

**RD Sharma**  
**Solutions Class**  
**12 Maths**  
**Chapter 22**  
**Ex 22.1**

### Differential Equations Ex 22.1 Q1

$$\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 = e^t$$

The highest order differential coefficient is  $\frac{d^3x}{dt^3}$  and its power is 1.

So, it is a non-linear differential equation with order 3 and degree 1.

### Differential Equations Ex 22.1 Q2

$$\frac{d^2y}{dx^2} + 4y = 0$$

It is a linear differential equation.

The highest order differential coefficient is  $\frac{d^2y}{dx^2}$  and its power is 1.

So, it is a linear differential equation with order 2 and degree 1.

### Differential Equations Ex 22.1 Q3

$$\left(\frac{dy}{dx}\right)^2 + \frac{1}{\left(\frac{dy}{dx}\right)} = 2$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^3 + 1 = 2\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^3 - 2\left(\frac{dy}{dx}\right) + 1 = 0$$

This is a polynomial in  $\frac{dy}{dx}$ .

The highest order differential coefficient is  $\frac{dy}{dx}$  and its power is 3.

So, it is a non-linear differential equation with order 1 and degree 3.

### Differential Equations Ex 22.1 Q4

Consider the given differential equation,  $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \left(c \frac{d^2y}{dx^2}\right)^{\frac{1}{3}}$

Squaring on both the sides, we have

$$1 + \left(\frac{dy}{dx}\right)^2 = \left(c \frac{d^2y}{dx^2}\right)^{\frac{2}{3}}$$

Cubing on both the sides, we have

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left\{\left(c \frac{d^2y}{dx^2}\right)^{\frac{2}{3}}\right\}^3$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^6 + 3\left(\frac{dy}{dx}\right)^2 + 3\left(\frac{dy}{dx}\right)^4 = c^2 \left(\frac{d^2y}{dx^2}\right)^2$$

$$\Rightarrow c^2 \left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right)^6 - 3\left(\frac{dy}{dx}\right)^4 - 3\left(\frac{dy}{dx}\right)^2 - 1 = 0$$

The highest order differential coefficient in this

equation is  $\frac{d^2y}{dx^2}$  and its power is 2.

Therefore, the given differential equation is a

non – linear differential equation of second order and second degree.

### Differential Equations Ex 22.1 Q5

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + xy = 0$$

The highest order differential coefficient is  $\frac{d^2y}{dx^2}$  and its power is 1.

So, it is a non-linear differential equation with order 2 and degree 1.

### Differential Equations Ex 22.1 Q6

Consider the given differential equation,

$$\sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$$

Cubing on both the sides of the above equation, we have

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^{\frac{3}{2}}$$

Squaring on both the sides of the above equation, we have

$$\left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)^{\frac{3}{2}}\right]^2$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)\right]^3$$

$$\Rightarrow \left(\frac{d^2y}{dx^2}\right)^2 - \left[\left(\frac{dy}{dx}\right)\right]^3 = 0$$

The highest order differential coefficient in this equation is  $\frac{d^2y}{dx^2}$

and its power is 2.

Therefore, the given differential equation is a non – linear differential equation of second order and second degree.

#### Differential Equations Ex 22.1 Q7

$$\frac{d^4y}{dx^4} = \left[c + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^2 = \left[c + \left(\frac{dy}{dx}\right)^2\right]^3$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^2 = c^3 + \left(\frac{dy}{dx}\right)^6 + 3c\left(\frac{dy}{dx}\right)^2 + 3c^2\left(\frac{dy}{dx}\right)$$

$$\Rightarrow \left(\frac{d^4y}{dx^4}\right)^2 - \left(\frac{dy}{dx}\right)^6 - 3c\left(\frac{dy}{dx}\right)^2 - 3c^2\left(\frac{dy}{dx}\right) - c^3 = 0$$

The highest order differential coefficient is  $\left(\frac{d^4y}{dx^4}\right)$  and its power is 2.

It is a non-linear differential equation with order 4 and degree 2.

#### Differential Equations Ex 22.1 Q8

$$x + \left(\frac{dy}{dx}\right) = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \left(x + \frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow x^2 + \left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) = 1 + \left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow 2x\left(\frac{dy}{dx}\right) + x^2 - 1 = 0$$

$$\Rightarrow \frac{dy}{dx} + \frac{x}{2} - \frac{1}{2x} = 0$$

The highest order differential coefficient is  $\frac{dy}{dx}$  and power is 1.

