

RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.8

Indefinite Integrals Ex 19.9 Q1

$$\text{Let } I = \int \frac{\log x}{x} dx$$

Let $\log x = t$ then,

$$d(\log x) = dt$$

$$\Rightarrow \frac{1}{x} dx = dt$$

$$\Rightarrow dx = x dt$$

Putting $\log x = t$ and $dx = x dt$, we get

$$\begin{aligned} I &= \int \frac{t}{x} \times x dt \\ &= \int t dt \\ &= \frac{t^2}{2} + c \\ &= \frac{(\log x)^2}{2} + c \end{aligned}$$

$$\therefore I = \frac{(\log x)^2}{2} + c$$

Indefinite Integrals Ex 19.9 Q2

$$\text{Let } I = \int \frac{\log\left(1 + \frac{1}{x}\right)}{x(1+x)} dx \text{ ----- (i)}$$

$$\text{Let } \log\left(1 + \frac{1}{x}\right) = t \quad \text{then,}$$

$$d\left[\log\left(1 + \frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \frac{1}{1 + \frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{-x}{x^2(x+1)} dx = -dt$$

$$\Rightarrow \frac{dx}{x(x+1)} = -dt$$

Putting $\log\left(1 + \frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int t \times -dt \\ &= -\frac{t^2}{2} + c \\ &= -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + c \end{aligned}$$

$$\therefore I = -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + c$$

$$\text{Let } I = \int \frac{\log\left(1 + \frac{1}{x}\right)}{x(1+x)} dx \text{ ----- (i)}$$

$$\text{Let } \log\left(1 + \frac{1}{x}\right) = t \quad \text{then,}$$

$$d\left[\log\left(1 + \frac{1}{x}\right)\right] = dt$$

$$\Rightarrow \frac{1}{1 + \frac{1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{1}{\frac{x+1}{x}} \times \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{-x}{x^2(x+1)} dx = dt$$

$$\Rightarrow \frac{dx}{x(x+1)} = -dt$$

Putting $\log\left(1 + \frac{1}{x}\right) = t$ and $\frac{dx}{x(x+1)} = -dt$ in equation (i), we get

$$\begin{aligned} I &= \int t \times -dt \\ &= -\frac{t^2}{2} + c \\ &= -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + c \end{aligned}$$

$$\therefore I = -\frac{1}{2} \left[\log\left(1 + \frac{1}{x}\right) \right]^2 + c$$

$$\text{Let } I = \int \frac{(1 + \sqrt{x})^2}{\sqrt{x}} dx$$

$$\text{Let } (1 + \sqrt{x}) = t \quad \text{then,}$$

$$d(1 + \sqrt{x}) = dt$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow dx = dt \times 2\sqrt{x}$$

Putting $(1 + \sqrt{x}) = t$ and $dx = dt \times 2\sqrt{x}$, we get

$$I = \int \frac{t^2}{\sqrt{x}} \times dt \times 2\sqrt{x}$$

$$= 2 \int t^2 dt$$

$$= 2 \times \frac{t^3}{3} + c$$

$$= \frac{2}{3} [1 + \sqrt{x}]^3 + c$$

$$\therefore I = \frac{2}{3} (1 + \sqrt{x})^3 + c$$

$$\text{Let } I = \int \sqrt{1+e^x} e^x dx \text{ ----- (i)}$$

$$\text{Let } 1+e^x = t \quad \text{then,}$$

$$d(1+e^x) = dt$$

$$\Rightarrow e^x dx = dt$$

$$\Rightarrow dx = \frac{dt}{e^x}$$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$I = \int \sqrt{t} e^x \frac{dt}{e^x}$$

$$= \int t^{\frac{1}{2}} dt$$

$$= \frac{2}{3} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$= \frac{2}{3} (1+e^x)^{\frac{3}{2}} + c$$

$$\text{Let } I = \int \sqrt[3]{\cos^2 x} \sin x \, dx \text{ ----- (i)}$$

Let $\cos x = t$ then,

$$d(\cos x) = dt$$

$$\Rightarrow -\sin x \, dx = dt$$

$$\Rightarrow dx = -\frac{dt}{\sin x}$$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i), we get

$$I = \int \sqrt[3]{t^2} \sin x \times \frac{-dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}} \sin x \frac{dt}{\sin x}$$

$$= -\int t^{\frac{2}{3}} dt$$

$$= -\frac{3}{5} \times \frac{t^{\frac{5}{3}}}{\frac{5}{3}} + C$$

$$= -\frac{3}{5} (\cos x)^{\frac{5}{3}} + C$$

$$\therefore I = -\frac{3}{5} (\cos x)^{\frac{5}{3}} + C$$

$$\text{Let } I = \int \frac{e^x}{(1+e^x)^2} dx \text{ ----- (i)}$$

Let $1+e^x = t$ then,

$$d(1+e^x) = dt$$

$$\Rightarrow e^x dx = dt$$

$$\Rightarrow dx = \frac{dt}{e^x}$$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i), we get

$$I = \int \frac{e^x}{t^2} \times \frac{dt}{e^x}$$

$$= \int \frac{dt}{t^2}$$

$$= \int t^{-2} dt$$

$$= -t^{-1} + C$$

$$= -\frac{1}{t} + C$$

$$= -\frac{1}{1+e^x} + C$$

$$\therefore I = -\frac{1}{1+e^x} + C$$

$$\text{Let } I = \int \cot^3 x \operatorname{cosec}^2 x \, dx \text{ ----- (i)}$$

Let $\cot x = t$ then,

$$d(\cot x) = dt$$

$$\Rightarrow -\operatorname{cosec}^2 x \, dx = dt$$

$$\Rightarrow dx = -\frac{dt}{\operatorname{cosec}^2 x}$$

Putting $\cot x = t$ and $dx = -\frac{dt}{\operatorname{cosec}^2 x}$ in equation (i), we get

$$I = \int t^3 \operatorname{cosec}^2 x \times \frac{-dt}{\operatorname{cosec}^2 x}$$

$$= -\int t^3 dt$$

$$= -\frac{t^4}{4} + c$$

$$= -\frac{\cot^4 x}{4} + c$$

$$\therefore I = -\frac{\cot^4 x}{4} + c$$

$$\text{Let } I = \int \frac{\left\{e^{\sin^{-1} x}\right\}^2}{\sqrt{1-x^2}} dx \text{ ----- (i)}$$

Let $\sin^{-1} x = t$ then,

$$d(\sin^{-1} x) = dt$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\Rightarrow dx = \sqrt{1-x^2} dt$$

Putting $\sin^{-1} x = t$ and $dx = \sqrt{1-x^2} dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{\left(e^t\right)^2}{\sqrt{1-x^2}} \times \sqrt{1-x^2} dt \\ &= \int e^{2t} dt \\ &= \frac{e^{2t}}{2} + C \\ &= \frac{e^{2\sin^{-1} x}}{2} + C \end{aligned}$$

$$\therefore I = \frac{\left\{e^{\sin^{-1} x}\right\}^2}{2} + C$$

$$\text{Let } I = \int \frac{1 + \sin x}{\sqrt{x - \cos x}} dx \text{ ----- (i)}$$

Let $x - \cos x = t$ then,

$$d(x - \cos x) = dt$$

$$\Rightarrow [1 - (-\sin x)]dx = dt$$

$$\Rightarrow (1 + \sin x)dx = dt$$

Putting $x - \cos x = t$ and $(1 + \sin x)dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{\sqrt{t}} \\ &= \int t^{-\frac{1}{2}} dt \\ &= 2t^{\frac{1}{2}} + C \\ &= 2(x - \cos x)^{\frac{1}{2}} + C \end{aligned}$$

$$\therefore I = 2\sqrt{x - \cos x} + C$$

$$\text{Let } I = \int \frac{1}{\sqrt{1-x^2} (\sin^{-1} x)^2} dx \text{ ----- (i)}$$

