# RD Sharma Solutions Class 12 Maths Chapter 20 Ex 20.4

## Definite Integrals Ex 20.4A Q1

We know

$$\int_0^{2\Pi} f(x)dx = \int_0^{2\Pi} f(2\Pi - x)dx$$

Hence

$$\int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}}dx = \int_0^{2\Pi} \frac{e^{\sin(2\Pi - x)}}{e^{\sin(2\Pi - x)} + e^{-\sin(2\Pi - x)}}dx$$

We know

$$\sin(2\Pi - x) = -\sin x$$

$$\int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}}dx = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}}dx$$

If

$$I = \int_0^{2\Pi} \frac{e^{\sin x}}{e^{\sin x} + e^{-\sin x}}dx$$

Then also

$$I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}}dx$$

Hence

$$2I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}}dx + \int_0^{2\Pi} \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}}dx$$

$$2I = \int_0^{2\Pi} \frac{e^{-\sin x}}{e^{-\sin x} + e^{\sin x}} + \frac{e^{\sin x}}{e^{-\sin x} + e^{\sin x}}dx$$

$$2I = \int_0^{2\Pi} dx$$

$$2I = 2\Pi$$

$$I = \Pi$$

## Definite Integrals Ex 20.4A Q2

We know

$$\int_0^{2\pi} f(x)dx = \int_0^{2\pi} f(2\Pi - x)dx$$

Hence

$$\int_0^{2\pi} \log(\sec x + \tan x)dx = \int_0^{2\pi} \log(\sec(2\Pi - x) + \tan(2\Pi - x))dx$$

$$\int_0^{2\pi} \log(\sec x + \tan x)dx = \int_0^{2\pi} \log(\sec x - \tan x)dx$$

If

$$I = \int_0^{2\pi} \log(\sec x + \tan x)dx$$

Then

$$I = \int_0^{2\pi} \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\pi} \log(\sec x + \tan x)dx + \int_0^{2\pi} \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\pi} \log(\sec x + \tan x) + \log(\sec x - \tan x)dx$$

$$2I = \int_0^{2\pi} \log(\sec^2 x - \tan^2 x)dx$$

$$2I = \int_0^{2\pi} \log(1)dx$$

$$2I = 0$$

$$I = 0$$

## Definite Integrals Ex 20.4A Q3

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan(\frac{\Pi}{2}-x)}}{\sqrt{\tan(\frac{\Pi}{2}-x)}+\sqrt{\cot(\frac{\Pi}{2}-x)}}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

So

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}} + \frac{\sqrt{\cot x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

## Definite Integrals Ex 20.4A Q4

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin(\frac{\Pi}{2}-x)}}{\sqrt{\sin(\frac{\Pi}{2}-x)}+\sqrt{\cos(\frac{\Pi}{2}-x)}}dx$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

If

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Then

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

Hence

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}+\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 1dx$$

$$2I = \frac{\Pi}{6}$$

$$I = \frac{\Pi}{12}$$

## Definite Integrals Ex 20.4A Q5

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2(-x)}{1+e^{-x}}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}}dx$$

If

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x}dx$$

Then

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^{-x}}dx$$

So

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{\tan^2 x}{1+e^{-x}}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x}{1+e^x} + \frac{e^x \tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x)\tan^2 x}{1+e^x}dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\tan^2 x + e^x \tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{(1+e^x)\tan^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \frac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2 x dx$$

We know

If f(x) is even

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

If f(x) is odd

$$\int_{-a}^{a} f(x)dx = 0$$

Here

$$f(x) = \tan^2 x$$

f(x) is even, hence

$$I = \int_{0}^{\frac{\pi}{4}} \tan^2 x dx$$

$$I = \int_{0}^{\frac{\pi}{4}} \sec^2 x - 1 dx$$

$$I = \{\tan x - x\}_{0}^{\frac{\pi}{4}}$$

$$I = 1 - \frac{\pi}{4}$$

Note: Answer given in the book is incorrect.

## Definite Integrals Ex 20.4A Q6

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-a}^{a} \frac{1}{1+a^x} dx = \int_{-a}^{a} \frac{1}{1+a^{-x}} dx$$

If

$$I = \int_{-a}^{a} \frac{1}{1+a^x} dx$$

Then

$$I = \int_{-a}^{a} \frac{1}{1+a^{-x}} dx$$

So

$$2I = \int_{-a}^{a} \frac{1}{1+a^x} + \frac{1}{1+a^{-x}} dx$$

$$2I = \int_{-a}^{a} \frac{1}{1+a^x} + \frac{a^x}{1+a^x} dx$$

$$2I = \int_{-a}^{a} 1dx$$

$$2I = 2a$$

$$I = a$$

## Definite Integrals Ex 20.4A Q7

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\Pi}{3}}^{\frac{\Pi}{3}} \frac{1}{1+e^{\tan x}} dx = \int_{-\frac{\Pi}{3}}^{\frac{\Pi}{3}} \frac{1}{1+e^{-\tan x}} dx$$

If

$$I = \int_{-\frac{\Pi}{3}}^{\frac{\Pi}{3}} \frac{1}{1+e^{\tan x}} dx$$

Then

$$I = \int_{-\frac{\Pi}{3}}^{\frac{\Pi}{3}} \frac{1}{1+e^{-\tan x}} dx$$

So

$$2I = \int_{-\frac{\Pi}{3}}^{\frac{\Pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{1}{1+e^{-\tan x}} dx$$

$$2I = \int_{-\frac{\Pi}{3}}^{\frac{\Pi}{3}} \frac{1}{1+e^{\tan x}} + \frac{e^{\tan x}}{1+e^{\tan x}} dx$$

$$2I = \int_{-\frac{\Pi}{3}}^{\frac{\Pi}{3}} 1dx$$

$$2I = \frac{2\Pi}{3}$$

$$I = \frac{\Pi}{3}$$

**Definite Integrals Ex 20.4A Q8**

We know

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

Hence

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x}dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2(-x)}{1+e^{-x}}dx$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x}dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}}dx$$

If

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x}dx$$

Then

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^{-x}}dx$$

So

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} + \frac{\cos^2 x}{1+e^{-x}} dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos^2 x}{1+e^x} + \frac{e^x \cos^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(1+e^x)\cos^2 x}{1+e^x} dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 x dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx$$

$$I = \frac{1}{4}\left\{x + \frac{\sin 2x}{2}\right\}_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$I = \frac{1}{4}\left\{\left(\frac{\Pi}{2}\right) - \left(-\frac{\Pi}{2}\right)\right\}$$

$$I = \frac{\Pi}{4}$$

Note: Answer given in the book is incorrect.

