RD Sharma Solutions Class 12 Maths Chapter 2 Ex 2.1

Functions Ex 2.1 Q1(i)

Example of a function which is one-one but not only.

let $f : N \to N$ given by $f(x) = x^2$

Check for injectivity:

let $x, y \in N$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^2 = y^2$

$\Rightarrow \quad (x - y)(x + y) = 0 \qquad [\because x, y \in N \Rightarrow x + y > 0]$

$\Rightarrow \quad x - y = 0$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is one-one

Surjectivity: let $y \in N$ be arbitrary, then

$f(x) = y$

$\Rightarrow \quad x^2 = y$

$\Rightarrow \quad x = \sqrt{y} \notin N$ for non-perfect square value of y.

$\therefore$ No non-perfect square value of y has a pre image in domain N.

$\therefore$ $f : N \to N$ given by $f(x) = x^2$ is one-one but not onto.

Functions Ex 2.1 Q1(ii)

Example of a function which is onto but not one-one.

let $f : R \to R$ defined by $f(x) = x^3 - x$

Check for injectivity:

let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad x^3 - x = y^3 - y$

$\Rightarrow \quad x^3 - y^3 - (x - y) = 0$

$\Rightarrow \quad (x - y)(x^2 + xy + y^2 - 1) = 0$

$\because \quad x^2 + xy + y^2 \geq 0 \quad \Rightarrow \quad x^2 + xy + y^2 - 1 \geq -1$

$\therefore \quad x \neq y$ for some $x, y \in R$

$\therefore \quad f$ is not one-one.

Surjectivity: let $y \in R$ be arbitrary

then, $f(x) = y$

$\Rightarrow \quad x^3 - x = y$

$\Rightarrow \quad x^3 - x - y = 0$

we know that a degree 3 equation has a real root.

let $x = \alpha$ be that root

$\therefore \quad \alpha^3 - \alpha = y$

$\Rightarrow \quad f(\alpha) = y$

Thus for clearly $y \in R$. there exist $\alpha \in R$ such that $f(x) = y$

$\therefore \quad f$ is onto

$\therefore$ Hence $f : R \to R$ defined by $f(x) = x^3 - x$ is not one-one but onto.

Functions Ex 2.1 Q1(iii)

Example of a function which is neither one-one nor onto.

let $f : R \to R$ defined by $f(x) = 2$

We know that a constant function in neither one-one nor onto

Here $f(x) = 2$ is a constant function

$\therefore \quad f : R \to R$ defined by $f(x) = 2$ is neither one-one nor onto.

Functions Ex 2.1 Q2

i) $f_1 = \{(1,3),(2,5),(3,7)\}$

$A = \{1,2,3\}$, $B = \{3,5,7\}$

We can earily observe that in f_1 every element of A has different image from B.

$\therefore$ f_1 in one-one

also, each element of B is the image of some element of A.

$\therefore$ f_1 in onto.

ii)

$f_2 = \{(2,a),(3,b),(4,c)\}$

$A = \{2,3,4\}$ $B = \{a,b,c\}$

It in clear that different elements of A have different images in B

$\therefore$ f_2 in one-one

Again, each element of B is the image of some element of A.

$\therefore$ f_2 in onto

iii) $f_3 = \{(a,x),(b,x),(c,z)(d,z)\}$

$A = \{a,b,c,d\}$ $B = \{x,y,z\}$

Since, $f_3(a) = x = f_3(b)$ and $f_3(c) = z = f_3(d)$

$\therefore$ f_3 in not one-one

Again, $y \in B$ in not the image of any of the element of A

$\therefore$ f_3 in not onto

Functions Ex 2.1 Q3

We have, $f : N \to N$ defined by $f(x) = x^2 + x + 1$

Check for injectivity:

Let $x, y \in N$ such that

$$f(x) = f(y)$$
$$\Rightarrow \quad x^2 + x + 1 = y^2 + y + 1$$
$$\Rightarrow \quad x^2 - y^2 + x - y = 0$$
$$\Rightarrow \quad (x - y)(x + y + 1) = 0$$
$$\Rightarrow \quad x - y = 0 \qquad [\because x, y \in N \Rightarrow x + y + 1 > 0]$$
$$\Rightarrow \quad x = y$$

$\therefore$ f is one-one.

