RD Sharma Solutions Class 12 Maths Chapter 20 Ex 20.3

Definite Integrals Ex 20.3 Q1(i)

We have,

$$\int_1^4 f(x)dx$$

$$= \int_1^2 (4x+3)\,dx + \int_2^4 (3x+5)\,dx$$

$$= \left[\frac{4x^2}{2}+3x\right]_1^2 + \left[\frac{3x^2}{2}+5x\right]_2^4$$

$$= \left[\left(\frac{16}{2}+6\right)-\left(\frac{4}{2}+3\right)\right]+\left[\left(\frac{48}{2}+20\right)-\left(\frac{12}{2}+10\right)\right]$$

$$= [(14-5)]+[(44-16)]$$

$$= 9+28$$

$$= 37$$

Definite Integrals Ex 20.3 Q1(ii)

We have,

$$\int_0^9 f(x)dx$$

$$= \int_0^{\frac{\pi}{2}} \sin x\,dx + \int_{\frac{\pi}{2}}^3 1\,dx + \int_3^9 e^{x-3}dx$$

$$= [-\cos x]_0^{\frac{\pi}{2}} + [x]_{\frac{\pi}{2}}^3 + \left[e^{x-3}\right]_3^9$$

$$= \left[-\cos\frac{\pi}{2}+\cos 0\right]+\left[3-\frac{\pi}{2}\right]+\left[e^{9-2}-e^{3-3}\right]$$

$$= [0+1]+\left[3-\frac{\pi}{2}\right]+\left[e^6-e^0\right]$$

$$= 0+1+3-\frac{\pi}{2}+e^6-e^0$$

$$= 1+3-\frac{\pi}{2}+e^6-1$$

$$= 3-\frac{\pi}{2}+e^6$$

Definite Integrals Ex 20.3 Q1(iii)

We have,

$$\int_1^4 f(x)\,dx$$

$$= \int_1^3 (7x+3)\,dx + \int_3^4 8x\,dx$$

$$= \left[\frac{7x^2}{2}+3x\right]_1^3 + \left[\frac{8x^2}{2}\right]_3^4$$

$$= \left[\left(\frac{7\times 9}{2}+3\times 3\right)-\left(\frac{7\times 1}{2}+3\times 1\right)\right]+\left[\left(\frac{8\times 16}{2}-\frac{8\times 9}{2}\right)\right]$$

$$= \left[\frac{63}{2}+9-\frac{7}{2}-3\right]+\left[64-36\right]$$

$$= 34+28$$

$$= 62$$

Definite Integrals Ex 20.3 Q2

We have,

$$\int_{-4}^4 |x+2|\,dx$$

$$= \int_{-4}^{-2} -(x+2)\,dx + \int_{-2}^{4} (x+2)\,dx$$

$$= -\left[\frac{x^2}{2}+2x\right]_{-4}^{-2} + \left[\frac{x^2}{2}+2x\right]_{-2}^{4}$$

$$= -\left[\left(\frac{4}{2}-4\right)-\left(\frac{16}{2}-8\right)\right]+\left[\left(\frac{16}{2}+8\right)-\left(\frac{4}{2}-4\right)\right]$$

$$= -\left[(-2)-(0)\right]+\left[(16)-(-2)\right]$$

$$= -\left[-2\right]+\left[16+2\right]$$

$$= 2-18$$

$$= 20$$

$$\therefore \int_{-4}^4 |x+2|\,dx = 20$$

Definite Integrals Ex 20.3 Q3

We have,

$$\int_{-3}^{3} |x+1| dx$$

$$= \int_{-3}^{-1} -(x+1) dx + \int_{-1}^{3} (x+1) dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-3}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{3}$$

$$= -\left[\left(\frac{1}{2}-1\right)-\left(\frac{9}{2}-3\right)\right]+\left[\left(\frac{9}{2}+3\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}\right)-\left(1\frac{1}{2}\right)\right]+\left[\left(7\frac{1}{2}\right)-\left(-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{2}-1\frac{1}{2}\right]+\left[7\frac{1}{2}+\frac{1}{2}\right]$$