## Definite Integrals Ex 20.4A Q9

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11}-3x^9+5x^7-x^5+1}{\cos^2 x}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}+\frac{1}{\cos^2 x}dx$$

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}dx+\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x dx$$

If f(x) is even

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

If f(x) is odd

$$\int_{-a}^{a} f(x)dx = 0$$

Here

$\frac{x^{11}-3x^9+5x^7-x^5}{\cos^2 x}$ is odd and

$\sec^2 x$ is even. Hence

$$0+2\int_{0}^{\frac{\pi}{4}} \sec^2 x dx$$

$$2\{\tan x\}_0^{\frac{\pi}{4}}$$

$$2$$

**Definite Integrals Ex 20.4A Q10**

$$I = \int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}dx$$

$$I = \int_a^b \frac{(a+b-x)^{\frac{1}{n}}}{(a+b-x)^{\frac{1}{n}}+x^{\frac{1}{n}}}dx$$

$$2I = \int_a^b \frac{x^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}dx+\int_a^b \frac{(a+b-x)^{\frac{1}{n}}}{(a+b-x)^{\frac{1}{n}}+x^{\frac{1}{n}}}dx$$

$$2I = \int_a^b \frac{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}{x^{\frac{1}{n}}+(a+b-x)^{\frac{1}{n}}}dx$$

$$I = \frac{1}{2}\int_a^b dx$$

$$I = \frac{b-a}{2}$$

**Definite Integrals Ex 20.4A Q11**

We have,

$$I = \int_0^{\frac{\pi}{2}} \left(2\log\cos x - \log\sin 2x\right)dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\log\cos^2 x - \log\sin 2x\right)dx$$

$$= \int_0^{\frac{\pi}{2}} \log\frac{\cos^2 x}{\sin x}dx$$

$$= \int_0^{\frac{\pi}{2}} \log\frac{\cos^2 x}{2\sin x.\cos x}dx$$

$$= \int_0^{\frac{\pi}{2}} \log\frac{\cos x}{2\sin x}dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\log\cos x - \log\sin x - \log 2\right)dx$$

$$= \int_0^{\frac{\pi}{2}} \log\cos x\,dx - \int_0^{\frac{\pi}{2}} \log\sin x\,dx - \int_0^{\frac{\pi}{2}} \log 2$$

We know that $\int_0^{\frac{\pi}{2}} \log\cos x\,dx = \int_0^{\frac{\pi}{2}} \log\sin x\,dx$ $\quad -(\mathrm{i})$

Hence from equation (i)

$$I = -\int_0^{\frac{\pi}{2}} \log 2 = -\frac{\pi}{2}\log 2$$

## Definite Integrals Ex 20.4A Q12

Let $I = \int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(1)

It is known that, $\left(\int_0^a f(x)dx = \int_0^a f(a-x)dx\right)$

$I = \int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$ ...(2)

Adding (1) and (2), we obtain

$$2I = \int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx$$

$$\Rightarrow 2I = \int_0^a 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^a$$

$$\Rightarrow 2I = a$$

$$\Rightarrow I = \frac{a}{2}$$

**Definite Integrals Ex 20.4A Q13**

Let $I = \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}} dx$ --(i)

We know that $\int_0^a f(x) = \int_0^a f(a-x)$

So,

$$I = \int_0^5 \frac{\sqrt[4]{(5-x)+4}}{\sqrt[4]{(5-x)+4} + \sqrt[4]{9-(5-x)}} dx$$

$$I = \int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x} + \sqrt[4]{4+x}} dx \quad \text{--(ii)}$$

Adding (i) & (ii)

$$2I = \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}} dx + \int_0^5 \frac{\sqrt[4]{9-x}}{\sqrt[4]{9-x} + \sqrt[4]{4+x}} dx$$

$$2I = \int_0^5 \frac{\sqrt[4]{x+4} + \sqrt[4]{9-x}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}} dx$$

$$2I = \int_0^5 dx$$

$$2I = [x]_0^5$$

$$I = \frac{1}{2}[5-0] = \frac{5}{2}$$

$$\therefore \int_0^5 \frac{\sqrt[4]{x+4}}{\sqrt[4]{x+4} + \sqrt[4]{9-x}} dx = \frac{5}{2}$$

## Definite Integrals Ex 20.4A Q14

Let $I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$ --(i)

We know that $\int_0^a f(x) = \int_0^a f(a-x)$

Hence,

$I = \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx$ --(ii)

Adding (i) & (ii)

$$2I = \int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx + \int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x} + \sqrt[3]{x}} dx$$

$$2I = \int_0^7 \frac{\sqrt[3]{x} + \sqrt[3]{7-x}}{\sqrt[3]{x} + \sqrt[3]{7-x}} dx$$

$$2I = \int_0^7 dx$$

$$2I = [x]_0^7$$

$$I = \frac{7}{2}$$

## Definite Integrals Ex 20.4A Q15

Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}\,dx$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad --(i)$$

We know that $\int_a^b f(x) = \int_a^b f(a+b-x)\,dx$

Hence,

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$$

$$2I = \left[x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$I = \frac{\pi}{12}$$

**Definite Integrals Ex 20.4A Q16**

$$I = \int_a^b x f(x)\,dx$$

$$I = \int_a^b (a+b-x) f(a+b-x)\,dx$$

$$I = \int_a^b (a+b-x) f(x)\,dx \ldots\ldots\ldots [\because f(a+b-x) = f(x)]$$

$$I = \int_a^b (a+b) f(x)\,dx - \int_a^b f(x)\,dx$$

$$I = (a+b)\int_a^b f(x)\,dx - I$$

$$2I = (a+b)\int_a^b f(x)\,dx$$

$$I = \frac{(a+b)}{2}\int_a^b f(x)\,dx$$

$$\therefore \int_a^b x f(x)\,dx = \frac{(a+b)}{2}\int_a^b f(x)\,dx$$

## Definite Integrals Ex 20.4B Q1

We have,

$$\frac{1}{1+\tan x} = \frac{1}{1+\frac{\sin x}{\cos x}} = \frac{\cos x}{\cos x + \sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x} = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx \qquad --(\mathrm{I})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2} - x\right)}{\cos\left(\frac{\pi}{2} - x\right) + \sin\left(\frac{\pi}{2} - x\right)} dx \qquad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx \qquad --(\mathrm{II})$$

Hence, adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos x}{\cos x + \sin x} dx + \int_0^{\frac{\pi}{2}} \frac{\sin x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x + \sin x}{\cos x + \sin x} dx$$

$$= \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right] \quad \Rightarrow \quad I = \frac{\pi}{4}$$

## Definite Integrals Ex 20.4B Q2

We have,

$$\frac{1}{1+\cot x} = \frac{1}{1+\frac{\cos x}{\sin x}} = \frac{\sin x}{\sin x + \cos x}$$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{1}{1+\cot x}dx = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}dx \qquad --(\text{I})$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right) + \cos\left(\frac{\pi}{2}-x\right)}dx \qquad \left[\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x}dx \qquad --(\text{II})$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x}dx + \int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x}dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$= [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi}{4}$$

## Definite Integrals Ex 20.4B Q3

We have,

$$\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}=\frac{\sqrt{\frac{\cos x}{\sin x}}}{\sqrt{\frac{\cos x}{\sin x}}+\sqrt{\frac{\sin x}{\cos x}}}=\frac{\sqrt{\frac{\cos x}{\sin x}}}{\frac{\cos x+\sin x}{\sqrt{\sin x}.\sqrt{\cos x}}}=\sqrt{\frac{\cos x}{\sin x}}\times\frac{\sqrt{\sin x}.\sqrt{\cos x}}{\cos x+\sin x}$$

$$=\frac{\cos x}{\cos x+\sin x}$$

$$\therefore \int_0^{\frac{\pi}{2}}\frac{\sqrt{\cot x}}{\sqrt{\cot x}+\sqrt{\tan x}}dx=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx$$

Let

$$I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx \qquad --(\text{I})$$

So,

B $$I=\int_0^{\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}-x\right)}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx \qquad \left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\cos x+\sin x}dx \qquad --(\text{II})$$