Let $\sin^{-1} x = t$ then,

$$d(\sin^{-1} x) = dt$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{t^2} \\ &= \int t^{-2} dt \\ &= -1t^{-1} + C \\ &= \frac{-1}{t} + C \\ &= \frac{-1}{\sin^{-1} x} + C \end{aligned}$$

$$\therefore I = \frac{-1}{\sin^{-1} x} + C$$

$$\text{Let } I = \int \frac{\cot x}{\sqrt{\sin x}} dx \text{ ----- (i)}$$

Let $\sin x = t$ then,

$$d(\sin x) = dt$$

$$\Rightarrow \cos x dx = dt$$

$$\begin{aligned} \text{Now, } I &= \int \frac{\cot x}{\sqrt{\sin x}} dx \\ &= \int \frac{\cos x}{\sin x \sqrt{\sin x}} dx \\ &= \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx \end{aligned}$$

$$\Rightarrow = \int \frac{\cos x}{(\sin x)^{\frac{3}{2}}} dx \text{ ----- (ii)}$$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (ii), we get

$$\begin{aligned} I &= \int \frac{dt}{t^{\frac{3}{2}}} \\ &= \int t^{-\frac{3}{2}} dt \\ &= -2t^{-\frac{1}{2}} + C \\ &= \frac{-2}{\sqrt{t}} + C \\ &= \frac{-2}{\sqrt{\sin x}} + C \end{aligned}$$

$$\therefore I = \frac{-2}{\sqrt{\sin x}} + C$$

$$\text{Let } I = \int \frac{\tan x}{\sqrt{\cos x}} dx$$

$$\begin{aligned}\therefore I &= \int \frac{\sin x}{\cos x \sqrt{\cos x}} dx \\ &= \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx\end{aligned}$$

$$\Rightarrow I = \int \frac{\sin x}{(\cos x)^{\frac{3}{2}}} dx \text{ ----- (i)}$$

$$\begin{aligned}\text{Let } \cos x &= t \quad \text{then,} \\ d(\cos x) &= dt\end{aligned}$$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow \sin x dx = -dt$$

Putting $\cos x = t$ and $\sin x dx = -dt$ in equation (i), we get

$$\begin{aligned}I &= \int \frac{-dt}{t^{\frac{3}{2}}} \\ &= -\int t^{-\frac{3}{2}} dt \\ &= -\left[-2t^{-\frac{1}{2}} \right] + c \\ &= \frac{2}{t^{\frac{1}{2}}} + c \\ &= \frac{2}{\sqrt{\cos x}} + c\end{aligned}$$

$$\therefore I = \frac{2}{\sqrt{\cos x}} + c$$

$$\text{Let } I = \int \frac{\cos^3 x}{\sqrt{\sin x}} dx$$

$$\begin{aligned} \therefore I &= \int \frac{\cos^2 x \cos x}{\sqrt{\sin x}} dx \\ &= \int \frac{(1 - \sin^2 x) \cos x}{\sqrt{\sin x}} dx \end{aligned}$$

$$\therefore I = \int \frac{(1 - \sin^2 x)}{\sqrt{\sin x}} \cos x dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } \sin x &= t && \text{then,} \\ d(\sin x) &= dt \end{aligned}$$

$$\Rightarrow \cos x dx = dt$$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{1 - t^2}{\sqrt{t}} dt \\ &= \int \left(t^{\frac{-1}{2}} - t^2 \times t^{\frac{-1}{2}} \right) dt \\ &= \int \left(t^{\frac{-1}{2}} - t^{\frac{3}{2}} \right) dt \\ &= 2t^{\frac{1}{2}} - \frac{2}{5} t^{\frac{5}{2}} + c \end{aligned}$$

$$\Rightarrow I = 2(\sin x)^{\frac{1}{2}} - \frac{2}{5}(\sin x)^{\frac{5}{2}} + c$$

$$\therefore I = 2\sqrt{\sin x} - \frac{2}{5}(\sin x)^{\frac{5}{2}} + c$$

$$\text{Let } I = \int \frac{\sin^3 x}{\sqrt{\cos x}} dx$$

$$\therefore I = \int \frac{\sin^2 x \sin x}{\sqrt{\cos x}} dx$$

$$\Rightarrow I = \int \frac{(1 - \cos^2 x)}{\sqrt{\cos x}} \sin x dx \text{ ----- (i)}$$

$$\text{Let } \cos x = t \quad \text{then,} \\ d(\cos x) = dt$$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow \sin x dx = -dt$$

Putting $\cos x = t$ and $\sin x dx = -dt$ in equation (i), we get

$$I = \int \frac{(1 - t^2)}{\sqrt{t}} \times -dt$$

$$= \int \frac{t^2 - 1}{\sqrt{t}} dt$$

$$= \int \left(\frac{t^2}{t^{\frac{1}{2}}} - \frac{1}{t^{\frac{1}{2}}} \right) dx$$

$$= \int \left(t^{2 - \frac{1}{2}} - t^{-\frac{1}{2}} \right) dt$$

$$= \int \left(t^{\frac{3}{2}} - t^{-\frac{1}{2}} \right) dt$$

$$= \frac{2}{5} t^{\frac{5}{2}} - 2t^{\frac{1}{2}} + c$$

$$\therefore I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2 \cos^{\frac{1}{2}} x + c$$

$$\therefore I = \frac{2}{5} \cos^{\frac{5}{2}} x - 2\sqrt{\cos x} + c$$

$$\text{Let } I = \int \frac{1}{\sqrt{\tan^{-1} x} (1+x^2)} dx \text{ ----- (i)}$$

$$\text{Let } \tan^{-1} x = t \quad \text{then,}$$

$$d(\tan^{-1} x) = dt$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

Putting $\tan^{-1} x = t$ and $\frac{1}{1+x^2} dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{1}{\sqrt{t}} dt \\ &= \int t^{-\frac{1}{2}} dt \\ &= 2t^{\frac{1}{2}} + c \\ &= 2\sqrt{\tan^{-1} x} + c \end{aligned}$$

$$\therefore I = 2\sqrt{\tan^{-1} x} + c$$

Indefinite Integrals Ex 19.9 Q16

$$\begin{aligned} \text{Let } I &= \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx \\ &= \int \frac{\sqrt{\tan x} \times \cos x}{\sin x \cos x \times \cos x} dx \\ &= \int \frac{\sqrt{\tan x}}{\tan x \cos^2 x} dx \\ &= \int \frac{\sec^2 x dx}{\sqrt{\tan x}} \end{aligned}$$

$$\text{Let } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\begin{aligned} \therefore I &= \int \frac{dt}{\sqrt{t}} \\ &= 2\sqrt{t} + C \\ &= 2\sqrt{\tan x} + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q17

$$\text{Let } I = \int \frac{1}{x} (\log x)^2 dx \text{ ----- (i)}$$

$$\text{Let } \log x = t \quad \text{then,} \\ d(\log x) = dt$$

$$\Rightarrow \frac{1}{x} dx = dt$$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i), we get

$$I = \int t^2 dt \\ = \frac{t^3}{3} + c \\ = \frac{(\log x)^3}{3} + c$$

$$\therefore I = \frac{1}{3} (\log x)^3 + c$$

Indefinite Integrals Ex 19.9 Q18

$$\text{Let } I = \int \sin^5 x \cos x dx \text{ ----- (i)}$$

$$\text{Let } \sin x = t \quad \text{then,} \\ d(\sin x) = dt$$

$$\Rightarrow \cos x dx = dt$$

Putting $\sin x = t$ and $\cos x dx = dt$ in equation (i), we get

$$I = \int t^5 dt \\ = \frac{t^6}{6} + c \\ = \frac{\sin^6 x}{6} + c$$

$$\therefore I = \frac{1}{6} \sin^6 x + c$$

Indefinite Integrals Ex 19.9 Q19

$$\text{Let } I = \int \tan^{\frac{3}{2}} x \sec^2 x \, dx \text{ ----- (i)}$$

$$\text{Let } \tan x = t \quad \text{then,}$$
$$d(\tan x) = dt$$

$$\Rightarrow \sec^2 x \, dx = dt$$

Putting $\tan x = t$ and $\sec^2 x \, dx = dt$ in equation (i), we get

$$I = \int t^{\frac{3}{2}} dt$$
$$= \frac{2}{5} t^{\frac{5}{2}} + c$$
$$= \frac{2}{5} (\tan x)^{\frac{5}{2}} + c$$