Surjectivity:

Let $y \in N$, then
$$f(x) = y$$
$$\Rightarrow \quad x^2 + x + 1 - y = 0$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1 - 4(1 - y)}}{2} \notin N \quad \text{for } y > 1$$

$\therefore$ for $y > 1$, we do not have any pre-image in domain N.

$\therefore$ f is not onto.

Functions Ex 2.1 Q4.

We have, $A = \{-1, 0, 1\}$ and $f : A \to A$

defined by $f = \{(x, x^2) : x \in A\}$

clearly $f(1) = 1$ and $f(-1) = 1$

$\therefore$ $f(1) = f(-1)$

$\therefore$ f is not one-one

Again $y = -1 \in A$ in the co-domain does not have any pre image in domain A.

$\therefore$ f is not onto.

Functions Ex 2.1 Q5(i)

$f : N \to N$ given by $f(x) = x^2$

let $\quad x_1 = x_2 \quad$ for $\quad x_1, x_2 \in N$

$\Rightarrow \quad x_1^2 = x_2^2 \quad \Rightarrow \quad f(x_1) = f(x_2)$

$\therefore \quad f$ in one-one.

Surjectivity: Since f takes only square value like 1,4,9,16......
so, non-perfect square values in N (co-domain) do not have pre image in domain N.
Thus, f is not onto.

Functions Ex 2.1 Q5(ii)

$f : Z \to Z$ given by $f(x) = x^2$

Injectivity: let $x_1 \,\&\, -x_1 \in Z$

$\Rightarrow \quad x_1 \neq -x_1$

$\Rightarrow \quad x_1^2 = (-x_1)^2 \quad \Rightarrow \quad f(x_1) = f(-x_1)$

$\Rightarrow \quad f$ is not one-one.

Surjective: Again, f takes only square values 1,4,9,16,...
So, no non-perfect square values in Z have a pre image in domain Z.
$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(iii)

$f : N \to N$, given by $f(x) = x^3$

Injectivity: let $y, x \in N$ such that

$x = y$

$\Rightarrow \quad x^3 = y^3$

$\Rightarrow \quad f(x) = f(y)$

$\therefore \quad f$ is one-one

Surjective:

$\because \; f$ attain only cubic number like 1,8,27,64,...
So, no non-cubic values of N (co-domain) have pre image in N (Domain)
$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(iv)

$f : Z \to Z \quad$ given by $\quad f(x) = x^3$

Injectivity: let $x, y \in Z$ such that

$x = y$

$\Rightarrow \quad x^3 = y^3$

$\Rightarrow \quad f(x) = f(y)$

$\Rightarrow \quad f(x) = f(y)$

$\Rightarrow \quad f$ is one-one.

Surjective: Since f attains only cubic values like $\pm 1, \pm 8, \pm 27, \ldots\ldots\ldots\ldots$
so, no non-cubic values of Z (co-domain) have pre image in Z (domain)
$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(v)

$f: R \to R$ given by $f(x) = |x|$

Injectivity: let $x, y \in R$ such that

$x = y$ but if $y = -x$

$\Rightarrow$ $|x| = |y|$ $\Rightarrow$ $|y| = |-x| = x$

$\therefore$ f is not one-one.

Surjective: Since f attains only positive values, for negative real numbers in R, there is no pre-image in domain R.

$\therefore$ f is not onto.