Adding (I) & (II)

$$2I=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\cos x+\sin x}dx+\int_0^{\frac{\pi}{2}}\frac{\sin x}{\cos x+\sin x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}\frac{\cos x+\sin x}{\cos x+\sin x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}dx$$

$$2I=\left[x\right]_0^{\frac{\pi}{2}}$$

$$2I=\left[\frac{\pi}{2}-0\right]$$

$$I=\frac{\pi}{4}$$

**Definite Integrals Ex 20.4B Q4**

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(1)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)}{\sin^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right) + \cos^{\frac{3}{2}}\left(\frac{\pi}{2} - x\right)} dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx \qquad \ldots(2)$$

Adding (1) and (2), we obtain

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1 \cdot dx$$

$$\Rightarrow 2I = [x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 2I = \frac{\pi}{2}$$

$$\Rightarrow I = \frac{\pi}{4}$$

## Definite Integrals Ex 20.4B Q5

$$\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$$

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx \qquad --(i)$$

*So,*

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^n\left(\frac{\pi}{2} - x\right)}{\sin^n\left(\frac{\pi}{2} - x\right) + \cos^n\left(\frac{\pi}{2} - x\right)} dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x} \qquad --(II)$$

Adding (I) & (II)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n x}{\sin^n x + \cos^n x} dx + \int_0^{\frac{\pi}{2}} \frac{\cos^n x}{\sin^n x + \cos^n x}$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^n + \cos^n x}{\sin^n + \cos^n x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$2I = \left[\frac{\pi}{2} - 0\right]$$

$$I = \frac{\pi}{4}$$

## Definite Integrals Ex 20.4B Q6

We have,

$$\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

Let

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \qquad --(i)$$

So

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx + \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

## Definite Integrals Ex 20.4B Q7

$$Let\ I=\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$$

Let $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$

Now, $x = 0 \Rightarrow \theta = 0$

$x = a \Rightarrow \theta = \frac{\pi}{2}$

$$I = \int_0^{\frac{\pi}{2}} \frac{a\cos\theta\, d\theta}{a\sin\theta + a\cos\theta}$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sin\theta+\cos\theta} \qquad --(i)$$

So,

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)+\cos\left(\frac{\pi}{2}-\theta\right)} d\theta \qquad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)\,dx\right]$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sin\theta}{\cos\theta+\sin\theta} \qquad --(ii)$$

Adding (i) & (ii) we get

$$2I = \int_0^{\frac{\pi}{2}} \frac{\cos\theta+\sin\theta}{\cos\theta+\sin\theta} d\theta$$

$$2I = \int_0^{\frac{\pi}{2}} d\theta$$

$$2I = \frac{1}{2}[\theta]_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4}$$

## Definite Integrals Ex 20.4B Q8

Put $x = \tan\theta$

$\Rightarrow$ $dx = \sec^2\theta d\theta$

If $x = 0,\ \theta = 0$

If $x = \infty,\ \theta = \frac{\pi}{2}$

$$\therefore \quad I = \int_0^{\infty} \frac{\log x}{1+x^2} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\log(\tan\theta)\sec^2\theta d\theta}{1+\tan^2\theta}$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log(\tan\theta)\, d\theta \qquad ---(i)$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log\tan\left(\frac{\pi}{2}-\theta\right) d\theta$$

$$\Rightarrow \quad I = \int_0^{\frac{\pi}{2}} \log\cot(\theta)\, d\theta \qquad ---(ii)$$

Adding (i) and (ii), we get

$$2I = \int_0^{\frac{\pi}{2}} (\log\tan\theta + \log\cot\theta) d\theta$$

$$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \log 1 \times dx = \int_0^{\frac{\pi}{2}} 0 \times dx = 0$$

$$\Rightarrow \quad I = 0$$

## Definite Integrals Ex 20.4B Q9

Let $x = \tan\theta$

$\Rightarrow \quad dx = \sec^2\theta d\theta$

If $\quad x = 0, \ \theta = 0$

If $\quad x = 1, \ \theta = \frac{\pi}{4}$

$\therefore \quad \int_0^1 \frac{\log(1+x)}{1+x^2} dx$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log(1+\tan\theta) d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left\{1+\tan\left(\frac{\pi}{4}-\theta\right)\right\} d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left\{1+\frac{1-\tan\theta}{1+\tan\theta}\right\} d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan\theta}\right) d\theta$

$\Rightarrow \quad I = \int_0^{\frac{\pi}{4}} (\log 2 - \log(1+\tan\theta)) d\theta$

$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{4}} \log 2 \times d\theta = \frac{\pi}{4} \log 2$

$\Rightarrow \quad I = \frac{\pi}{8} \log 2$

## Definite Integrals Ex 20.4B Q10

$$I = \int_0^\infty \frac{x}{(1+x)(1+x^2)}dx$$

Let,

$$\frac{x}{(1+x)(1+x^2)} = \frac{A}{1+x} + \frac{Bx+C}{1+x^2}$$

$$\Rightarrow x = A(1+x^2) + (Bx+C)(1+x)$$

Equating coeffcients, we get

$$A + B = 0 \Rightarrow A = -B$$
$$B + C = 1 \Rightarrow -2A = 1$$
$$A + C = 0 \Rightarrow A = -C$$

$$\therefore A = -\frac{1}{2}, B = \frac{1}{2}, C = \frac{1}{2}$$

So,

$$I = \int_0^\infty \left( \frac{-\frac{1}{2}}{1+x} + \frac{1}{2}\frac{x+1}{x^2+1} \right) dx$$

$$= \int_0^\infty -\frac{1}{2}\frac{dx}{1+x} + \frac{1}{2}\int_0^\infty \frac{x}{x^2+1}dx + \frac{1}{2}\int_0^\infty \frac{dx}{1+x^2}$$

$$= \left[ -\frac{1}{2}\log|1+x| + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\tan^{-1}x \right]_0^\infty$$

$$= 0 + 0 + \frac{\pi}{4} + 0 - 0 - 0$$

$$= \frac{\pi}{4}$$

$$\because \int_0^\infty \frac{x}{(1+x)(1+x^2)}dx = \frac{\pi}{4}$$

## Definite Integrals Ex 20.4B Q11

We have,

$$I = \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx$$

$$I = \int_0^{\pi} \frac{x\left(\frac{\sin x}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right)} dx$$

$$I = \int_0^{\pi} x \sin^2 x \, dx \qquad \ldots(i)$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) dx \qquad \left[\because \int_0^a f(x) dx = \int_0^a f(a - x) dx\right]$$

$$I = \int_0^{\pi} (\pi - x) \sin^2 x \, dx \qquad --(ii)$$

Add (i) and (ii), we get

$$2I = \int_0^{\pi} (\pi) \sin^2 x \, dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} dx = \frac{\pi}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi} = \frac{\pi}{2}\left[\pi - 0 - 0 + 0\right] = \frac{\pi^2}{2}$$

$$\therefore \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x} dx = \frac{\pi^2}{4}$$

## Definite Integrals Ex 20.4B Q12

Let $I = \int_0^{\pi} x \sin x . \cos^4 x \, dx$ --(i)

So,

$$I = \int_0^{\pi} (\pi - x) \sin(\pi - x) . \cos^4(\pi - x) \, dx \qquad \left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$$

$$= \int_0^{\pi} (\pi - x) \sin x . \cos^4 x \, dx$$

$$= \int_0^{\pi} \pi \sin x . \cos^4 x \, dx - \int_0^{\pi} x \sin x . \cos^4 x \, dx$$