$$\therefore I = \frac{2}{5} \tan^{\frac{5}{2}} x + c$$

Indefinite Integrals Ex 19.9 Q20

$$\text{Let } I = \int \frac{x^3}{(x^2 + 1)^3} x \, dx \text{ ----- (i)}$$

$$\text{Let } 1 + x^2 = t \quad \text{then,}$$
$$d(1 + x^2) = dt$$

$$\Rightarrow 2x \, dx = dt$$

$$\Rightarrow x \, dx = \frac{dt}{2}$$

Putting $1 + x^2 = t$ and $x \, dx = \frac{dt}{2}$ in equation (i), we get

$$I = \int \frac{x^2}{t^3} \times \frac{dt}{2}$$
$$= \frac{1}{2} \int \frac{(t-1)}{t^3} dt \quad [\because 1 + x^2 = t]$$
$$= \frac{1}{2} \int \left[\frac{t}{t^3} - \frac{1}{t^3} \right] dt$$
$$= \frac{1}{2} \int (t^{-2} - t^{-3}) dt$$
$$= \frac{1}{2} \left[-1t^{-1} - \frac{t^{-2}}{-2} \right] + c$$

$$\begin{aligned}
&= \frac{1}{2} \left[-\frac{1}{t} + \frac{1}{2t^2} \right] + C \\
&= -\frac{1}{2t} + \frac{1}{4t^2} + C \\
&= -\frac{1}{2(1+x^2)} + \frac{1}{4(1+x^2)^2} + C \\
&= \frac{-2(1+x^2) + 1}{4(1+x^2)^2} + C \\
&= \frac{-2 - 2x^2 + 1}{4(1+x^2)^2} + C \\
&= \frac{-2x^2 - 1}{4(1+x^2)^2} + C \\
&= -\frac{(1+2x^2)}{4(x^2+1)^2} + C
\end{aligned}$$

$$\therefore I = \frac{-(1+2x^2)}{4(x^2+1)^2} + C$$

Indefinite Integrals Ex 19.9 Q21

Let $x^2 + x + 1 = t$

$$(2x + 1)dx = dt$$

$$\int (4x+2)\sqrt{x^2+x+1} dx$$

$$= \int 2\sqrt{t} dt$$

$$= 2 \int \sqrt{t} dt$$

$$= 2 \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right) + C$$

$$= \frac{4}{3} (x^2 + x + 1)^{\frac{3}{2}} + C$$

Indefinite Integrals Ex 19.9 Q22

$$\text{Let } I = \int \frac{4x+3}{\sqrt{2x^3+3x+1}} dx \text{ ----- (i)}$$

$$\text{Let } 2x^3+3x+1 = t \quad \text{then,}$$

$$d(2x^3+3x+1) = dt$$

$$\Rightarrow (4x+3) dx = dt$$

Putting $2x^3+3x+1 = t$ and $(4x+3) dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{\sqrt{t}} \\ &= \int t^{-\frac{1}{2}} dt \\ &= 2t^{\frac{1}{2}} + c \\ &= 2\sqrt{t} + c \end{aligned}$$

$$\therefore I = 2\sqrt{2x^3+3x+1} + c$$

$$\text{Let } I = \int \frac{1}{1+\sqrt{x}} dx \text{ ----- (i)}$$

$$\text{Let } x = t^2 \text{ then,}$$
$$dx = d(t^2)$$

$$\Rightarrow dx = 2t dt$$

Putting $x = t^2$ and $dx = 2t dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{2t}{1+\sqrt{t^2}} dt \\ &= \int \frac{2t}{1+t} dt \\ &= 2 \int \frac{t}{1+t} dt \\ &= 2 \int \frac{1+t-1}{1+t} dt \\ &= 2 \int \left[\frac{1+t}{1+t} - \frac{1}{1+t} \right] dt \\ &= 2 \int dt - 2 \int \frac{1}{1+t} dt \\ &= 2t - 2 \log(1+t) + c \\ &= 2\sqrt{x} - 2 \log(1+\sqrt{x}) + c \end{aligned}$$

$$\therefore I = 2\sqrt{x} - 2 \log(1+\sqrt{x}) + c$$

$$\text{Let } I = \int e^{\cos^2 x} \sin 2x \, dx \text{ --- (i)}$$

$$\text{Let } \cos^2 x = t \quad \text{then,}$$

$$d(\cos^2 x) = dt$$

$$\Rightarrow -2 \cos x \sin x \, dx = dt$$

$$\Rightarrow -\sin 2x \, dx = dt$$

$$\Rightarrow \sin 2x \, dx = -dt$$

Putting $\cos^2 x = t$ and $\sin 2x \, dx = -dt$ in equation (i),
we get

$$I = \int e^t (-dt)$$

$$= -e^t + C$$

$$= -e^{\cos^2 x} + C$$

$$\therefore I = -e^{\cos^2 x} + C$$

$$\text{Let } I = \int \frac{1 + \cos x}{(x + \sin x)^3} dx \text{ ----- (i)}$$

$$\text{Let } x + \sin x = t \quad \text{then,}$$
$$d(x + \sin x) = dt$$

$$\Rightarrow (1 + \cos x) dx = dt$$

Putting $x + \sin x = t$ and $(1 + \cos x) dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{t^3} \\ &= \int t^{-3} dt \\ &= \frac{t^{-2}}{-2} + C \\ &= -\frac{1}{2t^2} + C \\ &= \frac{-1}{2(x + \sin x)^2} + C \end{aligned}$$

$$\therefore I = \frac{-1}{2(x + \sin x)^2} + C$$

$$\frac{\cos x - \sin x}{1 + \sin 2x} = \frac{\cos x - \sin x}{(\sin^2 x + \cos^2 x) + 2 \sin x \cos x}$$

$$[\sin^2 x + \cos^2 x = 1; \sin 2x = 2 \sin x \cos x]$$

$$= \frac{\cos x - \sin x}{(\sin x + \cos x)^2}$$

Let $\sin x + \cos x = t$

$$\therefore (\cos x - \sin x) dx = dt$$

$$\begin{aligned}\Rightarrow \int \frac{\cos x - \sin x}{1 + \sin 2x} dx &= \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2} dx \\ &= \int \frac{dt}{t^2} \\ &= \int t^{-2} dt \\ &= -t^{-1} + C \\ &= -\frac{1}{t} + C \\ &= \frac{-1}{\sin x + \cos x} + C\end{aligned}$$

Indefinite Integrals Ex 19.9 Q27

$$\text{Let } I = \int \frac{\sin 2x}{(a + b \cos 2x)^2} dx \text{ ----- (i)}$$

$$\text{Let } a + b \cos 2x = t \quad \text{then,}$$
$$d(a + b \cos 2x) = dt$$

$$\Rightarrow b(-2 \sin 2x) dx = dt$$

$$\Rightarrow \sin 2x dx = -\frac{dt}{2b}$$

Putting $a + b \cos 2x = t$ and $\sin 2x dx = -\frac{dt}{2b}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{1}{t^2} \times \frac{-dt}{2b} \\ &= \frac{-1}{2b} \int t^{-2} dt \\ &= -\frac{1}{2b} (-1t^{-1}) + c \\ &= \frac{1}{2bt} + c \\ &= \frac{1}{2b(a + b \cos 2x)} + c \end{aligned}$$

$$\therefore I = \frac{1}{2b(a + b \cos 2x)} + c$$

$$\text{Let } I = \int \frac{\log x^2}{x} dx \text{ ----- (i)}$$

$$\text{Let } \log x = t \quad \text{then,}$$
$$d(\log x) = dt$$

$$\Rightarrow \frac{1}{x} dx = dt$$

$$\Rightarrow \frac{dx}{x} = dt$$

$$\text{Now, } I = \int \frac{\log x^2}{x} dx$$
$$= \int \frac{2 \log x}{x} dx$$
$$= 2 \int \frac{\log x}{x} dx \text{ ----- (ii)}$$

Putting $\log x = t$ and $\frac{dx}{x} = dt$ in equation (ii), we get

$$I = 2 \int t dt$$
$$= \frac{2t^2}{2} + c$$
$$= t^2 + c$$

$$\therefore I = (\log x)^2 + c$$

Let $I = \int \frac{\sin x}{(1 + \cos x)^2} dx \dots \dots \dots$ (i)

Let $1 + \cos x = t$ then,
 $d(1 + \cos x) = dt$

$\Rightarrow -\sin x dx = dt$

$\Rightarrow \sin x dx = -dt$

Putting $1 + \cos x = t$ and $\sin x dx = -dt$ in equation (ii), we get

$$\begin{aligned} I &= \int \frac{-dt}{t^2} \\ &= -\int t^{-2} dt \\ &= -(-1t^{-1}) + C \\ &= \frac{1}{t} + C \\ &= \frac{1}{1 + \cos x} + C \end{aligned}$$