$$= [-2]+[8]$$

$$= 2+8$$

$$= 10$$

$$\therefore \int_{-3}^{3} |x+1| dx = 10$$

Definite Integrals Ex 20.3 Q4

We have,

$$\int_{-1}^{1} |2x+1| dx$$

$$= \int_{-1}^{-\frac{1}{2}} -(2x+1) dx + \int_{-\frac{1}{2}}^{1} (2x+1) dx$$

$$= -\left[\frac{2x^2}{2}+x\right]_{-1}^{-\frac{1}{2}} + \left[\frac{2x^2}{2}+x\right]_{-\frac{1}{2}}^{1}$$

$$= -\left[\left(\frac{2}{8}-\frac{1}{2}\right)-\left(\frac{2}{2}-1\right)\right]+\left[\left(\frac{2}{2}+1\right)-\left(\frac{2}{8}-\frac{1}{2}\right)\right]$$

$$= -\left[\left(\frac{1}{4}-\frac{1}{2}\right)-(1-1)\right]+\left[(1+1)-\left(\frac{1}{4}-\frac{1}{2}\right)\right]$$

$$= -\left[-\frac{1}{4}\right]+\left[2+\frac{1}{4}\right]$$

$$= \frac{1}{4}+2+\frac{1}{4}$$

$$= 2\frac{1}{2}$$

$$\therefore \int_{-1}^{1} |2x+1| dx = \frac{5}{2}$$

Definite Integrals Ex 20.3 Q5

(i)

$$\int_{-2}^{2}|2x+3|dx$$

$$= \int_{-2}^{-\frac{3}{2}} -(2x+3)\,dx + \int_{-\frac{3}{2}}^{2} (2x+3)\,dx$$

$$= -\left[\frac{2x^2}{2}+3x\right]_{-2}^{-\frac{3}{2}} + \left[\frac{2x^2}{2}+3x\right]_{-\frac{3}{2}}^{2}$$

$$= -\left[\left(\frac{2\times 9}{2\times 4}-\frac{9}{2}\right)-\left(\frac{2\times 4}{2}-6\right)\right]+\left[\left(\frac{2\times 4}{2}+6\right)-\left(\frac{2\times 9}{2\times 4}-\frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{18}{8}-\frac{9}{2}\right)-\left(\frac{8}{2}-6\right)\right]+\left[\left(\frac{8}{2}+6\right)-\left(\frac{18}{8}-\frac{9}{2}\right)\right]$$

$$= -\left[\left(\frac{9}{4}-\frac{9}{2}\right)-(-2)\right]+\left[(10)-\left(\frac{9}{4}-\frac{9}{2}\right)\right]$$

$$= \left[-\frac{9}{4}+2\right]+\left[10+\frac{9}{4}\right]$$

$$= \frac{9}{4}-2+10+\frac{9}{4}$$

$$\Rightarrow 8\frac{9}{2}$$

$$= 12\frac{1}{2}$$

$$\therefore \int_{-2}^{2}|2x+3|dx = \frac{25}{2}$$

Definite Integrals Ex 20.3 Q6

(ii)

We have,

$$f(x) = |x^2 - 3x + 2|$$

$$= |(x-1)(x-2)|$$

$$= \begin{cases} x^2 - 3x + 2 & 0 \le x \le 1 \\ -(x^2 - 3x + 2) & 1 \le x \le 2 \end{cases}$$