So from equation (i)

$$I = \int_0^{\pi} \pi \sin x . \cos^4 x \, dx - I$$

$$2I = \pi \int_0^{\pi} \sin x . \cos^4 x \, dx$$

Let $t = \cos x \, dx$

$dt = -\sin \ x \, dx$

*As,*

$x = 0 \quad t = 1$

$x = \pi \quad t = -1$

Hence

$$2I = \pi \int_{-1}^{+1} t^4 \, dt = \pi \left[\frac{t^5}{5}\right]_{-1}^{1} = \pi \left[\frac{1}{5} + \frac{1}{5}\right]$$

$$I = \frac{\pi}{5}$$

## Definite Integrals Ex 20.4B Q13

Let $I = \int_0^\pi x \sin^3 x\, dx$

$= \int_0^\pi (\pi - x) \sin^3 (\pi - x) dx$  $\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$

$= \int_0^\pi \pi \sin^3 x\, dx - \int_0^\pi x \sin^3 x\, dx$

$\therefore I = \int_0^\pi \pi \sin^3 x\, dx - I$

$\Rightarrow 2I = \pi \int_0^\pi \sin^3 x\, dx$

$\Rightarrow 2I = \pi \int_0^\pi \frac{3 \sin x - \sin 3x}{4} dx$

$= \frac{\pi}{4} \int_0^\pi (3 \sin x - \sin 3x) dx$

$= \frac{\pi}{4} \left[-3 \cos x + \frac{\cos 3x}{3}\right]_0^\pi$

$= \frac{\pi}{4} \left[\left(-3 \cos \pi + \frac{\cos 3\pi}{3}\right) - \left(-3 \cos 0 + \frac{\cos 0}{3}\right)\right]$

$= \frac{\pi}{4} \left[\left(3 - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right]$

$= \frac{\pi}{4} \left[3 - \frac{1}{3} + 3 - \frac{1}{3}\right]$

$\frac{\pi}{4} \left[6 - \frac{2}{3}\right]$

$= \frac{\pi}{4} \times \frac{16}{3} \quad = \quad \frac{4\pi}{3}$

$\therefore I = \frac{2\pi}{3}$

**Definite Integrals Ex 20.4B Q14**

We have,

$$I = \int_0^{\pi} x \log \sin x \, dx = \int_0^{\pi} (\pi - x) \log \sin (\pi - x) \, dx$$

$$I = \pi \int_0^{\pi} \log \sin (x) dx - \int_0^{\pi} x \log \sin x \, dx$$

$$2I = \pi \int_0^{\pi} \log \sin x \, dx$$

Since $f(x) = f(-x)$, f(x) is an even function.

$$\therefore \; 2I = 2\pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

$$I = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx \qquad \ldots(i)$$

$$\Rightarrow I = \pi \int_0^{\frac{\pi}{2}} \log \sin \left( \frac{\pi}{2} - x \right) dx = \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx \quad \ldots(ii)$$

Now adding (i) & (ii) we get

$$2I = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx + \pi \int_0^{\frac{\pi}{2}} \log \cos x \, dx = \pi \int_0^{\frac{\pi}{2}} (\log \sin x + \log \cos x) \, dx = \pi \int_0^{\frac{\pi}{2}} \log \sin x . \cos x \, dx$$

$$\Rightarrow 2I = \pi \int_0^{\frac{\pi}{2}} \log \left( \frac{2 \sin x . \cos x}{2} \right) dx = \pi \int_0^{\frac{\pi}{2}} \log \left( \frac{\sin 2x}{2} \right) dx = \pi \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx - \pi \int_0^{\frac{\pi}{2}} \log 2 \, dx \qquad \ldots(iii)$$

Now let $I = \int_0^{\frac{\pi}{2}} \log \sin 2x \, dx$

Putting $2x = t$ we get

$$I_1 = \int_0^{\pi} \log \sin t \frac{dt}{2} = \frac{1}{2} \int_0^{\pi} \log \sin t . dt = \frac{1}{2} \times 2\pi \int_0^{\frac{\pi}{2}} \log \sin t \, dt = \pi \int_0^{\frac{\pi}{2}} \log \sin x \, dx = I$$

So from (iii) we get

$$2I = I - \pi \frac{\pi}{2} \log 2$$

$$I = -\frac{\pi^2}{2} \log 2$$

## Definite Integrals Ex 20.4B Q15

Let $I = \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$

$= \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\sin x} dx \qquad \left[\because \int_0^{a} f(x)dx = \int_0^{a} f(a-x)\,dx\right]$

$I = \int_0^{\pi} \frac{\pi \sin x}{1+\sin x} dx - \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$

$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\sin x} dx$

$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\sin x} \times \frac{(1-\sin x)}{(1-\sin x)} dx$

$2I = \pi \int_0^{\pi} \frac{\sin x - \sin^2 x}{1+\sin^2 x} dx$

$2I = \pi \int_0^{\pi} \frac{(\sin x - \sin^2 x)}{\cos^2 x} dx$

$2I = \pi \int_0^{\pi} \left(\tan x . \sec x - \tan^2 x\right) dx$

$2I = \pi \int_0^{\pi} \left[\tan x . \sec x - (\sec^2 x - 1)\right] dx$

$2I = \pi \int_0^{\pi} \left(\sec x . \tan x - \sec^2 x + 1\right) dx$

$2I = \pi \int_0^{\pi} \left(\sec x . \tan x - \sec^2 x + 1\right) dx$

$2I = \pi \left[\sec x - \tan x + x\right]_0^{\pi}$

$2I = \pi \left[(\sec \pi - \tan \pi + \pi) - (\sec 0 - \tan 0 + 0)\right]$

$2I = \pi \left[(-1 - 0 + \pi) - (1 - 0 + 0)\right]$

$2I = \pi (\pi - 1 - 1)$

$I = \frac{\pi}{2}(\pi - 2)$

$\therefore \int_0^{\pi} \frac{x \sin x}{1+\sin x} dx = \pi\left(\frac{\pi}{2} - 1\right)$

## Definite Integrals Ex 20.4B Q16

We have

$$I = \int_0^{\pi} \frac{x\,dx}{1+\cos\alpha \sin x} \qquad --(i)$$

$$\therefore \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\cos\alpha \sin(\pi - x)} = \int_0^{\pi} \frac{(\pi - x)\,dx}{1+\cos\alpha \sin x} \qquad --(ii)$$