Functions Ex 2.1 Q5(vi)

$f: Z \to Z$ given by $f(x) = x^2 + x$

Injective: let $x, y \in Z$ such that

$f(x) = f(y)$

$\Rightarrow$ $x^2 + x = y^2 + y$

$\Rightarrow$ $x^2 - y^2 + x - y = 0$

$\Rightarrow$ $(x - y)(x + y + 1) = 0$

$\Rightarrow$ either $x - y = 0$ or $x + y + 1 = 0$

Case I: if $x - y = 0$

$\Rightarrow$ $x = y$

$\therefore$ f is injective

Case II if $x + y + 1 = 0$

$\Rightarrow$ $x + y = -1$

$\Rightarrow$ $x \neq y$

$\therefore$ f is not one to one

Thus, in general, f is not one-one

Surjective:

Since $1 \in Z$ (co-domain)

Now, we wish to find if there is any pre-image in domain Z.

let $x \in Z$ such that $f(x) = 1$

$\Rightarrow$ $x^2 + x = 1$ $\Rightarrow$ $x^2 + x - 1 = 0$

$\Rightarrow$ $x = \dfrac{-1 \pm \sqrt{1 + 4}}{2} \notin Z.$

So, f is not onto.

Functions Ex 2.1 Q5(viI)

$f : Z \to Z$ given by $f(x) = x - 5$

Injective: let $x, y \in Z$ such that

$f(x) = f(y)$

$\Rightarrow \quad x - 5 = y - 5$

$\Rightarrow \quad x = y$

$\therefore \quad f$ is one-one.

Surjective: let $y \in Z$ be an arbitrary element

then $\quad f(x) = y$

$\Rightarrow \quad x - 5 = y$

$\Rightarrow \quad x = y + 5 \quad \in Z$ (domain)

Thus, for each element in co-domain Z there exists an element in domain Z such that $f(x) = y$

$\therefore \quad f$ in onto.

Since, f in one-one and onto,

$\therefore \quad f$ in bijective.

Functions Ex 2.1 Q5(viii)

$f : R \to R$ given by $f(x) = \sin x$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad \sin x = \sin y$

$\Rightarrow \quad x = n\pi + (-1)^n y$

$\Rightarrow \quad x \neq y$

$\therefore \quad f$ is not one-one.

Surjective: let $y \in R$ be arbitrary such that

$f(x) = y$

$\Rightarrow \quad \sin x = y$

$\Rightarrow \quad x = \sin^{-1} y$

Now, for $y > 1$ $x \notin R$ (domain)

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q5(ix)

$f : R \to R$ difined by $f(x) = x^3 + 1$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow$ $x^3 + 1 = y^3 + 1$

$\Rightarrow$ $x^3 = y^3$

$\Rightarrow$ $x = y$

$\therefore$ f is one-one.

Surjective:

let $y \in R$, then

$f(x) = y$

$\Rightarrow$ $x^3 + 1 = y \Rightarrow x^3 + 1 - y = 0$

We know that degree 3 equation has atleast one real root.

$\therefore$ let $x = \alpha$ be the real root.

$\therefore$ $\alpha^3 + 1 = y$

$\Rightarrow$ $f(\alpha) = y$

Thus, for each $y \in R$, there exist $\alpha \in R$ such that $f(\alpha) = y$

$\therefore$ f is onto.

Since f is one-one and onto, f is bijective.

Functions Ex 2.1 Q5(x)

$f : R \to R$ defined by $f(x) = x^3 - x$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow x^3 - x = y^3 - y$

$\Rightarrow x^3 - y^3 - (x - y) = 0$

$\Rightarrow (x - y)(x^2 + xy + y^2 - 1) = 0$

$\because x^2 + xy + y^2 \geq 0 \Rightarrow x^2 + xy + y^2 - 1 \geq -1$

$\therefore x^2 + xy + y^2 - 1 \neq 0$

$\Rightarrow x - y = 0 \Rightarrow x = y$

$\therefore$ f is one-one.