So, it is a linear differential equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q9

$$y \frac{d^2x}{dy^2} = y^2 + 1$$

$$\frac{d^2x}{dy^2} - y - \frac{1}{y} = 0$$

The differential coefficient is  $\frac{d^2x}{dy^2}$  and its power is 1.

So, it is a linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q10

$$s^2 \frac{d^2t}{ds^2} + st \frac{dt}{ds} = s$$

The differential coefficient of highest order is  $\frac{d^2t}{ds^2}$  and power is 1.

So, it is a non-linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q11

$$x^2 \left(\frac{d^2y}{dx^2}\right)^3 + y \left(\frac{dy}{dx}\right)^4 + y^4 = 0$$

The highest order differential coefficient is  $\frac{d^2y}{dx^2}$  and its power is 3.

So, it is a non-linear differential equation with order 2 and degree 3.

Differential Equations Ex 22.1 Q12

$$\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right) + 4y = \sin x$$

The highest order differential coefficient is  $\frac{d^3y}{dx^3}$  and its power is 1.

So, it is a non-linear differential equation with order 3 and degree 1.

Differential Equations Ex 22.1 Q13

$$(xy^2 + x)dx + (y - x^2y)dy = 0$$

$$(y - x^2y)\frac{dy}{dx} + xy^2 + x = 0$$

$$y(1 - x^2)\frac{dy}{dx} + x(y^2 + 1) = 0$$

The highest order differential coefficient is  $\frac{dy}{dx}$  and its power is 1.

So, it is a non-linear differential equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q14

$$\sqrt{1 - y^2}dx + \sqrt{1 - x^2}dy = 0$$

$$\sqrt{1 - x^2}\frac{dy}{dx} + \sqrt{1 - y^2} = 0$$

$$\frac{dy}{dx} + \sqrt{\frac{1 - y^2}{1 - x^2}} = 0$$

The highest order differential coefficient is  $\frac{dy}{dx}$  and its power is 1.

So, it is a non-linear differential equation with order 1 and degree 1.

Differential Equations Ex 22.1 Q15

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

$$\left(\frac{d^2y}{dx^2}\right)^3 = \left(\frac{dy}{dx}\right)^2$$

$$\left(\frac{d^2y}{dx^2}\right)^3 - \left(\frac{dy}{dx}\right)^2 = 0$$

The highest order differential coefficient is  $\frac{d^2y}{dx^2}$  and its power is 3.

So, it is a non-linear differential equation with order 2 and degree 3.

Differential Equations Ex 22.1 Q16

$$2\frac{d^2y}{dx^2} + 3\sqrt{1 - \left(\frac{dy}{dx}\right)^2} - y = 0$$

$$2\frac{d^2y}{dx^2} = -3\sqrt{1 - \left(\frac{dy}{dx}\right)^2} - y$$

Squaring both the sides,

$$4\left(\frac{d^2y}{dx^2}\right)^2 = 9\left(1 - \left(\frac{dy}{dx}\right)^2 - y\right)$$

$$4\left(\frac{d^2y}{dx^2}\right)^2 + 9\left(\frac{dy}{dx}\right)^2 + 9y - 9 = 0$$

The highest order differential coefficient is  $\frac{d^2y}{dx^2}$  and its power is 2.

So, it is a non-linear differential equation with order 2 and degree 2.

Differential Equations Ex 22.1 Q17

$$5 \frac{d^2y}{dx^2} = \left\{ 1 + \left( \frac{dy}{dx} \right)^2 \right\}^{\frac{3}{2}}$$

$$\left\{ 5 \left( \frac{d^2y}{dx^2} \right)^2 \right\} = \left\{ 1 + \left( \frac{dy}{dx} \right)^2 \right\}^3$$

$$25 \left( \frac{d^2y}{dx^2} \right)^2 = 1 + \left( \frac{dy}{dx} \right)^6 + 3 \left( \frac{dy}{dx} \right)^2 + 3 \left( \frac{dy}{dx} \right)^4$$

$$25 \left( \frac{d^2y}{dx^2} \right)^2 - \left( \frac{dy}{dx} \right)^6 - 3 \left( \frac{dy}{dx} \right)^4 - 3 \left( \frac{dy}{dx} \right)^2 - 1 = 0$$

The highest order differential coefficient is  $\frac{d^2y}{dx^2}$  and its power is 2.

