2x+3y)\right)$$

$$\frac{dy}{dx} = 2x + 3y$$

$$\frac{dy}{dx} - 3y = 2x$$

It is a linear differetial equation comparing it with $\frac{dy}{dx} + Py = Q$

$P = -3, Q = 2x$

$$I.F. = e^{\int P dx}$$

$$= e^{-\int 3 dx}$$

$$= e^{-3x}$$

Solution of the equation on given by

$$y(\text{I.F.}) = \int Q(I.F.)dx + c$$

$$y(e^{-3x}) = \int 2xe^{-3x}dx + c$$

$$= 2\left[x\left(\frac{-e^{-3x}}{3}\right) - \int 1.\left(\frac{-e^{-3x}}{3}\right)dx\right] + c$$

$$= -\frac{2}{3}xe^{-3x} + \frac{2}{3}\int e^{-3x}dx + c$$

$$y(e^{-3x}) = -\frac{2}{3}xe^{-3x} + \frac{2}{9}e^{-3x} + c$$

$$y = -\frac{2}{3}x - \frac{2}{9} + ce^{3x} \quad ----- (1)$$

It is passing through $(1,2)$,

$$2 = -\frac{2}{3} - \frac{2}{9} + ce^3$$

$$2 = -\frac{8}{9} + ce^3$$

$$\frac{26}{9} = ce^3$$

$$c = \frac{26}{9}e^{-3}$$

Now equation (1) becomes,

$$ye^{-3x} = \left(-\frac{2}{3}x - \frac{2}{9}\right)e^{-3x} + \frac{26}{9}e^{-3}$$

Differential Equations Ex 22.11 Q19

Let $P(x,y)$ be the point of contact of tangent whit curve $y = f(x)$ equatin of tangent at $P(x,y)$ is

$$Y - y = \frac{dy}{dx}(X - x)$$

Put Y = 0

$$-y = \frac{dy}{dx}(X - x)$$

$$X = X - \frac{ydx}{dx}$$

Coordinate of $B = \left(x - y\frac{dx}{dy}, 0\right)$

Given, (intercept on x - axis) = $4x$

$$x - y\frac{dx}{dy} = 2x$$

$$-y\frac{dx}{dy} = 2x - x$$

$$-y\frac{dx}{dy} = x$$

$$-\frac{dx}{x} = \frac{dy}{y}$$

$$-\int\frac{dx}{x} = \int\frac{dy}{y}$$

$$-\log x = \log y + c \text{ - - - } (1)$$

It is passing through $(1,2)$

$$-\log 1 = \log 2 + c$$

$$c = -\log 2$$

Put c in equation (1)