Adding (i) & (ii) we get

$$2I = \pi \int_0^{\pi} \frac{\pi}{1+\cos\alpha . \sin x} dx$$

$$\text{Substituting } \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$$

$$2I = \pi \int_0^{\pi} \frac{\sec^2\frac{x}{2}}{1+\tan^2\frac{x}{2}\, 2\cos\alpha . \tan\frac{x}{2}} dx = \pi \int_0^{\pi} \frac{\sec^2\frac{x}{2} dx}{1-\cos^2\alpha + \left(\cos\alpha . \tan\frac{x}{2}\right)^2} dx$$

$$\text{Let } \tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

When $x = 0 \quad t = 0$

$\pi \;\; \Rightarrow t = \alpha$

$$2I = \int_0^{\alpha} \frac{dt}{(1+\cos^2\alpha) + (\cos\alpha + t)^2} dx = 2\pi . \frac{1}{\sqrt{1+\cos^2\alpha}} . \left[\tan^{-1}\left(\frac{\cos\alpha + 1}{\sqrt{1+\cos^2\alpha}}\right)\right]_0^{\alpha}$$

$$= \frac{2\pi}{\sin\alpha}\left[\frac{\pi}{2} - \tan^{-1}\cot\alpha\right]$$

$$= \frac{2\pi}{\sin\alpha}\left[\cot^{-1}(\cot\alpha)\right]$$

$$= \frac{2\pi}{\sin\alpha} . \alpha$$

$$\Rightarrow I = \frac{\pi\alpha}{\sin\alpha}$$

**Definite Integrals Ex 20.4B Q17**

Let $I = \int_0^{\pi} x \cos^2 x \, dx$

$I = \int_0^{\pi} (\pi - x) \cos^2 (\pi - x) \, dx$ $\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx\right]$

$I = \pi \int_0^{\pi} \cos^2 x \, dx - \int_0^{\pi} x \cos^2 x \, dx$

$2I = \pi \int_0^{\pi} \cos^2 x \, dx$

$= \pi \int_0^{\pi} \left(\frac{1 + \cos 2x}{2}\right) dx$ Since $\cos^2 x = \frac{1 + \cos 2x}{2}$

$= \frac{\pi}{2} \int_0^{\pi} (1 + \cos 2x) \, dx$

$= \frac{\pi}{2} \left[x + \left(-\frac{\sin 2x}{2}\right)\right]_0^{\pi}$

$\therefore 2I = \frac{\pi}{2}\left[\pi - \frac{\sin 2.\pi}{2} - 0 + \frac{\sin 0}{2}\right]$

$\Rightarrow 2I = \frac{\pi}{2}[\pi - 0 - 0 + 0]$

$I = \frac{\pi^2}{4}$

## Definite Integrals Ex 20.4B Q18

$I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\cot^{3/2} x} dx$

$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$

$I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2}\left(\frac{\pi}{2} - x\right)}{\sin^{3/2}\left(\frac{\pi}{2} - x\right) + \cos^{3/2}\left(\frac{\pi}{2} - x\right)} dx = \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x) + \sin^{3/2}(x)} dx$

$\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx + \int_{\pi/6}^{\pi/3} \frac{\cos^{3/2}(x)}{\cos^{3/2}(x) + \sin^{3/2}(x)} dx$

$2I = \int_{\pi/6}^{\pi/3} \frac{\sin^{3/2} x + \cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$

$I = \frac{1}{2} \int_{\pi/6}^{\pi/3} dx$

$I = \frac{\pi}{12}$

## Definite Integrals Ex 20.4B Q19

$$I=\int_0^{\frac{\pi}{2}}\frac{\tan^7 x}{\tan^7 x+\cot^7 x}dx$$

$$I=\int_0^{\frac{\pi}{2}}\frac{\tan^7\left(\frac{\Pi}{2}-x\right)}{\tan^7\left(\frac{\Pi}{2}-x\right)+\cot^7\left(\frac{\Pi}{2}-x\right)}dx$$

$$I=\int_0^{\frac{\pi}{2}}\frac{\cot^7 x}{\tan^7 x+\cot^7 x}dx$$

Hence

$$2I=\int_0^{\frac{\pi}{2}}\frac{\tan^7 x}{\tan^7 x+\cot^7 x}+\frac{\cot^7 x}{\tan^7 x+\cot^7 x}dx$$

$$2I=\int_0^{\frac{\pi}{2}}1dx$$

$$2I=\frac{\Pi}{2}$$

$$I=\frac{\Pi}{4}$$

## Definite Integrals Ex 20.4B Q20

$$I=\int_2^8\frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$I=\int_2^8\frac{\sqrt{10-(8+2-x)}}{\sqrt{(8+2-x)}+\sqrt{10-(8+2-x)}}dx$$

$$I=\int_2^8\frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$2I=\int_2^8\frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}+\frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$$

$$2I=\int_2^8 1dx$$

$$2I=6$$

$$I=3$$

## Definite Integrals Ex 20.4B Q21

$$\int_0^{\pi} x\sin x\cos^2 x\,dx = \int_0^{\pi} (\Pi - x)\sin(\Pi - x)\cos^2(\Pi - x)\,dx$$

$$\int_0^{\pi} x\sin x\cos^2 x\,dx = \int_0^{\pi} (\Pi - x)\sin x\cos^2 x\,dx$$

$$\int_0^{\pi} x\sin x\cos^2 x\,dx = \int_0^{\pi} \Pi\sin x\cos^2 x\,dx - \int_0^{\pi} x\sin x\cos^2 x\,dx$$

$$2\int_0^{\pi} x\sin x\cos^2 x\,dx = \int_0^{\pi} \Pi\sin x\cos^2 x\,dx$$

$$\int_0^{\pi} x\sin x\cos^2 x\,dx = \frac{\Pi}{2}\int_0^{\pi} \sin x\cos^2 x\,dx$$

Now

$$\int_0^{\pi} \sin x\cos^2 x\,dx$$

Let cosx=t

$$\sin x\,dx = -dt$$

$$-\int_1^{-1} t^2\,dt$$

$$\int_{-1}^{1} t^2\,dt$$

$$\left\{\frac{t^3}{3}\right\}_{-1}^{1}$$

$$\frac{2}{3}$$

$$\therefore \int_0^{\pi} x\sin x\cos^2 x\,dx = \frac{\pi}{2}\times\frac{2}{3} = \frac{\pi}{3}$$

## Definite Integrals Ex 20.4B Q22

We have,

$$I = \int_0^{\frac{\pi}{2}} \frac{x \sin x . \cos x}{\sin^4 x + \cos^4 x} dx \qquad --(i)$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\left(\frac{\pi}{2} - x\right) \cos x . \sin x}{\cos^4 x + \sin^4 x} dx \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \frac{\pi}{2} \int_0^{\frac{\pi}{2}} \frac{\sin x . \cos x}{\cos^4 x + \sin^4 x} dx$$

$$2I = \frac{\pi}{4} \int_0^{\frac{\pi}{2}} \frac{2 \sin x . \cos x}{\cos^4 x + \sin^4 x} dx$$

Let $t = \sin^2 x$

$$\Rightarrow 2I = \frac{\pi}{4} \int_0^1 \frac{1}{(1-t)^2 + t^2} dt$$

$$\Rightarrow 2I = \frac{\pi}{8} \int_0^1 \frac{1}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dt$$

$$\Rightarrow 2I = \frac{\pi}{8} \times 2 \left[ \tan^{-1}(2t-1) \right]_0^1$$

$$\Rightarrow I = \frac{\pi}{8} \left[ \frac{\pi}{4} + \frac{\pi}{4} \right] = \frac{\pi^2}{16}$$

## Definite Integrals Ex 20.4B Q23

Let $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x\, dx$

$$f(-x) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3(-x)\, dx$$

$$= -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x\, dx$$

Here $f(x) = -f(+x)$

Hence $f(x)$ is odd function.