$\therefore I = \frac{1}{1 + \cos x} + C$

Indefinite Integrals Ex 19.9 Q30

Let $\log \sin x = t$

$\Rightarrow \frac{1}{\sin x} \cdot \cos x dx = dt$

$\therefore \cot x dx = dt$

$$\begin{aligned} \Rightarrow \int \cot x \log \sin x dx &= \int t dt \\ &= \frac{t^2}{2} + C \\ &= \frac{1}{2} (\log \sin x)^2 + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q31

$$\text{Let } I = \int \sec x \cdot \log(\sec x + \tan x) dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } \log(\sec x + \tan x) &= t \text{ then,} \\ d[\log(\sec x + \tan x)] &= dt \end{aligned}$$

$$\Rightarrow \sec x dx = dt \quad \left[\because \frac{d}{dx}(\log(\sec x + \tan x)) = \sec x \right]$$

Putting $\log(\sec x + \tan x) = t$ and $\sec x dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int t dt \\ &= \frac{t^2}{2} + c \\ &= \frac{1}{2} [\log(\sec x + \tan x)]^2 + c \end{aligned}$$

$$\therefore I = \frac{1}{2} [\log(\sec x + \tan x)]^2 + c$$

Indefinite Integrals Ex 19.9 Q32

$$\text{Let } I = \int \operatorname{cosec} x \log(\operatorname{cosec} x - \cot x) dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } \log(\operatorname{cosec} x - \cot x) &= t \quad \text{then,} \\ dx [\log(\operatorname{cosec} x - \cot x)] &= dt \end{aligned}$$

$$\Rightarrow \operatorname{cosec} x dx = dt \quad \left[\because \frac{d}{dx}(\log(\operatorname{cosec} x - \cot x)) = \operatorname{cosec} x \right]$$

Putting $\log(\operatorname{cosec} x - \cot x) = t$ and $\operatorname{cosec} x dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int t dt \\ &= \frac{t^2}{2} + c \end{aligned}$$

$$\therefore I = \frac{1}{2} [\log(\operatorname{cosec} x - \cot x)]^2 + c$$

Indefinite Integrals Ex 19.9 Q33

$$\text{Let } I = \int x^3 \cos x^4 dx \text{ ----- (i)}$$

$$\text{Let } x^4 = t \text{ then,}$$
$$dx(x^4) = dt$$

$$\Rightarrow 4x^3 dx = dt$$

$$\Rightarrow x^3 = \frac{dt}{4}$$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \cos t \frac{dt}{4}$$
$$= \frac{1}{4} \sin t + c$$

$$\therefore I = \frac{1}{4} \sin x^4 + c$$

Indefinite Integrals Ex 19.9 Q34

$$\text{Let } I = \int x^3 \sin x^4 dx \text{ ----- (i)}$$

$$\text{Let } x^4 = t \text{ then,}$$
$$d(x^4) = dt$$

$$\Rightarrow 4x^3 dx = dt$$

$$\Rightarrow x^3 = \frac{dt}{4}$$

Putting $x^4 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$
$$= \frac{1}{4} \int \sin t dt$$
$$= -\frac{1}{4} \cos t + c$$
$$= -\frac{1}{4} \cos x^4 + c$$

$$\therefore I = -\frac{1}{4} \cos x^4 + c$$

Indefinite Integrals Ex 19.9 Q35

$$\text{Let } I = \int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}} dx \text{ ---- (i)}$$

$$\text{Let } \sin^{-1} x^2 = t \quad \text{then,}$$
$$d(\sin^{-1} x^2) = dt$$

$$\Rightarrow 2x \times \frac{1}{\sqrt{1-x^4}} dx = dt$$

$$\Rightarrow \frac{x}{\sqrt{1-x^4}} dx = \frac{dt}{2}$$

Putting $\sin^{-1} x^2 = t$ and $\frac{x}{\sqrt{1-x^4}} dx = \frac{dt}{2}$ in equation (i),

we get

$$\begin{aligned} I &= \int t \frac{dt}{2} \\ &= \frac{1}{2} \times \frac{t^2}{2} + c \\ &= \frac{1}{4} (\sin^{-1} x^2)^2 + c \end{aligned}$$

$$\therefore I = \frac{1}{4} (\sin^{-1} x^2)^2 + c$$

Indefinite Integrals Ex 19.9 Q36

$$\text{Let } I = \int x^3 \sin(x^4 + 1) dx \text{ ----- (i)}$$

$$\text{Let } x^4 + 1 = t \quad \text{then,}$$
$$d(x^4 + 1) = dt$$

$$\Rightarrow x^4 dx = dt$$

$$\Rightarrow x^3 dx = \frac{dt}{4}$$

Putting $x^4 + 1 = t$ and $x^3 dx = \frac{dt}{4}$ in equation (i), we get

$$I = \int \sin t \frac{dt}{4}$$
$$= -\frac{1}{4} \cos t + c$$
$$= -\frac{1}{4} \cos(x^4 + 1) + c$$

$$\therefore I = -\frac{1}{4} \cos(x^4 + 1) + c$$

$$\text{Let } I = \int \frac{(x+1)e^x}{\cos^2(xe^x)} dx \text{ ----- (i)}$$

$$\text{Let } xe^x = t \quad \text{then,}$$
$$d(xe^x) = dt$$

$$\Rightarrow (e^x + xe^x) dx = dt$$

$$\Rightarrow (x+1)e^x dx = dt$$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{\cos^2 t} \\ &= \int \sec^2 t dt \\ &= \tan t + c \\ &= \tan(xe^x) + c \end{aligned}$$

$$\therefore I = \tan(xe^x) + c$$

$$\text{Let } I = \int x^2 e^{x^3} \cos(e^{x^3}) dx \text{ ----- (i)}$$

$$\text{Let } e^{x^3} = t \quad \text{then,}$$
$$d(e^{x^3}) = dt$$

$$\Rightarrow 3x^2 e^{x^3} dx = dt$$

$$\Rightarrow x^2 e^{x^3} dx = \frac{dt}{3}$$

Putting $e^{x^3} = t$ and $x^2 e^{x^3} dx = \frac{dt}{3}$ in equation (i), we get

$$I = \int \cos t \frac{dt}{3}$$
$$= \frac{\sin t}{3} + c$$
$$= \frac{\sin(e^{x^3})}{3} + c$$

$$\therefore I = \frac{1}{3} \sin(e^{x^3}) + c$$

Indefinite Integrals Ex 19.9 Q39

$$\text{Let } I = \int 2x \sec^3(x^2 + 3) \tan(x^2 + 3) dx \text{ ----- (i)}$$

$$\text{Let } \sec(x^2 + 3) = t \quad \text{then,}$$
$$d[\sec(x^2 + 3)] = dt$$

$$\Rightarrow 2x \sec(x^2 + 3) \tan(x^2 + 3) dx = dt$$

Putting $\sec(x^2 + 3) = t$ and $2x \sec(x^2 + 3) \tan(x^2 + 3) dx = dt$ in equation (i), we get

$$I = \int t^2 dt$$
$$= \frac{t^3}{3} + c$$
$$= \frac{1}{3} [\sec(x^2 + 3)]^3 + c$$

$$\therefore I = \frac{1}{3} [\sec(x^2 + 3)]^3 + c$$

Indefinite Integrals Ex 19.9 Q40

$$\frac{(x+1)(x+\log x)^2}{x} = \left(\frac{x+1}{x}\right)(x+\log x)^2 = \left(1+\frac{1}{x}\right)(x+\log x)^2$$

$$\text{Let } (x+\log x) = t$$

$$\Rightarrow \left(1+\frac{1}{x}\right)dx = dt$$

$$\begin{aligned}\Rightarrow \int \left(1+\frac{1}{x}\right)(x+\log x)^2 dx &= \int t^2 dt \\ &= \frac{t^3}{3} + C \\ &= \frac{1}{3}(x+\log x)^3 + C\end{aligned}$$