$$-\log x = \log y - \log 2$$

$$\frac{1}{x} = \frac{y}{2}$$

$$xy = 2$$

Differential Equations Ex 22.11 Q20

$$x(x+1)\frac{dy}{dx} - y = x(x+1)$$

$$\frac{dy}{dx} - \frac{y}{x(x+1)} = \frac{x(x+1)}{x(x+1)}$$

$$\frac{dy}{dx} - \frac{y}{x(x+1)} = 1$$

It is linear differential equation coparing it with $\frac{dy}{dx} + Py = Q$

$$P = -\frac{1}{x(x+1)}, \qquad Q = 1$$

$$I.F. = e^{\int \frac{1}{x(x+1)}dx}$$

$$= e^{\int\left(\frac{1}{x} - \frac{1}{(x+1)}\right)dx}$$

$$= e^{-\log|x| + \log|x+1|}$$

$$= e^{\log\left(\frac{x+1}{x}\right)}$$

$$= \frac{x+1}{x}$$

Solution of the equation is given by

$$y(I.F.) = \int Q(I.F.)dx + c$$

$$y\left(\frac{x+1}{x}\right) = \int\left(\frac{x+1}{x}\right)dx + c$$

$$y\left(\frac{x+1}{x}\right) = \int\left(1 + \frac{1}{x}\right)dx + c$$

$$y\left(\frac{x+1}{x}\right) = x + \log|x| + c \,---(1)$$

It is passing through $(1,0)$, so

$$0 = 1 + \log(1) + c$$

$$-1 = c$$

Now,equation (1) becomes,

$$y\left(\frac{x+1}{x}\right) = x + \log|x| - 1$$

$$y(x+1) = x(x + \log x - 1)$$

Differential Equations Ex 22.11 Q22

Slope of the curve $= \frac{2y}{x}$

$$\frac{dy}{dx} = \frac{2y}{x}$$

$$\frac{dy}{y} = \frac{2}{x}dx$$

$$\int \frac{dy}{y} = 2\int \frac{1}{x}dx$$

$$\log|y| = 2\log|x| + \log|c|$$

$$y = x^2c \text{ --- } (1)$$

It is passing through $(3, -4)$ so,

$$-4 = (3)^2 c$$

$$-4 = 9c$$

$$c = -\frac{4}{9}$$

Now, equation (1) becomes,

$$y = -\frac{4}{9}x^2$$

$$9y = -4x^2$$

$$9y + 4x^2 = 0$$

Differential Equations Ex 22.11 Q22

Given,

$$\text{Slope of the equation} = x + 3y - 1$$

$$\frac{dy}{dx} = x + 3y - 1$$

$$\frac{dy}{dx} - 3y = x - 1$$

It is a linear differential equation. Camparing it with $\frac{dy}{dx} + Py = Q$

$P = -3, Q = x - 1$

$$I.F. = e^{\int P dx}$$

$$= e^{\int -3dx}$$

$$= e^{-3x}$$

Solution of the equation is given by,

$$y\left(I.F.\right) = \int Q\left(I.F.\right)dx + c$$

$$y\left(e^{-3x}\right) = \int \left(x - 1\right)\left(e^{-3x}\right)dx + c$$

$$y\left(e^{-3x}\right) = \left(x - 1\right)\left(-\frac{1}{3}e^{-3x}\right) - \int \left(1\right)\left(\frac{-e^{-3x}}{3}\right)dx + c$$

$$y\left(e^{-3x}\right) = -\frac{\left(x - 1\right)}{3}e^{-3x} + \left(-\frac{e^{-3x}}{9}\right) + c$$

$$y = -\frac{x}{3} + \frac{1}{3} - \frac{1}{9} + ce^{3x}$$

$$y = -\frac{x}{3} + \frac{2}{9} + ce^{3x}$$

It is passing through origin, so

$$0 = 0 + \frac{2}{9} + ce^{3(0)}$$

$$0 = \frac{2}{9} + c$$

$$c = -\frac{2}{9}$$

Now, equation (1) becomes,

$$y = -\frac{x}{3} + \frac{2}{9} - \frac{2}{9}e^{3x}$$

$$9y = -3x + 2 - 2e^{3x}$$

$$3\left(3y + x\right) = 2\left(1 - e^{3x}\right)$$

Differential Equations Ex 22.11 Q23

Given,

$$\text{Slope at point } (x, y) = x + xy$$

$$\frac{dy}{dx} = x(y+1)$$

$$\frac{dy}{y+1} = x\,dx$$

$$\int \frac{dy}{y+1} = \int x\,dx$$

$$\log|y+1| = \frac{x^2}{2} + c \text{ - - - - - - } (1)$$

It is passing through $(0, 1)$, so,

$$\log z = 0 + c$$

$$c = \log 2$$

Now, equation (2) becomes,

$$\log|y+1| = \frac{x^2}{2} + \log 2$$

$$y + 1 = 2e^{\frac{x^2}{2}}$$

Differential Equations Ex 22.11 Q24

$$y^2 - 2xy\frac{dy}{dx} - x^2 = 0$$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

It is a homeganeous equation.

put, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now,

$$x\frac{dv}{dx} + v = \frac{v^2x^2 - x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$x\frac{dv}{dx} = \frac{-v^2 - 1}{2v}$$

$$\int \frac{2v}{v^2+1}dv = -\int \frac{dx}{x}$$

$$\log|v^2 + 1| = -\log|x| + \log|c|$$

$$v^2 + 1 = \frac{c}{x}$$

$$\frac{y^2 + x^2}{x^2} = \frac{c}{x}$$

$$y^2 + x^2 = cx$$

$$y^2 + x^2 - cx = 0$$

Differentiating it with respect to x,

$$2x + 2y\frac{dy}{dx} - c = 0$$

$$\frac{dy}{dx} = \frac{c - 2x}{2y}$$

Let (h,k) be the point where tangent passes through origin and length is equal to h. so, equation of tangent at (h,k) is