Surjective:

let $y \in R$, then

$f(x) = y$

$\Rightarrow x^3 - x - y = 0$

We know that a degree 3 equation has atleast one real solution.

let $x = \alpha$ be that real solution

$\therefore \alpha^3 - \alpha = y$

$\Rightarrow f(\alpha) = y$

$\therefore$ For each $y \in R$, there exist $x = \alpha \in R$ such that $f(\alpha) = y$

$\therefore$ f is onto.

Functions Ex 2.1 Q5(xi)

$f : R \to R$ defined by $f(x) = \sin^2 x + \cos^2 x$.

Injective: since $f(x) = \sin^2 x + \cos^2 x = 1$

$\Rightarrow$ $f(x) = 1$ which is a constant function we know that a constant function in neither injective nor surjective

$\therefore$ f is not one-one and not onto.

Functions Ex 2.1 Q5(xii)

$f : Q - [3] \to Q$ defined by $f(x) = \dfrac{2x+3}{x-3}$

Injective: let $x, y \in Q - [3]$ such that

$f(x) = f(y)$

$\Rightarrow \quad \dfrac{2x+3}{x-3} = \dfrac{2y+3}{y-3}$

$\Rightarrow \quad 2xy - 6x + 3y - 9 = 2xy + 3x - 6y - 9$

$\Rightarrow \quad -6x + 3y - 3x + 6y = 0$

$\Rightarrow \quad -9(x - y) = 0$

$\Rightarrow \quad x = y$

$\Rightarrow \quad f$ is one-one.

Surjective:

let $y \in Q$ be arbitrary. then

$f(x) = y$

$\Rightarrow \quad \dfrac{2x+3}{x-3} = y$

$\Rightarrow \quad 2x + 3 = xy - 3y$

$\Rightarrow \quad x(2 - y) = -3(y + 1)$

$\therefore \quad x = \dfrac{-3(y+1)}{2-y} \notin Q - [3]$ for $y = 2$

$\therefore \quad f$ is not onto

Functions Ex 2.1 Q5(xiii)

$f : Q \to Q$ defined by $f(x) = x^3 + 1$

Injective: let $x, y \in Q$ such that

$f(x) = f(y)$

$\Rightarrow$ $x^3 + 1 = y^3 + 1$

$\Rightarrow$ $(x^3 - y^3) = 0$

$\Rightarrow$ $(x - y)(x^2 + xy + y^2) = 0$

but $x^2 + xy + y^2 \geq 0$

$\therefore$ $x - y = 0$

$\Rightarrow$ $x = y$

$\therefore$ f is injective.

Surjective: let $y \in Q$ be arbitrary, then

$f(x) = y$

$\Rightarrow$ $x^3 + 1 - y = 0$

we know that a degree 3 equation has alteast one real solution.

let $x = \alpha$ be that solution

$\therefore$ $\alpha^3 + 1 = y$

$\therefore$ $f(\alpha) = y$

$\therefore$ f is onto.

Functions Ex 2.1 Q5(xiv)

$f : R \to R$ defined by $f(x) = 5x^3 + 4$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow$ $5x^3 + 4 = 5y^3 + 4$

$\Rightarrow$ $5(x^3 - y^3) = 0$

$\Rightarrow$ $5(x - y)(x^2 + xy + y^2) = 0$

but $5(x^2 + xy + y^2) \geq 0$

$\Rightarrow$ $x - y = 0 \Rightarrow x = y$

$\therefore$ f is one-one

Surjective: let $y \in R$ be arbitrary, then

$f(x) = y$

$\Rightarrow$ $5x^3 + 4 = y$

$\Rightarrow$ $5x^3 + 4 - y = 0$

we know that a degree 3 equation has alteast one real solution.

let $x = \alpha$ be that real solution

$\therefore$ $5\alpha^3 + 4 = y$

$\therefore$ $f(\alpha) = y$

$\therefore$ For each $y \in Q$, there $\alpha \in R$ such that $f(\alpha) = y$

$\therefore$ f is onto

Since f in one-one and onto

$\therefore$ f in bijective.