Hence,

$$\int_0^2 |x^2 - 3x + 2|\,dx$$

$$= \int_0^1 (x^2 - 3x + 2)\,dx + \int_1^2 -(x^2 - 3x + 2)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right]_1^2$$

$$= \left[\frac{1}{3} - \frac{3}{2} + 2 - 0\right] - \left[\frac{8}{3} - \frac{12}{2} + 4 - \frac{1}{3} + \frac{3}{2} + 2\right]$$

$$= \left[\frac{1}{6}\right] - \left[-\frac{5}{6}\right]$$

$$= \frac{1}{6} + \frac{5}{6}$$

$$= 1$$

$$\therefore \int_0^2 |x^2 - 3x + 2|\,dx = 1$$

Definite Integrals Ex 20.3 Q7

$$\int_0^3 |3x-1|\,dx = \int_0^{\frac{1}{3}} -(3x-1)\,dx + \int_{\frac{1}{3}}^3 (3x-1)\,dx$$

$$= -\left[\frac{3x^2}{2} - x\right]_0^{\frac{1}{3}} + \left[\frac{3x^2}{2} - x\right]_{\frac{1}{3}}^3$$

$$= -\left[\left(\frac{3}{9\times 2} - \frac{1}{3}\right) - (0)\right] + \left[\left(\frac{3\times 9}{2} - 3\right) - \left(\frac{3}{9\times 2} - \frac{1}{3}\right)\right]$$

$$= -\left[\left(\frac{1}{6} - \frac{1}{3}\right)\right] + \left[\left(\frac{27}{2} - 3\right) - \left(\frac{1}{6} - \frac{1}{3}\right)\right]$$

$$= -\left[\left(-\frac{1}{6}\right)\right] + \left[\left(10\frac{1}{2}\right) - \left(\frac{-1}{6}\right)\right]$$

$$= -\left[\left(-\frac{1}{6}\right)\right] + \left[10\frac{1}{2} + \frac{1}{6}\right]$$

$$= \frac{1}{6} + 10\frac{1}{2} + \frac{1}{6}$$

$$= \frac{1}{3} + \frac{21}{2} = \frac{2+63}{6} = \frac{65}{6}$$

$$= \frac{65}{6}$$

$$\therefore \int_0^3 |3x-1|\,dx = \frac{65}{6}$$

Definite Integrals Ex 20.3 Q8

$$\int_{-6}^{6} |x+2|\,dx$$

$$= \int_{-6}^{-2} -(x+2)\,dx + \int_{-2}^{6} (x+2)\,dx$$

$$= -\left[\frac{x^2}{2} + 2x\right]_{-6}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{6}$$

$$= -\left[\left(\frac{4}{2} + 2(-2)\right) - \left(\frac{36}{2} - 12\right)\right] + \left[\left(\frac{36}{2} + 12\right) - \left(\frac{4}{2} - 4\right)\right]$$

$$= -\left[(2-4) - (18-12)\right] + \left[(18+12) - (2-4)\right]$$

$$= -[-8] + [30+2]$$

$$= 8 + 32$$

$$= 40$$

$$\therefore \int_{-6}^{6} |x+2|\,dx = 40$$

Definite Integrals Ex 20.3 Q9

$$\int_{-2}^{2} |x+1| dx = \int_{-2}^{-1} -(x+1) dx + \int_{-1}^{2} (x+1) dx$$

$$= -\left[\frac{x^2}{2}+x\right]_{-2}^{-1} + \left[\frac{x^2}{2}+x\right]_{-1}^{2}$$

$$= -\left[\left(\frac{1}{2}-1\right)-\left(\frac{4}{2}-2\right)\right]+\left[\left(\frac{4}{2}+2\right)-\left(\frac{1}{2}-1\right)\right]$$

$$= -\left[\left(-\frac{1}{2}\right)-0\right]+\left[4+\frac{1}{2}\right]$$

$$= \frac{1}{2}+4\frac{1}{2}$$

$$= 5$$

$$\therefore \int_{-2}^{2} |x+1| dx = 5$$

Definite Integrals Ex 20.3 Q10

$$\int_{1}^{2} |x-3| dx = \int_{1}^{2} -(x-3) dx \qquad [x-3<0 \text{ for } 1>x>2]$$