$$(y-k)=\left(\frac{dy}{dx}\right)_{(h,k)}(x-h)$$

$$(y-k)=\left(\frac{c-2h}{2k}\right)(x-h)$$

$$2ky-2k^2=xc-2hx-hc+2h^2$$

$$x(c-2h)-2ky+2k^2-hc+2h^2=0$$

$$x(c-2h)-2ky+2\left(k^2+h^2\right)-hc=0$$

$$x(c-2h)-2ky+2(ch)-hc=0$$

$\left[\text{Since } h^2+k^2=ch \text{ as } (h,k) \text{ is on the curve}\right]$

$$x(c-2h)-2ky+hc=0$$

length of perpendicular as tangent from origin is

$$L=\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$$

$$=\left|\frac{(0)(c-2h)+(0)(-2k)+hc}{\sqrt{(c-2h)^2+(-2k)^2}}\right|$$

$$=\frac{hc}{\sqrt{c^2+4h^2+4k^2-4ch}}$$

$$L=\frac{hc}{\sqrt{c^2+4\left(h^2+k^2-ch\right)}}$$

$$=\frac{hc}{\sqrt{c^2+4(0)}}$$

$$=\frac{hc}{c}$$

$$=c$$

Hence,

$x^2+y^2=cx$ is the required curve

Differential Equations Ex 22.11 Q25

Let $P(x,y)$ be the point of contact of tangent and curve $y = f(x)$. Equation tangent at $P(x,y)$ is

$$Y - y = \frac{dx}{dy}(X - x)$$

put $Y = 0$

$$-y = \frac{dx}{dy}(X - x)$$

$$-y = \frac{dx}{dy}(X - x)$$

$$X = x - y\frac{dx}{dy}$$

coordinate of $B = \left(x - y\frac{dx}{dy}, 0\right)$

Given,

Distance between foot of ordinate of the point of contact and the point of intersection of tangent and x – axis = $2x$

$$BC = 2x$$

$$\sqrt{\left(x - y\frac{dx}{dy} - x\right)^2 + (0)^2} = 2x$$

$$y\frac{dx}{dy} = 2x$$

$$y\frac{dx}{x} = 2\frac{dy}{y}$$

$$\int\frac{dx}{x} = 2\int\frac{dy}{y}$$

$$\log x = 2\log y + \log c \quad ---(1)$$

It is passing through $(1,2)$,

$$\log 1 = 2\log 2 + \log c$$

$$-2\log 2 = \log c$$

$$\log\left(\frac{1}{4}\right) = \log c$$

$$c = \frac{1}{4}$$

Put value of c in equation (1),

$$\log x = 2\log y + \log\left(\frac{1}{4}\right)$$

$$x = \frac{y^2}{4}$$

$$y^2 = 4x$$

Differential Equations Ex 22.11 Q26

Equation of normal on point (x, y) on the curve

$$Y - y = \frac{-dx}{dy}(X - x)$$

Itis passing through $(3,0)$

$$0 - y = \frac{-dx}{dy}(3 - x)$$

$$y = \frac{dx}{dy}(3 - x)$$

$$ydy = (3 - x)dx$$

$$\int y\, dy = \int (3 - x)dx$$

$$\frac{y^2}{2} = 3x - \frac{x^2}{2} + c \text{ - - - } (1)$$

It passing through $(3,4)$, so,

$$\frac{16}{2} = 9 - \frac{9}{2} + c$$

$$\frac{16}{2} = \frac{9}{2} + c$$

$$c = 7$$

Put $c = 7$ is equation (1)

$$\frac{y^2}{2} = 3x - \frac{x^2}{2} + \frac{7}{2}$$

$$y^2 = 6x - x^2 + 7$$

Differential Equations Ex 22.11 Q27

Let A be the quantity of bacteria present in culture at any time t and initial quantity of bacteria is A_0.