Functions Ex 2.1 Q5(xv)

$f : R \to R$ defined by $f(x) = 3 - 4x$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow$ $3 - 4x = 3 - 4y$

$\Rightarrow$ $-4(x - y) = 0$

$\Rightarrow$ $x = y$

$\therefore$ f is one-one.

Surjective: let $y \in R$ be arbitrary, such that

$f(x) = y$

$\Rightarrow$ $3 - 4x = y$

$\Rightarrow$ $x = \frac{3 - y}{4} \in R$

Thus for each $y \in R$, there exist $x \in R$ such that

$f(x) = y$

$\therefore$ f is onto.

Hence, f is one-one and onto and therefore bijective.

Functions Ex 2.1 Q5(xvi)

$f : R \to R$ defined by $f(x) = 1 + x^2$

Injective: let $x, y \in R$ such that

$f(x) = f(y)$

$\Rightarrow \quad 1 + x^2 = 1 + y^2$

$\Rightarrow \quad x^2 - y^2 = 0$

$\Rightarrow \quad (x - y)(x + y) = 0$

either $x = y$ or $x = -y$ or $x \neq y$

$\therefore \quad f$ is not one-one.

Surjective: let $y \in R$ be arbitrary, then

$f(x) = y$

$\Rightarrow \quad 1 + x^2 = y$

$\Rightarrow \quad x^2 + 1 - y = 0$

$\therefore \quad x = \pm\sqrt{y - 1} \notin R$ for $y < 1$

$\therefore \quad f$ is not onto.

Functions Ex 2.1 Q6

Given, $f : A \to B$ is injective such that range$(f) = \{a\}$

We know that in injective map different elements have different images.

$\therefore$ A has only one element.

Functions Ex 2.1 Q7

A = **R** − {3}, B = **R** − {1}

f: A → B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let $x, y \in$ A such that $f(x) = f(y)$.

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow (x-2)(y-3) = (y-2)(x-3)$

$\Rightarrow xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$\Rightarrow -3x - 2y = -3y - 2x$

$\Rightarrow 3x - 2x = 3y - 2y$

$\Rightarrow x = y$

Therefore, f is one-one.

Let $y \in B = \mathbf{R} - \{1\}$.

Then, $y \neq 1$.

The function f is onto if there exists $x \in A$ such that $f(x) = y$.

Now,

$$f(x) = y$$
$$\Rightarrow \frac{x-2}{x-3} = y$$
$$\Rightarrow x - 2 = xy - 3y$$
$$\Rightarrow x(1-y) = -3y + 2$$
$$\Rightarrow x = \frac{2-3y}{1-y} \in A \qquad [y \neq 1]$$

Thus, for any $y \in B$, there exists $\frac{2-3y}{1-y} \in A$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right) - 2}{\left(\frac{2-3y}{1-y}\right) - 3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y.$$

$\therefore f$ is onto.

Hence, function f is one-one and onto.

Functions Ex 2.1 Q8

We have $f : R \to R$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$\because f(x) = x - [x] \Rightarrow f(x) = 0$ for $x \in Z$

$\therefore$ Range of $f = [0, 1] \neq R$

$\therefore$ f is not one-one, where as many-one

Again, Range of $f = [0, 1] \neq R$

$\therefore$ f is an into function

Functions Ex 2.1 Q9

Suppose $f(n_1) = f(n_2)$
If n_1 is odd and n_2 is even, then we have
$n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2$, not possible
If n_1 is even and n_2 is odd, then we have
$n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2$, not possible
Therefore, both n_1 and n_2 must be either odd or even.
Suppose both n_1 and n_2 are odd.
Then, $f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$
Suppose both n_1 and n_2 are even.
Then, $f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$
Thus, f is one – one.
Also, any odd number $2r + 1$ in the co – domain **N** will have an even number as image in domain **N** which is
$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$
any even number $2r$ in the co – domain **N** will have an odd number as image in domain **N** which is
$f(n) = 2r \Rightarrow n + 1 = 2r \Rightarrow n = 2r - 1$
Thus, f is onto.