So,

$I = 0$

## Definite Integrals Ex 20.4B Q24

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\,dx = 2\int_0^{\frac{\pi}{2}} \sin^4 x\,dx \qquad \left[\because \sin^4 x \text{ is an even function}\right]$$

$$= 2\int_0^{\frac{\pi}{2}} \left(\sin^2 x\right)^2 dx$$

$$= 2\int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(1-\cos 2x\right)^2 dx$$

$$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \left(1+\cos^2 2x - 2\cos 2x\right) dx\right]$$

$$= \frac{1}{2}\left[\int_0^{\frac{\pi}{2}} \left(1-2\cos 2x + \frac{1+\cos 4x}{2}\right) dx\right]$$

$$= \frac{1}{4}\left[\int_0^{\frac{\pi}{2}} \left(3-4\cos 2x + \cos 4x\right) dx\right]$$

$$= \frac{1}{4}\left[3x - \frac{4\sin 2x}{2} + \frac{\sin 4x}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}\left[\left\{\frac{3\pi}{2} - 2\sin\pi + \frac{1}{4}\sin 2\pi\right\} - \left\{0-0+0\right\}\right]$$

$$= \frac{1}{4}\left[\frac{3\pi}{2} - 0 + 0\right] = \frac{1}{4}\times\frac{3\pi}{2}$$

$$= \frac{3\pi}{8}$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^4 x\,dx = \frac{3\pi}{8}$$

## Definite Integrals Ex 20.4B Q25

We have,

$$I = \int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$$

*Since,* $\log\left\{\frac{2-(-x)}{2+(-x)}\right\} = -\log\left(\frac{2-x}{2+x}\right)$ $\therefore$ This is an odd function.

Hence,

$I = 0$

## Definite Integrals Ex 20.4B Q26

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx$$

$\sin^2 x$ is even function.

Hence,

$$I = 2\int_{0}^{\frac{\pi}{4}} \sin^2 x \, dx = 2\int_{0}^{\frac{\pi}{4}} \left(\frac{1-\cos 2x}{2}\right) dx = \frac{2}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}} = \frac{1}{2}\left[\frac{2\pi}{4} - \sin\frac{\pi}{2} - 0 + \sin 0\right]$$

$$= \frac{1}{2}\left[\frac{2\pi}{4} - 1\right]$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx = \frac{\pi}{4} - \frac{1}{2}$$

## Definite Integrals Ex 20.4B Q27

$$I = \int_0^{\pi} \log(1-\cos x)\,dx$$

$$= \int_0^{\pi} \log\left(2\sin^2\frac{x}{2}\right)dx$$

$$= \int_0^{\pi} \log 2\,dx + \int_0^{\pi} \log\sin^2\frac{x}{2}\,dx$$

$$= \int_0^{\pi} \log 2\,dx + 2\int_0^{\pi} \log\sin\frac{x}{2}\,dx$$

$$I = \log 2[x]_0^{\pi} + 4\int_0^{\frac{\pi}{2}} \log\sin t\,dt \qquad \left[\text{Put} \quad t = \frac{x}{2} \Rightarrow dt = \frac{1}{2}dx\right]$$

$$I = \pi\log 2 + 4I_1 \qquad \ldots(\text{i})$$

$$I_1 = \int_0^{\frac{\pi}{2}} \log\sin t\,dt \qquad \ldots(\text{ii})$$

$$= \int_0^{\frac{\pi}{2}} \log\cos t\,dt \qquad \ldots(\text{iii})$$

Adding (ii) & (iii) we get

$$2I_1 = \int_0^{\frac{\pi}{2}} \log\sin t.\cos t\,dt = \int_0^{\frac{\pi}{2}} \log\left(\frac{\sin 2t}{2}\right)dt = \int_0^{\frac{\pi}{2}} \log\sin 2t\,dt - \frac{\pi}{2}\log 2$$

We know the property $\int_a^b f(x) = \int_a^b f(t)$

$$2I_1 = I_1 - \frac{\pi}{2}\log 2$$

$$\Rightarrow I_1 = -\frac{\pi}{2}\log 2 \qquad \ldots(\text{iv})$$

Putting the value from (iv) to (i)

$$I = \pi\log 2 + 4\left(-\frac{\pi}{2}\log 2\right) = \pi\log 2 - 2\pi\log 2 = -\pi\log 2$$

$$I = -\pi\log 2$$

## Definite Integrals Ex 20.4B Q28

We have,

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2 - \sin x}{2 + \sin x}\right) dx$$

Let $f(x) = \log\left(\frac{2 - \sin x}{2 + \sin x}\right)$

Then,

$$f(-x) = \log\left(\frac{2 - \sin(-x)}{2 + \sin(-x)}\right) = -\log\left(\frac{2 - \sin x}{2 + \sin x}\right) = -f(x)$$

Thus, f(x) is an odd function.

$$\therefore I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \log\left(\frac{2 - \sin x}{2 + \sin x}\right) dx = 0$$

## Definite Integrals Ex 20.4B Q29

$$I = \int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$$

$$I = \int_{-\pi}^{\pi} \frac{2x}{1 + \cos^2 x} dx + \int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx$$

$$I = 0 + \int_{-\pi}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx \ldots\ldots\ldots \left[\because \frac{2x}{1 + \cos^2 x} \text{ is an odd function}\right]$$

$$I = 2\int_{0}^{\pi} \frac{2x \sin x}{1 + \cos^2 x} dx \ldots\ldots\ldots \left[\because \frac{2x \sin x}{1 + \cos^2 x} \text{ is an even function}\right]$$

$$I = 4\int_{0}^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$I = 2\pi\int_{0}^{\pi} \frac{\sin x}{1 + \cos^2 x} dx \ldots\ldots\ldots \left[\because \int_{0}^{a} x f(x) dx = \frac{a}{2}\int_{0}^{a} f(x) dx\right]$$

Put $\cos x = t$ then $-\sin x \, dx = dt$

$$I = -2\pi\int_{1}^{-1} \frac{1}{1 + t^2} dt$$

$$I = -2\pi\left[\tan^{-1} t\right]_{1}^{-1}$$

$$I = \pi^2$$

## Definite Integrals Ex 20.4B Q30

$$I = \int_{-a}^{a} \log\left(\frac{a - \sin\theta}{a + \sin\theta}\right) d\theta$$

$$\text{Let } f(\theta) = \log\left(\frac{a - \sin\theta}{a + \sin\theta}\right)$$

$$f(-\theta) = \log\left(\frac{a - \sin(-\theta)}{a + \sin(-\theta)}\right) = -\log\left(\frac{a - \sin\theta}{a + \sin\theta}\right) = -f(\theta)$$

$$\therefore\ f(\theta) = \log\left(\frac{a - \sin\theta}{a + \sin\theta}\right) \text{ is an odd function.}$$

$$\therefore\ I = \int_{-a}^{a} \log\left(\frac{a - \sin\theta}{a + \sin\theta}\right) d\theta = 0$$