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = \lambda A$$

$$\frac{dA}{A} = \lambda dt$$

$$\int \frac{dA}{A} = \lambda \int dt$$

$$\log A = \lambda t + c \text{ - - - - } (1)$$

Initially, $A = A_0, t = 0$

$$\log A_0 = 0 + c$$

$$\log A_0 = c$$

Now eqution (1) becomes,

$$\log A = \lambda t + \log A_0$$

$$\log\left(\frac{A}{A_0}\right) = \lambda t \text{ - - - - - } (2)$$

Given $A = 2A_0$ when $t = 6$ hours

$$\log\left(\frac{A}{A_0}\right) = 6\lambda$$

$$\frac{\log 2}{6} = \lambda$$

Now equation (2) becomes,

$$\log\left(\frac{A}{A_0}\right) = \frac{\log 2}{6}t$$

Now, $A = 8A_0$

so, $$\log\left(\frac{8A_0}{A_0}\right) = \frac{\log 2}{6}t$$

$$\log 2^3 = \frac{\log 2}{6}t$$

$$3\log 2 = \frac{\log 2}{6}t$$

$$18 = t$$

Therefore,

Bacteria becomes 8 times in 18 hours

Differential Equations Ex 22.11 Q28

Let A be the quantity of radium present at time t and A_0 be the initial quantity of radium.

$$\frac{dA}{dt} \propto A$$

$$\frac{dA}{dt} = -\lambda A$$

$$\frac{dA}{A} = -\lambda dt$$

$$\int \frac{dA}{A} = -\lambda \int dt$$

$$\log A = -\lambda t + c ----(2)$$

Now, $A = A_0$ when $t = 0$

$$\log A_0 = 0 + c$$

$$c = \log A_0$$

Put value of c in equation

$$\log A = -\lambda t + \log A_0$$

$$\log\left(\frac{A}{A_0}\right) = -\lambda t ---(2)$$

Given that,

In 25 years bacteria decomposes 1.1%, so

$$A = (100 - 1.1)\% = 98.9\% = 0.989A_0, t = 5$$

$$\log\left(\frac{0.989A_0}{A_0}\right) = -\lambda 25$$

$$\log(0.989) = -25\lambda$$

$$\lambda = -\frac{1}{25}\log(0.989)$$

Now, equation (2) becomes,

$$\log\left(\frac{A}{A_0}\right) = \left\{\frac{1}{25}\log(0.989)\right\}t$$

Now $A = \frac{1}{2}A_0$

$$\log\left(\frac{A}{2A}\right) = \frac{1}{25}\log(0.989)t$$

$$\frac{-\log 2 \times 25}{\log(0.989)} = t$$

$$-\frac{0.6931 \times 25}{0.01106} = t$$

$$t = 1567 \text{ years.}$$

Required time = 1567 years

Differential Equations Ex 22.11 Q29

Given,

$$\text{Slope of tangent} = \frac{x^2 + y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

It is a homeganeous equation.

put, $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now,

$$v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{2xvx}$$

$$x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$

$$x\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{v}$$

$$x\frac{dv}{dx} = \frac{1 - v^2}{v}$$

$$\frac{v}{1 - v^2}dv = \frac{dx}{x}$$

$$\int \frac{v}{1 - v^2}dv = \int \frac{dx}{x}$$

$$\int \frac{-2v}{1 - v^2}dv = \int \frac{-2dx}{x}$$

$$\log\left|1 - v^2\right| = -2\log x + \log c$$

$$1 - \frac{y^2}{x^2} = \frac{c}{x^2}$$

$$\frac{x^2 - y^2}{x^2} = \frac{c}{x^2}$$

$$x^2 - y^2 = c$$

It is equation of rectangular hyperbola.

Differential Equations Ex 22.11 Q30

Given,

$$\text{Slope of tangent at } (x, y) = x + y$$

$$\frac{dy}{dx} = x + y$$

$$\frac{dy}{dx} - y = x$$

It is a linear differential equation.Comparing it with $\frac{dy}{dx} + Py = Q$

$P = -1, Q = x$

$$I.F. = e^{\int P dx}$$

$$= e^{\int (-1) dx}$$

$$= e^{-x}$$

Solution of equation is given by,

$$y\left(I.F.\right) = \int Q\left(I.F.\right) dx + c$$

$$y\left(e^{-x}\right) = \int x e^{-x} dx + c$$

$$ye^{-x} = x\left(e^{-x}\right) + \int\left(1 \times e^{-x}\right) dx + c$$

[Using integration by parts]

$$ye^{-x} = -xe^{-x} - e^{-x} + c$$

$$y = -x - 1 + ce^{x} \text{ - - - - - } (1)$$

It is passing through origin

$$0 = 0 - 1 + ce^{0}$$

$$1 = c$$

Put $c = 1$ is equation

$$y = -x - 1 + e^{x}$$

$$y + x + 1 = e^{x}$$

Differential Equations Ex 22.11 Q31

We know that the slope of the tangent to the curve is $\frac{dy}{dx}$.