$$= -\left[\frac{x^2}{2}-3x\right]_{1}^{2}$$

$$= -\left[\left(\frac{4}{2}-6\right)-\left(\frac{1}{2}-3\right)\right]$$

$$= -\left[(-4)-\left(-2\frac{1}{2}\right)\right]$$

$$= -\left[-4+2\frac{1}{2}\right]$$

$$= -\left[-\frac{3}{2}\right]$$

$$= \frac{3}{2}$$

$$\therefore \int_{1}^{2} |x-3| dx = \frac{3}{2}$$

Definite Integrals Ex 20.3 Q11

$$\int_0^{\frac{\pi}{2}} \left|\cos 2x\right| dx$$

$$= \int_0^{\frac{\pi}{4}} -\cos 2x\, dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} +\cos 2x\, dx$$

$$= \left[\frac{+\sin 2x}{2}\right]_0^{\frac{\pi}{4}} + \left[\frac{-\sin 2x}{2}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\sin\frac{\pi}{2} - \sin 0\right] + \frac{1}{2}\left[\sin\pi + \sin\frac{\pi}{2}\right]$$

$$= \frac{1}{2}[1] + \frac{1}{2}[1]$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$\therefore \int_0^{\frac{\pi}{2}} \left|\cos 2x\right| dx = 1$$

Definite Integrals Ex 20.3 Q12

$$\int_0^{2\pi} |\sin x| dx = \int_0^{\pi} \sin x dx + \int_{\pi}^{2\pi} -\sin x dx$$

$$= [-\cos x]_0^{\pi} + [\cos x]_{\pi}^{2\pi}$$

$$= [1+1] + [1+1]$$

$$\int_0^{2\pi} |\sin x| dx = 4$$

Definite Integrals Ex 20.3 Q13

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x|\,dx$$

$$= \int_{-\frac{\pi}{4}}^{0} -\sin x\,dx + \int_{0}^{\frac{\pi}{4}} \sin x\,dx$$

$$= [\cos x]_{-\frac{\pi}{4}}^{0} + [-\cos x]_{0}^{\frac{\pi}{4}}$$

$$= \left(1 - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= \left(2 - \sqrt{2}\right)$$

$$\therefore \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x|\,dx = 2 - \sqrt{2}$$

Definite Integrals Ex 20.3 Q14

We have,

$$I = \int_{2}^{8} |x - 5|\,dx$$

We have,

$$|x - 5| = \begin{cases} x - 5 & \text{if } x \in (5, 8) \\ -(x - 5) & \text{if } x \in (2, 5) \end{cases}$$

Hence,

$$I = \int_{2}^{5} -(x - 5)\,dx + \int_{5}^{8} (x - 5)\,dx$$

$$= -\left[\frac{x^2}{2} - 5x\right]_{2}^{5} + \left[\frac{x^2}{2} - 5x\right]_{5}^{8}$$

$$= -\left[\left(\frac{25}{2} - 25\right) - \left(\frac{4}{2} - 10\right)\right] + \left[\left(\frac{64}{2} - 40\right) - \left(\frac{25}{2} - 25\right)\right]$$

$$= -\left[-\frac{25}{2} + 8\right] + \left[(-8) + \left(\frac{25}{2}\right)\right]$$

$$= \frac{25}{2} - 8 - 8 + \frac{25}{2}$$

$$= 25 - 16 = 9$$

$$\therefore \int_{2}^{8} |x - 5|\,dx = 9$$

Definite Integrals Ex 20.3 Q15

We have,

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx$$

Let $f(x) = \sin|x| + \cos|x|$

Then, $f(x) = f(-x)$

$\therefore$ f(x) is an even function.

So, $$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx = 2\int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = 2[-\cos x + \sin x]_0^{\frac{\pi}{2}} = 4$$

$$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \{\sin|x| + \cos|x|\} dx = 4$$