**Definite Integrals Ex 20.4B Q31**

$$I = \int_{-2}^{2} \frac{3x^3 + 2|x| + 1}{x^2 + |x| + 1}\, dx$$

$$I = \int_{-2}^{2} \frac{3x^3}{x^2 + |x| + 1}\, dx + \int_{-2}^{2} \frac{2|x| + 1}{x^2 + |x| + 1}\, dx$$

$$I = 0 + \int_{-2}^{2} \frac{2|x| + 1}{x^2 + |x| + 1}\, dx \ldots\ldots\ldots \left[\because \frac{3x^3}{x^2 + |x| + 1} \text{ is an odd function}\right]$$

$$I = 2\int_{0}^{2} \frac{2|x| + 1}{x^2 + |x| + 1}\, dx \ \ldots\ldots\ldots \left[\because \frac{2|x| + 1}{x^2 + |x| + 1} \text{ is an even function}\right]$$

$$I = 2\left[\log\left(x^2 + |x| + 1\right)\right]_0^2$$

$$I = 2\left[\log(4 + 2 + 1) - \log(1)\right]$$

$$I = 2\log_e(7)$$

**Definite Integrals Ex 20.4B Q32**

$$I = \int_{-3\pi/2}^{-\pi/2} \left\{\sin^2(3\pi + x) + (\pi + x)^3\right\} dx$$

Substitute $\pi + x = u$ then $dx = du$

$$I = \int_{-\pi/2}^{\pi/2} \left\{\sin^2(2\pi + u) + (u)^3\right\} du$$

$$I = \int_{-\pi/2}^{\pi/2} \left\{\sin^2(u) + (u)^3\right\} du$$

$$I = \left[\frac{1}{2}\left(u - \frac{1}{2}\sin(2u)\right) + \frac{u^4}{4}\right]_{-\pi/2}^{\pi/2}$$

$$I = \frac{\pi}{2}$$

**Definite Integrals Ex 20.4B Q33**

$$\text{Let } I = \int_0^2 x\sqrt{2-x}\,dx$$

$$I = \int_0^2 (2-x)\sqrt{x}\,dx \qquad \left(\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$

$$= \int_0^2 \left\{2x^{\frac{1}{2}} - x^{\frac{3}{2}}\right\} dx$$

$$= \left[2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) - \frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right]_0^2$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}}\right]_0^2$$

$$= \frac{4}{3}(2)^{\frac{3}{2}} - \frac{2}{5}(2)^{\frac{5}{2}}$$

$$= \frac{4 \times 2\sqrt{2}}{3} - \frac{2}{5} \times 4\sqrt{2}$$

$$= \frac{8\sqrt{2}}{3} - \frac{8\sqrt{2}}{5}$$

$$= \frac{40\sqrt{2} - 24\sqrt{2}}{15}$$

$$= \frac{16\sqrt{2}}{15}$$

**Definite Integrals Ex 20.4B Q34**

$$\text{Let } I = \int_0^1 \log\left(\frac{1}{x} - 1\right)dx$$

$$= \int_0^1 \log\left(\frac{1-x}{x}\right)dx$$

$$= \int_0^1 \log(1-x)dx - \int_0^1 \log(x)dx$$

$$\text{Applying the property, } \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$\text{Thus, } I = \int_0^1 \log(1-(1-x))dx - \int_0^1 \log(x)dx$$

$$= \int_0^1 \log(1-1+x)dx - \int_0^1 \log(x)dx$$

$$= \int_0^1 \log(x)dx - \int_0^1 \log(x)dx$$

$$= 0$$

**Definite Integrals Ex 20.4B Q35**

$$I = \int_{-1}^{1} |x \cos \pi x|\, dx$$

Let $f(x) = |x \cos \pi x|$

$$f(-x) = |-x \cos(-\pi x)| = |-x \cos(\pi x)| = |x \cos \pi x| = f(x)$$

$$\therefore I = \int_{-1}^{1} |x \cos \pi x|\, dx = 2\int_{0}^{1} |x \cos \pi x|\, dx$$

Now,

$$f(x) = |x \cos \pi x| = \begin{cases} x \cos \pi x \ , \text{ if } 0 \le x \le \frac{1}{2} \\ -x \cos \pi x \ , \text{ if } \frac{1}{2} < x < 1 \end{cases}$$

$$\therefore I = 2\int_{0}^{1} |x \cos \pi x|\, dx$$

$$\Rightarrow I = 2\left[\int_{0}^{\frac{1}{2}} x \cos \pi x \ dx + \int_{\frac{1}{2}}^{1} -x \cos \pi x\, dx\right]$$

$$\Rightarrow I = 2\left\{\left[\frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x\right]_0^{\frac{1}{2}} - \left[\frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x\right]_{\frac{1}{2}}^{1}\right\}$$

$$\Rightarrow I = 2\left\{\left[\frac{1}{2\pi} - \frac{1}{\pi^2}\right] - \left[-\frac{1}{\pi^2} - \frac{1}{2\pi}\right]\right\}$$

$$\Rightarrow I = \frac{2}{\pi}$$

**Definite Integrals Ex 20.4B Q36**

$$I=\int_0^{\Pi}\left(\frac{x}{1+\sin^2 x}+\cos^7 x\right)dx$$

$$I=\int_0^{\Pi}\left(\frac{\Pi-x}{1+\sin^2(\Pi-x)}+\cos^7(\Pi-x)\right)dx$$

$$I=\int_0^{\Pi}\left(\frac{\Pi-x}{1+\sin^2 x}-\cos^7 x\right)dx$$

$$2I=\int_0^{\Pi}\left(\frac{\Pi}{1+\sin^2 x}\right)dx$$

$$2I=\Pi\int_0^{\Pi}\frac{1}{1+\sin^2 x}dx$$

$$2I=\Pi\int_0^{\Pi}\frac{1}{1+2\tan^2 x}\sec^2 x dx$$

$$I=\pi\int_0^{\frac{\pi}{2}}\frac{1}{1+2\tan^2 x}\sec^2 x dx \ldots\ldots\ldots\left[\because \int_0^{2a} f(x)dx=2\int_0^{a} f(x)dx, \text{ if } f(2a-x)=f(x)\right]$$

Let $\tan x = v$

$dv = \sec^2 x \, dx$

$$\Rightarrow I=\pi\int_0^{\infty}\frac{1}{1+2v^2}dv$$

$$\Rightarrow I=\pi\left[\frac{\tan^{-1}\left(\sqrt{2}v\right)}{\sqrt{2}}\right]_0^{\infty}$$

$$\Rightarrow I=\pi\left[\frac{\pi}{2\sqrt{2}}\right]$$

$$\Rightarrow I=\frac{\pi^2}{2\sqrt{2}}$$

## Definite Integrals Ex 20.4B Q37

$$I = \int_0^{\pi} \frac{x}{1+\cos\alpha \sin x} dx$$

Then,

$$I = \int_0^{\pi} \frac{(\pi - x)}{1+\cos\alpha \sin(\pi - x)} dx$$

$$I = \int_0^{\pi} \frac{(\pi - x)}{1+\cos\alpha \sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{1}{1+\cos\alpha \sin x} dx$$

$$2I = \pi \int_0^{\pi} \frac{1+\tan^2\left(\frac{x}{2}\right)}{\left(1+\tan^2\left(\frac{x}{2}\right)\right) + 2\cos\alpha \tan\left(\frac{x}{2}\right)} dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sec^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) + 2\cos\alpha \tan\left(\frac{x}{2}\right) + 1} dx$$

Put $\tan\left(\frac{x}{2}\right) = t$ then $\sec^2\left(\frac{x}{2}\right) dx = 2dt$

