

RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.6

Evaluate the integral as follows

$$\begin{aligned} 1 \quad \int \cos^4 2x dx &= \int (\cos^2 2x)^2 dx \\ &= \int \left(\frac{1}{2}(\cos 4x + 1) \right)^2 dx \\ &= \int \left(\frac{1}{4}(\cos^2 4x) + \frac{1}{4} + \frac{\cos 4x}{2} \right) dx \\ &= \int \left(\frac{1}{4} \left(\frac{1}{2}(\cos 8x + 1) \right) + \frac{1}{4} + \frac{\cos 4x}{2} \right) dx \\ &= \int \frac{1}{8} \left(\cos 8x + \frac{3}{8} + \frac{\cos 4x}{2} \right) dx \\ &= \frac{1}{64} \sin 8x + \frac{3}{8}x + \frac{1}{8} \sin 4x + C \end{aligned}$$

Indefinite Integrals Ex 19.6 Q4

Let $I = \int \sin^2 bx dx$. Then,

$$\begin{aligned} I &= \int \frac{1 - \cos 2bx}{2} dx \\ &= \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2bx dx \\ &= \frac{1}{2}x - \frac{1}{2} \frac{\sin(2bx)}{2b} + c \end{aligned}$$

$$\therefore I = \frac{x}{2} - \frac{\sin 2bx}{4b} + c$$

Indefinite Integrals Ex 19.6 Q5

Let $I = \int \sin^2 \frac{x}{2} dx$. Then,

$$\begin{aligned} I &= \frac{1}{2} \int 2 \sin^2 \frac{x}{2} dx \\ &= \frac{1}{2} \int (1 - \cos x) dx && [\because \cos 2x = 1 - 2 \sin^2 x] \\ &= \frac{1}{2} \int dx - \frac{1}{2} \int \cos x dx \\ &= \frac{1}{2} \times x - \frac{1}{2} \times \sin x + c \\ &= \frac{1}{2} (x - \sin x) + c \end{aligned}$$

$$\therefore I = \frac{1}{2} (x - \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q6

We have,

$$\begin{aligned} \int \cos^2 \frac{x}{2} dx &= \frac{1}{2} \int 2 \cos^2 \frac{x}{2} dx \\ &= \frac{1}{2} \int (1 + \cos x) dx \\ &= \frac{1}{2} \int dx + \frac{1}{2} \int \cos x dx \\ &= \frac{1}{2} \times x + \frac{1}{2} \sin x + c \\ &= \frac{1}{2} (x + \sin x) + c \end{aligned}$$

$$\therefore \int \cos^2 \frac{x}{2} = \frac{1}{2} (x + \sin x) + c.$$

Indefinite Integrals Ex 19.6 Q7

Let $I = \int \cos^2 nx dx$. Then,

$$\begin{aligned} I &= \frac{1}{2} \int 2 \cos^2 nx dx \\ &= \frac{1}{2} \int [1 + \cos 2nx] dx \\ &= \frac{1}{2} \left[x + \frac{\sin 2nx}{2n} \right] + c \\ &= \frac{x}{2} + \frac{1}{4n} \times \sin 2nx + c \end{aligned}$$

$$\therefore I = \frac{x}{2} + \frac{1}{4n} \times \sin 2nx + c.$$

Indefinite Integrals Ex 19.6 Q8

Let $I = \int \sin \sqrt{1 - \cos 2x} dx$. Then,

$$\begin{aligned} I &= \int \sin x \times \sqrt{2 \sin^2 x} \times dx \\ &= \int \sin x \times \sqrt{2} \times \sin x dx \\ &= \sqrt{2} \int \sin^2 x \times dx \\ &= \frac{\sqrt{2}}{2} \int 2 \sin^2 x \times dx \\ &= \frac{\sqrt{2}}{2} \left[x - \frac{\sin 2x}{2} \right] + c \\ &= \frac{\sqrt{2}x}{2} - \frac{\sqrt{2}}{4} \times \sin 2x + c \\ &= \frac{1}{\sqrt{2}} \times x - \frac{\sin 2x}{2\sqrt{2}} + c \end{aligned}$$

$$\therefore I = \frac{1}{\sqrt{2}} \times x - \frac{\sin 2x}{2\sqrt{2}} + c.$$