Indefinite Integrals Ex 19.9 Q41

$$\text{Let } I = \int \tan x \sec^2 x \sqrt{1 - \tan^2 x} \, dx \text{ ----- (i)}$$

$$\text{Let } 1 - \tan^2 x = t \text{ then,}$$
$$d(1 - \tan^2 x) = dt$$

$$\Rightarrow -2 \tan x \sec^2 x \, dx = dt$$

$$\Rightarrow \tan x \sec^2 x \, dx = \frac{-dt}{2}$$

Putting $1 - \tan^2 x = t$ and $\tan x \sec^2 x \, dx = -\frac{dt}{2}$ in equation (i),
we get

$$I = \int \sqrt{t} \times \frac{-dt}{2}$$

$$= -\frac{1}{2} \int t^{\frac{1}{2}} dt$$

$$= -\frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= -\frac{1}{3} t^{\frac{3}{2}} + C$$

$$\therefore I = -\frac{1}{3} [1 - \tan^2 x]^{\frac{3}{2}} + C$$

$$\text{Let } I = \int \log x \frac{\sin(1 + (\log x)^2)}{x} dx \text{ ----- (i)}$$

$$\text{Let } 1 + (\log x)^2 = t \text{ then,}$$
$$d(1 + (\log x)^2) = dt$$

$$\Rightarrow 2 \log x \frac{1}{x} dx = dt$$

$$\Rightarrow \frac{\log x}{x} dx = \frac{dt}{2}$$

Putting $1 + (\log x)^2 = t$ and $\frac{\log x}{x} dx = \frac{dt}{2}$ in equation (i),
we get

$$I = \int \sin t \times \frac{dt}{2}$$

$$= \frac{1}{2} \int \sin t dt$$

$$\therefore I = -\frac{1}{2} \cos t + c$$

$$= -\frac{1}{2} \cos[1 + (\log x)^2] + c$$

$$\therefore I = -\frac{1}{2} \cos[1 + (\log x)^2] + c$$

$$\text{Let } I = \int \frac{1}{x^2} \cos^2\left(\frac{1}{x}\right) dx \text{ ----- (i)}$$

$$\text{Let } \frac{1}{x} = t \text{ then,}$$

$$d\left(\frac{1}{x}\right) = dt$$

$$\Rightarrow \frac{-1}{x^2} dx = dt$$

$$\Rightarrow \frac{1}{x^2} dx = -dt$$

Putting $\frac{1}{x} = t$ and $\frac{1}{x^2} dx = -dt$ in equation (i),
we get

$$I = \int \cos^2 t (-dt)$$

$$= -\int \cos^2 t dt$$

$$= -\int \frac{\cos^2 2t + 1}{2} dt$$

$$= -\frac{1}{2} \int \cos 2t dt - \frac{1}{2} \int dt$$

$$= -\frac{1}{2} \times \frac{\sin 2t}{2} - \frac{1}{2} t + c$$

$$\therefore I = -\frac{1}{4} \sin 2t - \frac{1}{2} t + c$$

$$= -\frac{1}{4} \sin 2 \times \frac{1}{x} - \frac{1}{2} \times \frac{1}{x} + c$$

$$\therefore I = -\frac{1}{4} \sin\left(\frac{2}{x}\right) - \frac{1}{2} \left(\frac{1}{x}\right) + c$$

$$\text{Let } I = \int \sec^4 x \tan x \, dx \text{ --- (i)}$$

$$\text{Let } \tan x = t \quad \text{then,} \\ d(\tan x) = dt$$

$$\Rightarrow \sec^2 x \, dx = dt$$

$$\Rightarrow dx = \frac{dt}{\sec^2 x}$$

Putting $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$ in equation (i),
we get

$$\begin{aligned} I &= \int \sec^4 x \tan x \frac{dt}{\sec^2 x} \\ &= \int \sec^2 x \, t \, dt \\ &= \int (1 + \tan^2 x) \, t \, dt \\ &= \int (1 + t^2) \, t \, dt \\ &= \int (t + t^3) \, dt \\ &= \frac{t^2}{2} + \frac{t^4}{4} + c \\ &= \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c \end{aligned}$$

$$\therefore I = \frac{1}{2} \tan^2 x + \frac{1}{4} \tan^4 x + c$$

$$\text{Let } I = \int \frac{e^{\sqrt{x}} \cos(e^{\sqrt{x}})}{\sqrt{x}} dx \text{ ----- (i)}$$

$$\text{Let } e^{\sqrt{x}} = t \quad \text{then,}$$
$$d(e^{\sqrt{x}}) = dt$$

$$\Rightarrow \frac{e^{\sqrt{x}}}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$$

Putting $e^{\sqrt{x}} = t$ and $\frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2dt$ in equation (i),

we get

$$\begin{aligned} I &= \int \cos t \times 2dt \\ &= 2 \int \cos t dt \\ &= 2 \sin t + c \\ &= 2 \sin(e^{\sqrt{x}}) + c \end{aligned}$$

$$\therefore I = 2 \sin(e^{\sqrt{x}}) + c$$

$$\text{Let } I = \int \frac{\cos^5 x}{\sin x} dx \text{ ---- (i)}$$

$$\text{Let } \sin x = t \quad \text{then,} \\ d(\sin x) = dt$$

$$\Rightarrow \cos x dx = dt$$

$$\Rightarrow dx = \frac{dt}{\cos x}$$

Putting $\sin x = t$ and $dx = \frac{dt}{\cos x}$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{\cos^5 x}{t} \times \frac{dt}{\cos x} \\ &= \int \frac{\cos^4 x}{t} dt \\ &= \int \frac{(1 - \sin^2 x)^2}{t} dt \\ &= \int \frac{(1 - t^2)^2}{t} dt \\ &= \int \frac{1 + t^4 - 2t^2}{t} dt \\ &= \int \frac{1}{t} dt + \int \frac{t^4}{t} dt - 2 \int \frac{t^2}{t} dt \\ &= \log|t| + \frac{t^4}{4} - \frac{2t^2}{2} + c \\ &= \log|\sin x| + \frac{\sin^4 x}{4} - \sin^2 x + c \end{aligned}$$

$$\therefore I = \frac{1}{4} \sin^4 x - \sin^2 x + \log|\sin x| + c$$

$$\begin{aligned} \text{Let } \sqrt{x} &= t \\ \Rightarrow \frac{1}{2\sqrt{x}} dx &= dt \\ \Rightarrow \frac{1}{\sqrt{x}} dx &= 2dt \end{aligned}$$

$$\begin{aligned} \text{Therefore,} \\ \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx \\ &= 2 \int \sin t dt \\ &= -2 \cos t + C \\ &= -2 \cos \sqrt{x} + C \end{aligned}$$

Indefinite Integrals Ex 19.9 Q48

$$\text{Let } I = \int \frac{(x+1)e^x}{\sin^2(xe^x)} dx \text{ ----- (i)}$$

$$\begin{aligned} \text{Let } xe^x &= t && \text{then,} \\ d(xe^x) &= dt \end{aligned}$$

$$\Rightarrow (xe^x + e^x) dx = dt$$

$$\Rightarrow (x+1)e^x dx = dt$$

Putting $xe^x = t$ and $(x+1)e^x dx = dt$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{dt}{\sin^2 t} \\ &= \int \operatorname{cosec}^2 t dt \\ &= -\cot t + c \\ &= -\cot(xe^x) + c \end{aligned}$$

$$\therefore I = -\cot(xe^x) + c$$

Indefinite Integrals Ex 19.9 Q49

$$\text{Let } I = \int 5^{x+\tan^{-1}x} \left(\frac{x^2+2}{x^2+1} \right) dx \text{ ----- (i)}$$

$$\text{Let } x + \tan^{-1}x = t \quad \text{then,}$$
$$d(x + \tan^{-1}x) = dt$$

$$\Rightarrow \left(1 + \frac{1}{1+x^2} \right) dx = dt$$

$$\Rightarrow \left(\frac{1+x^2+1}{1+x^2} \right) dx = dt$$

$$\Rightarrow \frac{(x^2+2)}{(x^2+1)} dx = dt$$

Putting $x + \tan^{-1}x = t$ and $\left(\frac{x^2+2}{x^2+1} \right) dx = dt$ in equation (i),
we get

$$I = \int 5^t dt$$
$$= \frac{5^t}{\log 5} + C$$
$$= \frac{5^{x+\tan^{-1}x}}{\log 5} + C$$

$$\therefore I = \frac{5^{x+\tan^{-1}x}}{\log 5} + C$$

$$\text{Let } I = \int \frac{e^{m \sin^{-1} x}}{\sqrt{1-x^2}} dx \text{ ----- (i)}$$

$$\text{Let } m \sin^{-1} x = t \text{ then,}$$
$$d(m \sin^{-1} x) = dt$$

$$\Rightarrow m \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$\Rightarrow \frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$$