$x = 0 \Rightarrow t = 0$ and $x = \pi \Rightarrow t = \infty$

$$I = \frac{\pi}{2} \int_0^{\infty} \frac{2}{t^2 + 2t\cos\alpha + 1} dt$$

$$I = \pi \int_0^{\infty} \frac{1}{(t+\cos\alpha)^2 + (1-\cos^2\alpha)} dt$$

$$I = \pi \int_0^{\infty} \frac{1}{(t+\cos\alpha)^2 + \sin^2\alpha} dt$$

$$I = \frac{\pi}{\sin\alpha}\left[\tan^{-1}\left(\frac{t+\cos\alpha}{\sin\alpha}\right)\right]_0^{\infty}$$

$$I = \frac{\pi\alpha}{\sin\alpha}$$

**Definite Integrals Ex 20.4B Q38**

We know

$$\int_0^{2a} f(x)dx = \int_0^{a} f(x) + \int_0^{a} f(2a-x)dx$$

Also here

$$f(x) = f(2\Pi - x)$$

So

$$I = \int_0^{2\pi} \sin^{100} x \cos^{101} x dx = 2\int_0^{\pi} \sin^{100} x \cos^{101} x dx$$

$$I = 2\int_0^{\pi} \sin^{100}(\Pi - x)\cos^{101}(\Pi - x)dx$$

$$I = -2\int_0^{\pi} \sin^{100} x \cos^{101} x dx$$

Hence

$$2I = 0$$

$$I = 0$$

**Definite Integrals Ex 20.4B Q39**

$$I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x} dx$$

Then,

$$I = \int_0^{\pi/2} \frac{a\sin\left(\frac{\pi}{2} - x\right) + b\cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x} dx$$

$$2I = \int_0^{\pi/2} \frac{a\sin x + b\cos x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{a\cos x + b\sin x}{\cos x + \sin x} dx$$

$$2I = (a+b)\int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$I = \frac{(a+b)}{2}\int_0^{\pi/2} 1dx$$

$$I = \frac{(a+b)\pi}{4}$$

**Definite Integrals Ex 20.4B Q40**

We have,

$$I = \int_0^{2a} f(x)\,dx$$

Then

$$I = \int_0^{a} f(x)\,dx + \int_a^{2a} f(x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + I_1$$

where, $I_1 = \int_a^{2a} f(x)\,dx$

Let $2a - t = x$ then $dx = -dt$

If $t = a \Rightarrow x = a$

If $t = 2a \Rightarrow x = 0$

$$I_1 = \int_0^{2a} f(x)\,dx = \int_a^{0} f(2a - t)(-dt) = -\int_a^{0} f(2a - t)\,dt$$

$$I_1 = \int_0^{a} f(2a - t)\,dt = \int_0^{a} f(2a - x)\,dx$$

$$\therefore\ I = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a - x)\,dx$$

$$I = \int_0^{a} f(x)\,dx + \int_0^{a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \quad [f(2a - x) = f(x)]$$

Hence Proved.

## Definite Integrals Ex 20.4B Q41

We have,

$$I = \int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_a^{2a} f(x)dx$$

$$I = \int_0^{a} f(x)dx + I_1$$

Let $2a - t = x$ then $dx = -dt$

$t = a,\ x = a$

$t = 2a \quad x = 0$

$$I_1 = \int_0^{2a} f(x) = \int_a^{0} f(2a - t)(-dt)$$

$$= -\int_a^{0} f(2a - t)dt$$

$$I_1 = \int_0^{a} f(2a - t)dt = \int_0^{a} f(2a - x)dx$$

$$I = \int_0^{a} f(x)dx + \int_0^{a} f(2a - x)dx$$

$$I = \int_0^{a} f(x)dx - \int_0^{a} f(x) \qquad [\because f(2a - x) = -f(x)]$$

$$I = 0$$

Hence,

$$\int_0^{2a} f(x)dx = 0$$

## Definite Integrals Ex 20.4B Q42

(i) We have,

$I = \int_{-a}^{a} f\left(x^2\right)dx$

Clearly $f\left(x^2\right)$ is an even function.

So,

$\int_{-a}^{a} f\left(t\right) = 2\int_{0}^{a} f\left(t\right)$

$I = 2\int_{0}^{a} f\left(x^2\right)dx$

(ii) We have,

$I = \int_{-a}^{a} xf\left(x^2\right)dx$

Clearly, $xf\left(x^2\right)$ is odd function.

So, $I = 0$

$\therefore \int_{-a}^{a} xf\left(x^2\right)dx = 0$

## Definite Integrals Ex 20.4B Q43

We have from LHS,

$I = \int_{0}^{2a} f\left(x\right)dx = \int_{0}^{a} f\left(x\right)dx + \int_{a}^{2a} f\left(x\right)dx \quad \ldots(i)$

Let $x = 2a - t$, then $dx = -dt$

$x = a \Rightarrow t = a$, and $x = 2a \Rightarrow t = 0$

$\therefore \int_{0}^{2a} f\left(x\right)dx = -\int_{a}^{0} f\left(2a - t\right)dt$

$\Rightarrow \int_{0}^{2a} f\left(x\right)dx = \int_{0}^{a} f\left(2a - t\right)dt$

$\Rightarrow \int_{0}^{2a} f\left(x\right)dx = \int_{0}^{a} f\left(2a - x\right)dx$

Substituting $\int_{0}^{2a} f\left(x\right)dx = \int_{0}^{a} f\left(2a - x\right)dx$ in (i)

we get,

$\int_{0}^{2a} f\left(x\right)dx = \int_{0}^{a} f\left(x\right)dx + \int_{0}^{a} f\left(2a - x\right)dx$

$\Rightarrow \int_{0}^{2a} f\left(x\right)dx = \int_{0}^{a} \left\{f\left(x\right) + f\left(2a - x\right)\right\}dx$

## Definite Integrals Ex 20.4B Q44

$$I = \int_a^b x f(x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b-x) f(a+b-x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b-x) f(x)\,dx \ldots\ldots\ldots\ldots [\text{Given that } f(a+b-x) = f(x)]$$

$$\Rightarrow I = \int_a^b (a+b) f(x)\,dx - \int_a^b x f(x)\,dx$$

$$\Rightarrow I = \int_a^b (a+b) f(x)\,dx - I$$

$$\Rightarrow 2I = \int_a^b (a+b) f(x)\,dx$$

$$\Rightarrow I = \frac{a+b}{2}\int_a^b f(x)\,dx$$

## Definite Integrals Ex 20.4B Q45

We have,

$$I = \int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_{0}^{a} f(x)\,dx$$

Let $x = -t$ then $dx = -dt$

$x = -a \Rightarrow t = a$

$x = 0 \Rightarrow t = 0$

$$\therefore \int_{-a}^{a} f(x)\,dx = \int_{a}^{0} f(-t)(-dt) = -\int_{a}^{0} f(-t)\,dt$$

$$\Rightarrow \int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(-t)\,dt$$

$$\Rightarrow \int_{-a}^{0} f(x)\,dx = \int_{0}^{a} f(-x)\,dx$$

$$\therefore \int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(-x)\,dx + \int_{0}^{a} f(x)\,dx$$

Hence,

$$\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} \{f(-x) + f(x)\}\,dx$$

Proved

## Definite Integrals Ex 20.4B Q46

$$I = \int_0^{\pi} x f(\sin x)dx$$

$$I = \int_0^{\pi} (\Pi - x) f(\sin(\Pi - x))dx$$

$$I = \int_0^{\pi} (\Pi - x) f(\sin x)dx$$

$$2I = \int_0^{\pi} \Pi f(\sin x)dx$$

$$I = \frac{\Pi}{2}\int_0^{\pi} f(\sin x)dx$$