So, it is a non-linear differential equation with order 2 and degree 2

Differential Equations Ex 22.1 Q18

$$y = x \frac{dy}{dx} + a \sqrt{1 + \left( \frac{dy}{dx} \right)^2}$$

$$\left( y - x \frac{dy}{dx} \right)^2 = \left( a \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \right)^2$$

$$y^2 + x^2 \left( \frac{dy}{dx} \right)^2 - 2xy \frac{dy}{dx} = a^2 \left( 1 + \left( \frac{dy}{dx} \right)^2 \right)$$

$$x^2 \left( \frac{dy}{dx} \right)^2 - 2xy \frac{dy}{dx} + y + a^2 \left( \frac{dy}{dx} \right)^2 - a^2 = 0$$

$$(x^2 + a^2) \left( \frac{dy}{dx} \right)^2 - 2xy \frac{dy}{dx} + y - a^2 = 0$$

The highest order differential coefficient is  $\frac{dy}{dx}$  and power is 2.

So, it is a non-linear differential equation with order 1 and degree 2.

Differential Equations Ex 22.1 Q19

$$y = px + \sqrt{a^2 p^2 + b^2}, p = \frac{dy}{dx}$$

$$y - px = \sqrt{a^2 p^2 + b^2}$$

$$(y - px)^2 = (a^2 p^2 + b^2)$$

$$y^2 + p^2 x^2 - 2xyp = a^2 p^2 + b^2$$

$$x^2 p^2 - a^2 p^2 - 2xyp + y^2 - b^2 = 0$$

$$(x^2 - a^2) p^2 - 2xyp + (y^2 - b^2) = 0$$

$$(x^2 - a^2) \left( \frac{dy}{dx} \right)^2 - 2xy \left( \frac{dy}{dx} \right) + (y^2 - b^2) = 0$$

The highest order differential coefficient is  $\frac{dy}{dx}$  and its power is 2.

So, it is a non-linear differential equation of order 1 and degree 2

Differential Equations Ex 22.1 Q20

$$\frac{dy}{dx} + e^y = 0$$

The highest order differential coefficient is  $\frac{dy}{dx}$  and its power is 1.

So, it is a non-linear differential equation of order 1 and degree 1.

Differential Equations Ex 22.1 Q21

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{d^2y}{dx^2}\right)$$

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 - x \sin\left(\frac{d^2y}{dx^2}\right) = 0$$

The highest order differential coefficient is  $\left(\frac{d^2y}{dx^2}\right)$  and it is not a polynomial of derivative,

So, it is a non-linear differential equation of order 2 but degree is not defined.

Differential Equations Ex 22.1 Q22

$$(y''')^2 + (y'')^3 + \sin y = 0$$

The highest order of differential coefficient is  $y'''$  and its power is 2,

So, it is a non-linear differential equation of order 3 and degree 2.

Differential Equations Ex 22.1 Q23

$$\frac{d^2y}{dx^2} + 5x\left(\frac{dy}{dx}\right)^2 - 6y = \log x$$

The highest order differential coefficient is  $\frac{d^2y}{dx^2}$  and its power is 1.

So, it is a non-linear differential equation with order 2 and degree 1.

Differential Equations Ex 22.1 Q24

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y \sin y = 0$$

The highest order differential coefficient is  $\frac{d^3y}{dx^3}$  and its power is 1.

So, it is a linear differential equation of order 3 and degree 1.

Differential Equations Ex 22.1 Q25

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 - x^2 \log\left(\frac{d^2y}{dx^2}\right) = 0$$

The highest order derivative is  $\frac{d^2y}{dx^2}$  but it is not a polynomial in  $\frac{dy}{dx}$ .

So, it is a non-linear differential equation of order 2 but degree is not defined.



## Differential Equations Ex 22.1 Q26

The order of a differential equation is the order of the highest order derivative appearing in the equation.

The degree of a differential equation is the degree of the highest order derivative.

Consider the given differential equation

$$\left(\frac{dy}{dx}\right)^3 - 4\left(\frac{dy}{dx}\right)^2 + 7y = \sin x$$

In the above equation, the order of the highest order derivative is 1.

So the differential equation is of order 1.

In the above differential equation, the power of the highest order derivative is 3.

Hence, it is a differential equation of degree 3.

Since the degree of the above differential equation is 3, more than one, it is a non-linear differential equation.