Definite Integrals Ex 20.3 Q16

$$I = \int_0^4 |x-1| dx$$

It can be seen that, $(x - 1) \leq 0$ when $0 \leq x \leq 1$ and $(x - 1) \geq 0$ when $1 \leq x \leq 4$

$$I = \int_0^1 |x-1| dx + \int_1^4 |x-1| dx \qquad \left(\int_a^b f(x) = \int_a^c f(x) + \int_c^b f(x)\right)$$

$$= \int_0^1 -(x-1) dx + \int_1^4 (x-1) dx$$

$$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$$

$$= 1 - \frac{1}{2} + \frac{(4)^2}{2} - 4 - \frac{1}{2} + 1$$

$$= 1 - \frac{1}{2} + 8 - 4 - \frac{1}{2} + 1$$

$$= 5$$

Definite Integrals Ex 20.3 Q17

$$Let\ I = \int_1^4 \{|x-1|+|x-2|+|x-4|\}dx$$

$$= \int_1^2 \{(x-1)-(x-2)-(x-4)\}dx + \int_2^4 \{(x-1)+(x-2)-(x-4)\}dx$$

$$= \int_1^2 \{(x-1-x+2-x+4)\}dx + \int_2^4 \{(x-1+x-2-x+4)\}dx$$

$$= \int_1^2 (5-x)dx + \int_2^4 (x+1)dx$$

$$= \left[5x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} + x\right]_2^4$$

$$= \left[10-2-5+\frac{1}{2}\right] + [8+4-2-2]$$

$$= \frac{7}{2} + 8$$

$$I = \frac{23}{2}$$

Definite Integrals Ex 20.3 Q18

We have,

$$I = \int_{-5}^{0} (|x| + |x+2| + |x+5|)\, dx = \int_{-5}^{0} |x|dx + \int_{-5}^{0} |x+2|dx + \int_{-5}^{0} |x+5|dx$$

$$\Rightarrow I = \int_{-5}^{0} -x\, dx + \int_{-5}^{-2} -(x+2)\, dx + \int_{-2}^{0} (x+2)\, dx + \int_{-5}^{0} (x+5)\, dx$$

$$= \left[\frac{-x^2}{2}\right]_{-5}^{0} + \left[\frac{-x^2}{2} - 2x\right]_{-5}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{0} + \left[\frac{x^2}{2} + 5x\right]_{-5}^{0}$$

$$= \left[+\frac{25}{2}\right] - \left[\frac{4}{2} - 4 - \frac{25}{2} + 10\right] + \left[0 + 0 - \frac{4}{2} + 4\right] + \left[0 + 0 - \frac{25}{2} + 25\right]$$

$$= \frac{25}{2} - \left[8 - \frac{25}{2}\right] + [2] + \left[25 - \frac{25}{2}\right]$$

$$= \frac{25}{2} - 8 + \frac{25}{2} + 2 + 25 - \frac{25}{2}$$

$$= 19 + \frac{25}{2} = 31\frac{1}{2}$$

$$I = \frac{63}{2}$$

Definite Integrals Ex 20.3 Q19

$$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x<0 \end{cases}$$

$$|x-2| = \begin{cases} x-2, & x \ge 2 \\ 2-x, & x<2 \end{cases}$$

$$|x-4| = \begin{cases} x-4, & x \ge 4 \\ 4-x, & x < 4 \end{cases}$$

Splitting the limits of the integral, we get

$$\int_0^4 \left(|x|+|x-2|+|x-4|\right)dx$$

$$= \int_0^2 \left(|x|+|x-2|+|x-4|\right)dx + \int_2^4 \left(|x|+|x-2|+|x-4|\right)dx$$

$$= \int_0^2 (x+2-x+4-x)\,dx + \int_2^4 (x+x-2+4-x)\,dx$$

$$= \int_0^2 (6-x)\,dx + \int_2^4 (2+x)\,dx$$

$$= \left[6x - \frac{x^2}{2}\right]_0^2 + \left[2x + \frac{x^2}{2}\right]_2^4$$

$$= [12-2]+[16-6]$$

$$= 10+10$$

$$= 20$$

Definite Integrals Ex 20.3 Q20

$$\int_{-1}^{2} |x+1|\,dx + \int_{-1}^{2} |x|\,dx + \int_{-1}^{2} |x-1|\,dx$$

$$\int_{-1}^{2} (x+1)dx - \int_{-1}^{0} xdx + \int_{0}^{2} xdx - \int_{-1}^{1} (x-1)dx + \int_{1}^{2} (x-1)dx$$