Putting $m \sin^{-1} x = t$ and $\frac{dx}{\sqrt{1-x^2}} = \frac{dt}{m}$ in equation (i),

we get

$$I = \int e^t \frac{dt}{m}$$
$$= \frac{1}{m} e^t + C$$
$$= \frac{1}{m} e^{m \sin^{-1} x} + C$$

$$\therefore I = \frac{1}{m} e^{m \sin^{-1} x} + C$$

Indefinite Integrals Ex 19.9 Q51

$$\text{Let } \sqrt{x} = t$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow \int \frac{\cos \sqrt{x}}{\sqrt{x}} dx = 2 \int \cos t dt$$
$$= 2 \sin t + C$$
$$= 2 \sin \sqrt{x} + C$$

Indefinite Integrals Ex 19.9 Q52

$$\text{Let } I = \int \frac{\sin(\tan^{-1} x)}{1+x^2} dx \text{ ----- (i)}$$

$$\text{Let } \tan^{-1} x = t \quad \text{then,}$$

$$d(\tan^{-1} x) = dt$$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

Putting $\tan^{-1} x = t$ and $\frac{dx}{1+x^2} = dt$ in equation (i),
we get

$$I = \int \sin t dt$$

$$= -\cos t + c$$

$$= -\cos(\tan^{-1} x) + c$$

$$\therefore I = -\cos(\tan^{-1} x) + c$$

Indefinite Integrals Ex 19.9 Q53

$$\text{Let } I = \int \frac{\sin(\log x)}{x} dx \text{ ----- (i)}$$

$$\text{Let } \log x = t \quad \text{then,}$$

$$d(\log x) = dt$$

$$\Rightarrow \frac{1}{x} dx = dt$$

Putting $\log x = t$ and $\frac{1}{x} dx = dt$ in equation (i),
we get

$$I = \int \sin t dt$$

$$= -\cos t + c$$

$$= -\cos(\log x) + c$$

$$\therefore I = -\cos(\log x) + c$$

Indefinite Integrals Ex 19.9 Q54

Let $\tan^{-1}x = t$

Differentiating the above function with respect to, w , we have,

$$\frac{1}{1+x^2} dx = dt$$

$$\Rightarrow \int \frac{e^{m \tan^{-1}x}}{1+x^2} = \int e^{mt} \times dt$$

$$\Rightarrow \int \frac{e^{m \tan^{-1}x}}{1+x^2} = \frac{e^{mt}}{m}$$

Resubstituting the value of t in the above solution, we have,

$$\Rightarrow \int \frac{e^{m \tan^{-1}x}}{1+x^2} = \frac{e^{m \tan^{-1}x}}{m} + C$$

Indefinite Integrals Ex 19.9 Q55

$$\text{Let } I = \int \frac{x}{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}} dx$$

$$\begin{aligned} \therefore I &= \int \frac{x}{\sqrt{x^2 + a^2} + \sqrt{x^2 - a^2}} \times \frac{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}}{\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2}} dx \\ &= \int \frac{x \left(\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2} \right)}{x^2 + a^2 - x^2 + a^2} dx \\ &= \int \frac{x}{2a^2} \left(\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2} \right) dx \\ \therefore I &= \frac{1}{2a^2} \int x \left(\sqrt{x^2 + a^2} - \sqrt{x^2 - a^2} \right) dx \text{ ----- (i)} \end{aligned}$$

$$\begin{aligned} \text{Let } x^2 &= t \quad \text{then,} \\ d(x^2) &= dt \end{aligned}$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

Putting $x^2 = t$ and $x dx = \frac{dt}{2}$ in equation (i),

we get

$$\begin{aligned} I &= \frac{1}{2a^2} \int \left(\sqrt{t + a^2} - \sqrt{t - a^2} \right) \frac{dx}{2} \\ &= \frac{1}{4a^2} \left[\frac{2}{3} (t + a^2)^{\frac{3}{2}} - \frac{2}{3} (t - a^2)^{\frac{3}{2}} \right] + c \\ \therefore I &= \frac{1}{4a^2} \left[\frac{2}{3} (x^2 + a^2)^{\frac{3}{2}} - \frac{2}{3} (x^2 - a^2)^{\frac{3}{2}} \right] + c \\ &= \frac{1}{6a^2} \left[(x^2 + a^2)^{\frac{3}{2}} - (x^2 - a^2)^{\frac{3}{2}} \right] + c \end{aligned}$$

$$\text{Let } I = \int x \frac{\tan^{-1} x^2}{1+x^4} dx \text{ ----- (i)}$$

$$\text{Let } \tan^{-1} x^2 = t \quad \text{then,}$$
$$d(\tan^{-1} x^2) = dt$$

$$\Rightarrow \frac{1 \times 2x}{1+(x^2)^2} dx = dt$$

$$\Rightarrow \frac{1 \times x}{1+x^4} dx = \frac{dt}{2}$$

Putting $\tan^{-1} x^2 = t$ and $\frac{x}{1+x^4} dx = \frac{dt}{2}$ in equation (i),
we get

$$I = \int t \frac{dx}{2}$$
$$= \frac{1}{2} \int t dt$$
$$= \frac{1}{2} \times \frac{t^2}{2} + c$$

$$\therefore I = \frac{t^2}{4} + c$$
$$= \frac{(\tan^{-1} x^2)^2}{4} + c$$

$$\therefore I = \frac{1}{4} (\tan^{-1} x^2)^2 + c$$

$$\text{Let } I = \int \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}} dx \text{ ---- (i)}$$

$$\text{Let } \sin^{-1} x = t \quad \text{then,}$$
$$d(\sin^{-1} x) = dt$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

Putting $\sin^{-1} x = t$ and $\frac{1}{\sqrt{1-x^2}} dx = dt$ in equation (i),

we get

$$I = \int t^3 dt$$
$$= \frac{t^4}{4} + c$$

$$\therefore I = \frac{1}{4} (\sin^{-1} x)^4 + c$$

$$\text{Let } I = \int \frac{\sin(2 + 3\log x)}{x} dx \text{ ---- (i)}$$

$$\text{Let } 2 + 3\log x = t \quad \text{then,}$$
$$d(2 + 3\log x) = dt$$

$$\Rightarrow 3 \frac{1}{x} dx = dt$$

$$\Rightarrow \frac{dx}{x} = \frac{dt}{3}$$

Putting $2 + 3\log x = t$ and $\frac{dx}{x} = \frac{dt}{3}$ in equation (i),
we get

$$I = \int \sin t \frac{dt}{3}$$
$$= \frac{1}{3} (-\cos t) + c$$
$$= -\frac{1}{3} \cos(2 + 3\log x) + c$$

$$\therefore I = -\frac{1}{3} \cos(2 + 3\log x) + c$$

$$\text{Let } I = \int x e^{x^2} dx \text{ ---- (i)}$$

$$\text{Let } x^2 = t \text{ then,}$$

$$d(x^2) = dt$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

Putting $x^2 = t$ and $x dx = \frac{dt}{2}$ in equation (i),

we get

$$\begin{aligned} I &= \int e^t \frac{dt}{2} \\ &= \frac{1}{2} e^t + c \\ &= \frac{1}{2} e^{x^2} + c \end{aligned}$$

$$\therefore I = \frac{1}{2} e^{x^2} + c$$

$$\text{Let } I = \int \frac{e^{2x}}{1+e^x} dx \text{ ---- (i)}$$

$$\text{Let } 1+e^x = t \quad \text{then,}$$
$$d(1+e^x) = dt$$

$$\Rightarrow e^x dx = dt$$

$$\Rightarrow dx = \frac{dt}{e^x}$$

Putting $1+e^x = t$ and $dx = \frac{dt}{e^x}$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{e^{2x}}{t} \times \frac{dt}{e^x} \\ &= \int \frac{e^x}{t} dt \\ &= \int \frac{t-1}{t} dt \\ &= \int \left(\frac{t}{t} - \frac{1}{t} \right) dt \\ &= t - \log|t| + c \\ &= (1+e^x) - \log|1+e^x| + c \end{aligned}$$