Functions Ex 2.1 Q10

We have $A = \{1, 2, 3\}$

All one-one functions from $A = \{1, 2, 3\}$ to itself are obtained by re-arranging elements of A.

Thus all possible one-one functions are:
i) $f(1) = 1, \quad f(2) = 2, \quad f(3) = 3$
ii) $f(1) = 2, \quad f(2) = 3, \quad f(3) = 1$
iii) $f(1) = 3, \quad f(2) = 1, \quad f(3) = 2$
iv) $f(1) = 1, \quad f(2) = 3, \quad f(3) = 2$
v) $f(1) = 3, \quad f(2) = 2, \quad f(3) = 1$
vi) $f(1) = 2, \quad f(2) = 1, \quad f(3) = 3$

Functions Ex 2.1 Q11

We have $f : R \to R$ given by $f(x) = 4x^3 + 7$
Let $x, y \in R$ such that
$$f(a) = f(b)$$
$$4a^3 + 7 = 4b^3 + 7$$
$$a = b$$
f is one-one.
Now let $y \in R$ be arbitrary, then
$$f(x) = y$$
$$4x^3 + 7 = y$$
$$x = (y - 7)^{\frac{1}{3}} \in R$$
f is onto.
Hence the function is a bijection

Functions Ex 2.1 Q12

We have $f : R \to R$ given by $f(x) = e^x$

let $x, y \in R$, such that

$$f(x) = f(y)$$

$\Rightarrow \quad e^x = e^y$

$\Rightarrow \quad e^{x-y} = 1 = e^0$

$\Rightarrow \quad x - y = 0$

$\Rightarrow \quad x = y$

$\therefore$ f is one-one

clearly range of $f = (0, \infty) \neq R$

$\therefore$ f is not onto

When co-domain in replaced by R_0^+ $i.e, (0, \infty)$ then f becomes an onto function.

Functions Ex 2.1 Q13

We have $f : R_0^+ \to R$ given by $f(x) = log_a x : a > 0$

let $x, y \in R_0^+$, such that

$f(x) = f(y)$

$\Rightarrow \quad log_a x = log_a y$

$\Rightarrow \quad log_a^x \left({}^{x}\!/\!{}_{y} \right) = 0$

$\Rightarrow \quad {}^{x}\!/\!{}_{y} = 1$

$\Rightarrow \quad x = y$

$\therefore$ f is one-one

Now, let $y \in R$ be arbitrany, then

$f(x) = y$

$\Rightarrow \quad log_a x = y \quad \Rightarrow \quad x = a^y \in R_0^+ \qquad \left[\because\ a > 0 \Rightarrow a^y > 0\right]$

Thus, for all $y \in R$, there exist $x = a^y$ such that $f(x) = y$

$\therefore$ f is onto

$\because$ f is one-one and onto $\quad \therefore$ f is bijective

Functions Ex 2.1 Q14

Since f is one-one, three elements of {1, 2, 3} must be taken to 3 different elements of the co-domain {1, 2, 3} under f.

Hence, f has to be onto.

Functions Ex 2.1 Q15

Suppose f is not one-one.

Then, there exists two elements, say 1 and 2 in the domain whose image in the co-domain is same.

Also, the image of 3 under f can be only one element.

Therefore, the range set can have at most two elements of the co-domain {1, 2, 3}

i.e f is not an onto function, a contradiction.

Hence, f must be one-one.

Functions Ex 2.1 Q16

Onto functions from the set {1, 2, 3, ... , n} to itself is simply a permutation on n symbols 1, 2, ..., n.

Thus, the total number of onto maps from {1, 2, ... , n} to itself is the same as the total number of permutations on n symbols 1, 2, ..., n, which is $n!$.

Functions Ex 2.1 Q17

Let $f_1 : R \to R$ and $f_2 : R \to R$ be two functions given by:

$$f_1(x) = x$$
$$f_2(x) = -x$$

We can earily verify that f_1 and f_2 are one-one functions.