$$\left\{\frac{x^2}{2}+x\right\}_{-1}^{2} - \left\{\frac{x^2}{2}\right\}_{-1}^{0} + \left\{\frac{x^2}{2}\right\}_{0}^{2} - \left\{\frac{x^2}{2}-x\right\}_{-1}^{1} + \left\{\frac{x^2}{2}-x\right\}_{1}^{2}$$

$$\left\{(4)-(-\frac{1}{2})\right\} - \left\{-\frac{1}{2}\right\} + \{2\} - \left\{(-\frac{1}{2})-(\frac{3}{2})\right\} + \left\{(0)-(-\frac{1}{2})\right\}$$

$$\left\{4+\frac{1}{2}\right\} + \left\{\frac{1}{2}\right\} + \{2\} + \{2\} + \left\{\frac{1}{2}\right\}$$

$$\frac{19}{2}$$

Definite Integrals Ex 20.3 Q21

$$\int_{-2}^{0} xe^{-x}dx + \int_{0}^{2} xe^{x}dx$$

For

$$\int_{-2}^{0} xe^{-x}dx$$

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = e^{-x}, g = x$$

$$f = -e^{-x}, g' = 1$$

$$\int_{-2}^{0} xe^{-x}dx = \left\{-xe^{-x}\right\}_{-2}^{0} + \int_{-2}^{0} e^{-x}dx$$

$$\int_{-2}^{0} xe^{-x}dx = \left\{-xe^{-x} - e^{-x}\right\}_{-2}^{0}$$

$$\int_{-2}^{0} xe^{-x}dx = \left\{(-1) - (2e^{2} - e^{2})\right\}$$

$$\int_{-2}^{0} xe^{-x}dx = \left\{-1 - e^{2}\right\}$$

For

$$\int_{0}^{2} xe^{x}dx$$

Using Integration By parts

$$\int f'g = fg - \int fg'$$

$$f' = e^{x}, g = x$$

$$f = e^{x}, g' = 1$$

$$\int_{0}^{2} xe^{x}dx = \left\{xe^{x}\right\}_{0}^{2} - \int_{0}^{2} e^{x}dx$$

$$\int_{0}^{2} xe^{x}dx = \left\{xe^{x} - e^{x}\right\}_{0}^{2}$$

$$\int_{0}^{2} xe^{x}dx = 2e^{2} - e^{2} + 1$$

$$\int_{0}^{2} xe^{x}dx = e^{2} + 1$$

Hence answer is,

$$\int_{-2}^{2} xe^{|x|}dx = -1 - e^{2} + e^{2} + 1 = 0$$

Definite Integrals Ex 20.3 Q22

$$-\int_{-\frac{\pi}{4}}^{0} \sin^2 x dx + \int_{0}^{\frac{\pi}{2}} \sin^2 x dx$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$-\int_{-\frac{\pi}{4}}^{0} \frac{1-\cos 2x}{2} dx + \int_{0}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$$

$$-\frac{1}{2}\left\{x - \frac{\sin 2x}{2}\right\}_{-\frac{\pi}{4}}^{0} + \frac{1}{2}\left\{x - \frac{\sin 2x}{2}\right\}_{0}^{\frac{\pi}{2}}$$

$$-\frac{1}{2}\left\{-(-\frac{\Pi}{4} + \frac{1}{2})\right\} + \frac{1}{2}\left\{\frac{\Pi}{2}\right\}$$

$$\left\{-\frac{\Pi}{8} + \frac{1}{4}\right\} + \left\{\frac{\Pi}{4}\right\}$$

$$\frac{\Pi}{8} + \frac{1}{4}$$

$$\frac{\Pi + 2}{8}$$

Definite Integrals Ex 20.3 Q23

$$\int_{0}^{\frac{\pi}{2}} \cos^2 x dx - \int_{\frac{\pi}{2}}^{\pi} \cos^2 x dx$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\int_{0}^{\frac{\pi}{2}} \frac{1+\cos 2x}{2} dx - \int_{\frac{\pi}{2}}^{\pi} \frac{1+\cos 2x}{2} dx$$