$$\therefore I = 1+e^x - \log|1+e^x| + c$$

$$\text{Let } I = \int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx \text{ ----- (i)}$$

$$\text{Let } \sqrt{x} = t \text{ then,}$$

$$d(\sqrt{x}) = dt$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

$$\Rightarrow dx = 2t dt \quad [\because \sqrt{x} = t]$$

Putting $\sqrt{x} = t$ and $dx = 2t dt$ in equation (i),
we get

$$I = \int \frac{\sec^2 t}{t} \times 2t dt$$

$$= 2 \int \sec^2 t dt$$

$$= 2 \tan t + c$$

$$= 2 \tan \sqrt{x} + c$$

$$\therefore I = 2 \tan \sqrt{x} + c$$

Indefinite Integrals Ex 19.9 Q62

$$\tan^3 2x \sec 2x = \tan^2 2x \tan 2x \sec 2x$$

$$= (\sec^2 2x - 1) \tan 2x \sec 2x$$

$$= \sec^2 2x \cdot \tan 2x \sec 2x - \tan 2x \sec 2x$$

$$\therefore \int \tan^3 2x \sec 2x dx = \int \sec^2 2x \tan 2x \sec 2x dx - \int \tan 2x \sec 2x dx$$

$$= \int \sec^2 2x \tan 2x \sec 2x dx - \frac{\sec 2x}{2} + C$$

$$\text{Let } \sec 2x = t$$

$$\therefore 2 \sec 2x \tan 2x dx = dt$$

$$\therefore \int \tan^3 2x \sec 2x dx = \frac{1}{2} \int t^2 dt - \frac{\sec 2x}{2} + C$$

$$= \frac{t^3}{6} - \frac{\sec 2x}{2} + C$$

$$= \frac{(\sec 2x)^3}{6} - \frac{\sec 2x}{2} + C$$

Indefinite Integrals Ex 19.9 Q63

$$\text{Let } I = \int \frac{x + \sqrt{x+1}}{x+2} dx \text{ ----- (i)}$$

$$\text{Let } x+1 = t^2 \quad \text{then,}$$

$$d(x+1) = d(t^2)$$

$$\Rightarrow dx = 2t dt$$

Putting $x+1 = t^2$ and $dx = 2t dt$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{x + \sqrt{t^2}}{x+2} 2t dt \\ &= 2 \int \frac{(t^2 - 1) + t}{(t^2 - 1) + 2} t dt \quad [\because x+1 = t^2] \\ &= 2 \int \frac{t^2 + t - 1}{t^2 + 1} t dt \\ &= 2 \int \frac{t^3 + t^2 - t}{t^2 + 1} dt \\ &= 2 \left[\int \frac{t^3}{t^2 + 1} dt + \int \frac{t^2}{t^2 + 1} dt - \int \frac{t}{t^2 + 1} dt \right] \end{aligned}$$

$$\therefore I = 2 \left[\int \frac{t^3}{t^2 + 1} dt + \int \frac{t^2}{t^2 + 1} dt - \int \frac{t}{t^2 + 1} dt \right] \text{ -----(ii)}$$

$$\text{Let } I_1 = \int \frac{t^3}{t^2 + 1} dt$$

$$I_2 = \int \frac{t^2}{t^2 + 1} dt$$

$$\text{and } I_3 = \int \frac{t}{t^2 + 1} dt$$

$$\begin{aligned} \text{Now, } I_1 &= \int \frac{t^3}{t^2 + 1} dt \\ &= \int \left(t - \frac{t}{t^2 + 1} \right) dt \\ &= \frac{t^2}{2} - \frac{1}{2} \log(t^2 + 1) \end{aligned}$$

$$\therefore I = \frac{t^2}{2} - \frac{1}{2} \log(t^2 + 1) + c_1 \text{ -----(iii)}$$

$$\begin{aligned} \text{Since, } I_2 &= \int \frac{t^2}{t^2 + 1} dt \\ &= \int \frac{t^2 + 1 - 1}{t^2 + 1} dt \\ &= \int \frac{t^2 + 1}{t^2 + 1} dt - \int \frac{1}{t^2 + 1} dt \\ &= \int dt - \int \frac{1}{t^2 + 1} dt \end{aligned}$$

$$\Rightarrow I_2 = t - \tan^{-1}(t^2) + c_2 \text{ -----(iv)}$$

$$\begin{aligned} \text{and, } I_3 &= \int \frac{t}{t^2+1} dt \\ &= \frac{1}{2} \log(1+t^2) + c_3 \text{----- (v)} \end{aligned}$$

Using equations (ii), (iii), (iv) and (v), we get

$$\begin{aligned} I &= 2 \left[\frac{t^2}{2} - \frac{1}{2} \log(t^2+1) + c_1 + t - \tan^{-1}(t^2) + c_2 - \frac{1}{2} \log(1+t^2) + c_3 \right] \\ &= 2 \left[\frac{t^2}{2} + t - \tan^{-1}(t^2) - \log(1+t^2) + c_1 + c_2 + c_3 \right] \\ &= 2 \left[\frac{t^2}{2} + t - \tan^{-1}(t^2) - \log(1+t^2) + c_4 \right] \quad [\text{Putting } c_1 + c_2 + c_3 = c_4] \\ &= t^2 + 2t - 2 \tan^{-1}(t^2) - 2 \log(1+t^2) + 2c_4 \\ &= (x+1) + 2\sqrt{x+1} - 2 \tan^{-1}(\sqrt{x+1}) - 2 \log(1+x+1) + 2c_4 \\ &= (x+1) + 2\sqrt{x+1} - 2 \tan^{-1}(\sqrt{x+1}) - 2 \log(x+2) + c \quad [\text{Putting } 2c_4 = c] \end{aligned}$$

$$\therefore I = (x+1) + 2\sqrt{x+1} - 2 \tan^{-1}(\sqrt{x+1}) - 2 \log(x+2) + c$$

$$\text{Let } I = \int 5^{5^x} 5^{5^x} 5^x dx \text{ ----- (i)}$$

$$\text{Let } 5^{5^x} = t \text{ then,}$$

$$d(5^{5^x}) = dt$$

$$\Rightarrow 5^{5^x} 5^{5^x} 5^x (\log 5)^3 dx = dt$$

$$\Rightarrow 5^{5^x} 5^{5^x} 5^x dx = \frac{dt}{(\log 5)^3}$$

Putting $5^{5^x} = t$ and $5^{5^x} 5^{5^x} 5^x dx = \frac{dt}{(\log 5)^3}$ in equation (i), we get

$$\begin{aligned} I &= \int \frac{dt}{(\log 5)^3} \\ &= \frac{1}{(\log 5)^3} \int dt \\ &= \frac{t}{(\log 5)^3} + c \end{aligned}$$

$$\therefore I = \frac{5^{5^x}}{(\log 5)^3} + c$$

$$\text{Let } I = \int \frac{1}{x\sqrt{x^4-1}} dx \text{ ----- (i)}$$

$$\text{Let } x^2 = t \text{ then,}$$
$$d(x^2) = dt$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow dx = \frac{dt}{2x}$$

Putting $x^2 = t$ and $dx = \frac{dt}{2x}$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{1}{x\sqrt{t^2-1}} \times \frac{dt}{2x} \\ &= \frac{1}{2} \int \frac{1}{x^2\sqrt{t^2-1}} dt \\ &= \frac{1}{2} \int \frac{1}{t\sqrt{t^2-1}} dt \\ &= \frac{1}{2} \sec^{-1} t + c \\ &= \frac{1}{2} \sec^{-1} x^2 + c \end{aligned}$$

$$\therefore I = \frac{1}{2} \sec^{-1}(x^2) + c$$

$$\text{Let } I = \int \sqrt{e^x - 1} dx \text{ ----- (i)}$$

$$\text{Let } e^x - 1 = t^2 \quad \text{then,}$$

$$d(e^x - 1) = dt(t^2)$$

$$\Rightarrow e^x dx = 2t dt$$

$$\Rightarrow dx = \frac{2t}{e^x} dt$$

$$\Rightarrow dx = \frac{2t}{t^2 + 1} dt \quad \left[\because e^x - 1 = t^2 \right]$$