Now,

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x - x = 0$$

$\therefore$ $f_1 + f_2 : R \to R$ is a function given by

$$(f_1 + f_2)(x) = 0$$

Since $f_1 + f_2$ is a constant function, it is not one-one.

Functions Ex 2.1 Q18

Let $f_1 : Z \to Z$ defined by $f_1(x) = x$ and

$f_2 : Z \to Z$ defined by $f_2(x) = -x$

Then f_1 *and* f_2 are surjective functions.

Now,

$f_1 + f_2 : Z \to Z$ is given by

$$(f_1 + f_2)(x) = f_1(x) + f_2(x) = x - x = 0$$

Since $f_1 + f_2$ is a constant function, it is not surjective.

Functions Ex 2.1 Q19

Let $f_1 : R \to R$ be defined by $f_1(x) = x$
and $f_2 : R \to R$ be defined by $f_2(x) = x$

clearly f_1 *and* f_2 are one-one functions.

Now,

$F = f_1 \times f_2 : R \to R$ *is* defined by

$F(X) = (f_1 \times f_2)(x) = f_1(x) \times f_2(x) = x^2$ .... (i)

Clearly, $F(-1) = 1 = F(1)$

$\therefore$ F is not one-one

Hence, $f_1 \times f_2 : R \to R$ is not one-one.

Functions Ex 2.1 Q20

Let $f_1 : R \to R$ and $f_2 : R \to R$ are two functions defined by $f_1(x) = x^3$ and

$f_2(x) = x$

clearly f_1 & f_2 are one-one functions.

Now,

$\frac{f_1}{f_2} : R \to R$ given by

$$\left(\frac{f_1}{f_2}\right)(x) = \frac{f_1(x)}{f_2(x)} = x^2 \text{ for all } x \in R.$$

let $\frac{f_1}{f_2} = f$

$\therefore$ $F = R \to R$ defined by $f(x) = x^2$

now, $F(1) = 1 = F(-1)$

$\therefore$ F is not one-one

$\therefore$ $\frac{f_1}{f_2} = R \to R$ is not one-one.

Functions Ex 2.1 Q22

We have $f : R \to R$ given by $f(x) = x - [x]$

Now,

check for injectivity:

$\because$ $f(x) = x - [x] \Rightarrow f(x) = 0$ for $x \in Z$

$\therefore$ Range of $f = [0, 1] \neq R$

$\therefore$ f is not one-one, where as many-one

Again, Range of $f = [0, 1] \neq R$

$\therefore$ f is an into function

Functions Ex 2.1 23

Suppose $f(n_1) = f(n_2)$

If n_1 is odd and n_2 is even, then we have

$n_1 + 1 = n_2 - 1 \Rightarrow n_2 - n_1 = 2$, not possible

If n_1 is even and n_2 is odd, then we have

$n_1 - 1 = n_2 + 1 \Rightarrow n_1 - n_2 = 2$, not possible

Therefore, both n_1 and n_2 must be either odd or even.

Suppose both n_1 and n_2 are odd.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 + 1 = n_2 + 1 \Rightarrow n_1 = n_2$

Suppose both n_1 and n_2 are even.

Then, $f(n_1) = f(n_2) \Rightarrow n_1 - 1 = n_2 - 1 \Rightarrow n_1 = n_2$

Thus, f is one – one.

Also, any odd number $2r + 1$ in the co – domain $\mathbf{N}$ will have an even number as image in domain $\mathbf{N}$ which is

$f(n) = 2r + 1 \Rightarrow n - 1 = 2r + 1 \Rightarrow n = 2r + 2$

any even number $2r$ in the co – domain $\mathbf{N}$ will have an odd number as image in domain $\mathbf{N}$ which is

$f(n) = 2r \Rightarrow n + 1 = 2r \Rightarrow n = 2r - 1$

Thus, f is onto.