$\therefore \qquad \frac{dy}{dx} = x + xy$

$\Rightarrow \qquad \frac{dy}{dx} - xy = y \qquad \text{-------(i)}$

This is a linear differential equation of the type $\frac{dy}{dx} + Py = Q$

where $P = -x$ and $Q = x$.

So, $\qquad I.F. = e^{\int -x dx} = e^{\frac{-x^2}{2}}$

$\therefore$ Solution of the given equation is given by

$$y.\, e^{\frac{-x^2}{2}} = \int x.e^{\frac{-x^2}{2}}\, dx + C \qquad \text{-------(ii)}$$

Let $\qquad I = \int x.e^{\frac{-x^2}{2}}\, dx$

Let $\frac{-x^2}{2} = t$, then $-x\, dx = dt$ or $x\, dx = -dt$

$\therefore \qquad I = \int x.e^{\frac{-x^2}{2}}\, dx = \int -e^t\, dt = -e^t = -e^{\frac{-x^2}{2}}$

Substituting the value of I in (ii), we get

$$y.\, e^{\frac{-x^2}{2}} = -e^{\frac{-x^2}{2}} + C$$

or $\qquad y = -1 + Ce^{\frac{x^2}{2}} \qquad \text{-------(iii)}$

This equation (iii) passes through (0,1)

$\therefore \qquad 1 = -1 + Ce^0 \quad \Rightarrow \quad C = 2$

Substituting the value of C in (iii), we get

$$y = -1 + 2e^{\frac{x^2}{2}}$$

which is the equation of the required curve.

Differential Equations Ex 22.11 Q32

Given,

Slope of tangent at $(x, y) = x^2$

$$\frac{dy}{dx} = x^2$$

$$dy = x^2 dx$$

$$\int dy = \int x^2 dx$$

$$y = \frac{x^3}{3} + c ----(1)$$

It is passing through $(-1, 1)$

$$1 = \frac{(-1)}{3} + c$$

$$1 = -\frac{1}{3} + c$$

$$c = 1 + \frac{1}{3}$$

$$c = \frac{4}{3}$$

Put is equation

$$y = \frac{x^3}{3} + \frac{4}{3}$$

$$3y = x^3 + 4$$

Differential Equations Ex 22.11 Q33

Given,

y (Slope of tangent) $= x$

$$y\frac{dy}{dx} = x$$

$$ydy = xdx$$

$$\int ydy = \int xdx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c ----(1)$$

It is passing through $(0, a)$

$$\frac{a^2}{2} = 0 + c$$

$$c = \frac{a^2}{2}$$

Put $c = \frac{a^2}{2}$ is equation (1)

$$\frac{y^2}{2} = \frac{x^2}{2} + \frac{a^2}{2}$$

$$y^2 = x^2 + a^2$$

Differential Equations Ex 22.11 Q34

Let $P(x,y)$ be the point on the curve $y = f(x)$ such that tangent at P cuts the coordinate axes at A and B.

The equation of tangent is,

$$Y - y = \frac{dy}{dx}(X - x)$$

Put $Y = 0$

$$-y = \frac{dy}{dx}(X - x)$$

$$-y\frac{dy}{dx} + x = X$$

Coordinate of $B = \left(-y\frac{dy}{dx} + x, 0\right)$

Here, x intercept of tangent $= y$

$$-y\frac{dx}{dy} + x = y$$

$$\frac{dx}{dy} - \frac{x}{y} = -1$$

It is a linear differential equation on comparing it with $\frac{dx}{dy} + py = Q$

$P = \frac{1}{y}, Q = -1$

$$I.F. = e^{\int\left(\frac{1}{y}\right)dy}$$

$$= e^{\log y}$$

$$= \frac{1}{y}$$

Solution of the equation is given by,

$$x\,(I.F.) = \int Q\,(I.F)\,dy + c$$

$$x\left(\frac{1}{y}\right) = \int(-1)\left(\frac{1}{y}\right)dy + c$$

$$x\left(\frac{1}{y}\right) = -\log y + c \quad ----(1)$$

It is passing through $(1,1)$

$$\frac{1}{1} = -\log 1 + c$$

$$c = 1$$

put $c = 1$ is equation (1),

$$\frac{x}{y} = -\log y + 1$$

$$x = y - y\log y$$

$$x + y\log y = y$$