$$\frac{1}{2}\left\{x + \frac{\sin 2x}{2}\right\}_{0}^{\frac{\pi}{2}} - \frac{1}{2}\left\{x + \frac{\sin 2x}{2}\right\}_{\frac{\pi}{2}}^{\pi}$$

$$\frac{\Pi}{4} - \frac{\Pi}{4}$$

$$0$$

Definite Integrals Ex 20.3 Q24

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{2}} (2\sin|x| + \cos|x|)\,dx$$

$$= \int_{-\frac{\pi}{4}}^{0} (-2\sin x + \cos x)\,dx + \int_{0}^{\frac{\pi}{2}} (2\sin x + \cos x)\,dx$$

$$= \left[2\cos x + \sin x\right]_{-\frac{\pi}{4}}^{0} + \left[-2\cos x + \sin x\right]_{0}^{\frac{\pi}{2}}$$

$$= 2 + 0 - 0 + 1 + 0 + 1 + 2 - 0$$

$$= 6$$

Definite Integrals Ex 20.3 Q25

$$\int_{-\frac{\Pi}{2}}^{\Pi} \sin^{-1}(\sin x)\,dx = \int_{-\frac{\Pi}{2}}^{\frac{\Pi}{2}} x\,dx + \int_{\frac{\Pi}{2}}^{\Pi} (\Pi - x)\,dx$$

$$\Rightarrow \left\{\frac{x^2}{2}\right\}_{-\frac{\Pi}{2}}^{\frac{\Pi}{2}} + \left\{\Pi x - \frac{x^2}{2}\right\}_{\frac{\Pi}{2}}^{\Pi}$$

$$\Rightarrow \left\{(\Pi^2 - \frac{\Pi^2}{2}) - \left(\frac{\Pi^2}{2} - \frac{\Pi^2}{8}\right)\right\}$$

$$\Rightarrow \left\{\frac{\Pi^2}{2} - \frac{3\Pi^2}{8}\right\}$$

$$\Rightarrow \frac{\Pi^2}{8}$$

Definite Integrals Ex 20.3 Q27

[x]=0 for 0
and [x]=1 for 1
Hence

$$\int_0^1 0 + \int_1^2 2x\,dx$$

$$\left\{x^2\right\}_1^2$$

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Definite Integrals Ex 20.3 Q18

$$\int_0^{2\pi} \cos^{-1}(\cos x)\,dx$$

$$= -\int_0^{\pi} \cos^{-1}(\cos x)\,dx + \int_{\pi}^{2\pi} \cos^{-1}(\cos x)\,dx$$

$$= -\int_0^{\pi} x\,dx + \int_{\pi}^{2\pi} x\,dx$$

$$= -\left[\frac{x^2}{2}\right]_0^{\pi} + \left[\frac{x^2}{2}\right]_{\pi}^{2\pi}$$

$$= -\frac{\pi^2}{2} + \frac{4\pi^2}{2} - \frac{\pi^2}{2}$$

$$= \pi^2$$

Definite Integrals Ex 20.3 Q33

Let $I = \int_a^b \frac{f(x)}{f(x)+f(a+b-x)}dx \qquad --(i)$

We know that $\int_a^b f(x) = \int_a^b f(a+b-x)dx$

Then

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x)+f\{a+b-(a+b-x)\}}dx$$

$$I = \int_a^b \frac{f(a+b-x)}{f(a+b-x)f(x)}dx \qquad --(ii)$$

Adding (i) & (ii)

$$2I = \int_a^b \frac{f(x)+f(a+b-x)}{f(x)+f(a+b-x)}dx$$

$$2I = \int_a^b dx$$

$$I = [x]_a^b$$

$$I = \frac{1}{2}[b-a]$$

$$I = \frac{b-a}{2}$$