Putting $e^x - 1 = t^2$ and $dx = \frac{2t dt}{t^2 + 1}$ in equation (i),

we get

$$\begin{aligned} I &= \int \sqrt{t^2} \times \frac{2t dt}{t^2 + 1} \\ &= 2 \int \frac{t \times t}{t^2 + 1} dt \\ &= 2 \int \frac{t^2}{t^2 + 1} dt \\ &= 2 \int \frac{t^2 + 1 - 1}{t^2 + 1} dt \\ &= 2 \int \left[\frac{t^2 + 1}{t^2 + 1} - \frac{1}{t^2 + 1} \right] dt \\ &= 2 \int dt - 2 \int \frac{1}{t^2 + 1} dt \\ &= 2t - 2 \tan^{-1}(t) + c \\ &= 2\sqrt{e^x - 1} - 2 \tan^{-1}(\sqrt{e^x - 1}) + c \end{aligned}$$

$$\therefore I = 2\sqrt{e^x - 1} - 2 \tan^{-1} \sqrt{e^x - 1} + c$$

$$I = \int \frac{1}{(x+1)(x^2+2x+2)} dx$$

$$= \int \frac{1}{(x+1)((x+1)^2+1)} dx$$

Let $x+1 = \tan u$

$$\Rightarrow dx = \sec^2 u du$$

$$\therefore I = \int \frac{\sec^2 u}{\tan u (\tan^2 u + 1)} du$$

$$= \int \frac{\cos u}{\sin u} du$$

$$= \log |\sin u| + C$$

$$= \log \left| \frac{\tan u}{\sec^2 u} \right| + C$$

$$= \log \left| \frac{x+1}{\sqrt{x^2+2x+2}} \right| + C$$

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$$\text{Let } I = \int \frac{x^5}{\sqrt{1+x^3}} dx \text{ ---- (i)}$$

$$\text{Let } 1+x^3 = t^2 \quad \text{then,}$$

$$d(1+x^3) = d(t^2)$$

$$\Rightarrow 3x^2 dx = dt \cdot 2t$$

$$\Rightarrow dx = \frac{dt}{3x^2} \cdot 2t$$

Putting $1+x^3 = t^2$ and $dx = \frac{2t}{3x^2} dt$ in equation (i),

we get

$$I = \int \frac{x^5}{\sqrt{t^2}} \times \frac{2t}{3x^2} dt$$

$$= \int \frac{x^5}{t} \times \frac{2t}{3x^2} dt$$

$$= \frac{2}{3} \int x^3 dt$$

$$= \frac{2}{3} \int (t^2 - 1) dt$$

$$= \frac{2}{3} \times \frac{t^3}{3} - \frac{2}{3} t + c$$

$$\therefore I = \frac{2}{9} (1+x^3)^{\frac{3}{2}} - \frac{2}{3} \sqrt{1+x^3} + c$$

$$\text{Let } I = \int 4x^3 \sqrt{5-x^2} dx \text{ ---- (i)}$$

$$\text{Let } 5-x^2 = t^2 \quad \text{then,}$$

$$d(5-x^2) = 2t^2$$

$$\Rightarrow -2x dx = 2t dt$$

$$\Rightarrow dx = \frac{-t}{x} dt$$

Putting $5-x^2 = t^2$ and $dx = \frac{-t}{x} dt$ in equation (i),

we get

$$I = \int 4x^3 \sqrt{t^2} \times \frac{-t}{x} dt$$

$$= -4 \int x^2 t \times t dt$$

$$= -4 \int (5-t^2) t^2 dt \quad [\because 5-x^2 = t^2]$$

$$= -4 \int (5t^2 - t^4) dt$$

$$= -20 \times \frac{t^3}{3} + 4 \frac{t^5}{5} + c$$

$$= \frac{-20}{3} \times t^3 + \frac{4}{5} \times t^5 + c$$

$$= \frac{-20}{3} \times (5-x^2)^{\frac{3}{2}} + \frac{4}{5} \times (5-x^2)^{\frac{5}{2}} + c$$

$$\therefore I = \frac{-20}{3} \times (5-x^2)^{\frac{3}{2}} + \frac{4}{5} \times (5-x^2)^{\frac{5}{2}} + c$$

$$\text{Let } I = \int \frac{1}{\sqrt{x} + x} dx \text{ ----- (i)}$$

$$\text{Let } \sqrt{x} = t \text{ then,}$$
$$d(\sqrt{x}) = dt$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

Putting $\sqrt{x} = t$ and $2\sqrt{x} dt = dx$ in equation (i),
we get

$$I = \int \frac{1}{t+t^2} 2t \times dt \quad \left[\begin{array}{l} \because \sqrt{x} = t \\ \Rightarrow x = t^2 \end{array} \right]$$

$$= \int \frac{2t}{t(1+t)} dt$$

$$= 2 \int \frac{t}{(1+t)} dt$$

$$= 2 \log|1+t| + c$$

$$= 2 \log|1+\sqrt{x}| + c$$

$$\therefore I = 2 \log|1+\sqrt{x}| + c$$

$$\frac{1}{x^2(x^4+1)^{\frac{3}{4}}}$$

Multiplying and dividing by x^{-3} , we obtain

$$\begin{aligned}\frac{x^{-3}}{x^2 \cdot x^{-3}(x^4+1)^{\frac{3}{4}}} &= \frac{x^{-3}(x^4+1)^{-\frac{3}{4}}}{x^2 \cdot x^{-3}} \\ &= \frac{(x^4+1)^{-\frac{3}{4}}}{x^5 \cdot (x^4)^{\frac{3}{4}}} \\ &= \frac{1}{x^5} \left(\frac{x^4+1}{x^4} \right)^{\frac{3}{4}} \\ &= \frac{1}{x^5} \left(1 + \frac{1}{x^4} \right)^{\frac{3}{4}}\end{aligned}$$

$$\text{Let } \frac{1}{x^4} = t \Rightarrow -\frac{4}{x^5} dx = dt \Rightarrow \frac{1}{x^5} dx = -\frac{dt}{4}$$

$$\begin{aligned}\therefore \int \frac{1}{x^2(x^4+1)^{\frac{3}{4}}} dx &= \int \frac{1}{x^5} \left(1 + \frac{1}{x^4} \right)^{\frac{3}{4}} dx \\ &= -\frac{1}{4} \int (1+t)^{\frac{3}{4}} dt \\ &= -\frac{1}{4} \left[\frac{(1+t)^{\frac{1}{4}}}{\frac{1}{4}} \right] + C \\ &= -\frac{1}{4} \frac{\left(1 + \frac{1}{x^4} \right)^{\frac{1}{4}}}{\frac{1}{4}} + C \\ &= -\left(1 + \frac{1}{x^4} \right)^{\frac{1}{4}} + C\end{aligned}$$

$$\text{Let } I = \int \frac{\sin^5 x}{\cos^4 x} dx \text{ --- (i)}$$

$$\text{Let } \cos x = t \quad \text{then,} \\ d(\cos x) = dt$$

$$\Rightarrow -\sin x dx = dt$$

$$\Rightarrow dx = -\frac{dt}{\sin x}$$

Putting $\cos x = t$ and $dx = -\frac{dt}{\sin x}$ in equation (i),
we get

$$\begin{aligned} I &= \int \frac{\sin^5 x}{t^4} \times -\frac{dt}{\sin x} \\ &= -\int \frac{\sin^4 x}{t^4} dt \\ &= -\int \frac{(1 - \cos^2 x)^2}{t^4} dt \\ &= -\int \frac{(1 - t^2)^2}{t^4} dt \\ &= -\int \frac{1 + t^4 - 2t^2}{t^4} dt \\ &= -\int \left(\frac{1}{t^4} + \frac{t^4}{t^4} - \frac{2t^2}{t^4} \right) dt \\ &= -\int (t^{-4} + 1 - 2t^{-2}) dt \\ &= -\left[\frac{t^{-3}}{-3} + t - 2 \frac{t^{-1}}{-1} \right] + C \\ &= -\left[-\frac{1}{3} \times \frac{1}{t^3} + t + \frac{2}{t} \right] + C \\ &= \frac{1}{3} \times \frac{1}{t^3} - t - \frac{2}{t} + C \\ &= \frac{1}{3} \times \frac{1}{\cos^3 x} - \cos x - \frac{2}{\cos x} + C \end{aligned}$$

$$\therefore I = -\cos x - \frac{2}{\cos x} + \frac{1}{3 \cos^3 x} + C$$