

RD Sharma
Solutions Class
12 Maths
Chapter 19
Ex 19.29

Indefinite Integrals Ex 19.29 Q1

$$\text{Let } I = \int (x+1)\sqrt{x^2-x+1}dx \quad \text{--- (1)}$$

$$\begin{aligned} \text{Let } x+1 &= \lambda \frac{d}{dx}(x^2-x+1) + \mu \\ &= \lambda(2x-1) + \mu \end{aligned}$$

Equating similar terms, we get,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$-\lambda + \mu = 1$$

$$\Rightarrow \mu = 1 + \lambda = 1 + \frac{1}{2} = \frac{3}{2} \therefore \mu = \frac{3}{2}$$

So,

$$\begin{aligned} I &= \int \left(\frac{1}{2}(2x-1) + \frac{3}{2} \right) \sqrt{x^2-x+1} dx \\ &= \frac{1}{2} \int (2x-1)\sqrt{x^2-x+1} dx + \frac{3}{2} \int \sqrt{x^2-x+1} dx \end{aligned}$$

$$\text{Let } x^2-x+1 = t$$

$$\Rightarrow (2x-1)dx = dt$$

$$= \frac{1}{2} \int \sqrt{t} dt + \frac{3}{2} \int \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$= \frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2} \left[\frac{\left(x - \frac{1}{2}\right)}{2} \sqrt{x^2-x+1} + \frac{3}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2-x+1} \right| \right]$$

$$\Rightarrow I = \frac{1}{3} t^{\frac{3}{2}} + \frac{3}{8} (2x-1) \sqrt{x^2-x+1} + \frac{9}{16} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2-x+1} \right| + c$$

Hence,

$$I = \frac{1}{3} (x^2-x+1)^{\frac{3}{2}} + \frac{3}{8} (2x-1) \sqrt{x^2-x+1} + \frac{9}{16} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2-x+1} \right| + c$$

Indefinite Integrals Ex 19.29 Q2

$$\text{Let } I = \int (x+1)\sqrt{2x^2+3} dx$$

$$\begin{aligned}\text{Let } x+1 &= \lambda \frac{d}{dx}(2x^2+3) + \mu \\ &= \lambda(4x) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}4\lambda &= 1 \Rightarrow \lambda = \frac{1}{4} \\ \mu &= 1\end{aligned}$$

$$\therefore I = \int \frac{1}{4}(4x)\sqrt{2x^2+3} dx + \int 1\sqrt{2x^2+3} dx$$

$$\begin{aligned}\text{Let } 2x^2+3 &= t \\ \Rightarrow 4x dx &= dt\end{aligned}$$

$$\begin{aligned}I &= \frac{1}{4} \int \sqrt{t} dt + \sqrt{2} \int \sqrt{x^2 + \frac{3}{2}} dx \\ &= \frac{1}{4} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \sqrt{2} \left\{ \frac{x}{2} \sqrt{x^2 + \frac{3}{2}} + \frac{3}{4} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| \right\} + c\end{aligned}$$

Hence,

$$I = \frac{1}{6} (2x^2+3)^{\frac{3}{2}} + \frac{x}{2} \sqrt{2x^2+3} + \frac{3}{2\sqrt{2}} \log \left| x + \sqrt{x^2 + \frac{3}{2}} \right| + c$$

$$\text{Let } I = \int (2x - 5) \sqrt{2 + 3x - x^2} dx$$

$$\begin{aligned} \text{Let } 2x - 5 &= \lambda \frac{d}{dx} (2 + 3x - x^2) + \mu \\ &= \lambda (3 - 2x) + \mu \end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned} -2\lambda &= 2 & \Rightarrow & \lambda = -1 \\ 3\lambda + \mu &= -5 & \Rightarrow & \mu = -5 - 3\lambda = -2 \end{aligned}$$

$$\therefore \mu = -2$$

So,

$$\begin{aligned} I &= \int \{-1(3 - 2x) - 2\} \sqrt{2 + 3x - x^2} dx \\ &= -\int (3 - 2x) \sqrt{2 + 3x - x^2} dx - 2 \int \sqrt{2 + 3x - x^2} dx \end{aligned}$$

$$\text{Let } 2 + 3x - x^2 = t$$

$$\Rightarrow (3 - 2x) dx = dt$$

$$\begin{aligned} I &= -\int \sqrt{t} dt - 2 \int \sqrt{\frac{17}{4} - \left(\frac{9}{4} - 3x - x^2\right)} dx \\ &= -\frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2 \int \sqrt{\left(\frac{\sqrt{17}}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2} dx \end{aligned}$$

$$\Rightarrow I = -\frac{2}{3} (2 + 3x - x^2)^{\frac{3}{2}} - 2 \left\{ \frac{\left(x - \frac{3}{2}\right)}{2} \sqrt{2 + 3x - x^2} + \frac{17}{8} \sin^{-1} \left(\frac{x - \frac{3}{2}}{\frac{\sqrt{17}}{2}} \right) \right\} + c$$

Hence,

$$I = -\frac{2}{3} (2 + 3x - x^2)^{\frac{3}{2}} - \frac{(2x - 3)}{2} \sqrt{2 + 3x - x^2} - \frac{17}{8} \sin^{-1} \left(\frac{2x - 3}{\sqrt{17}} \right) + c$$

$$\text{Let } I = \int (x+2)\sqrt{x^2+x+1} dx$$

$$\begin{aligned}\text{Let } x+2 &= \lambda \frac{d}{dx}(x^2+x+1) + \mu \\ &= \lambda(2x+1) + \mu\end{aligned}$$

Equating similar terms, we get,

$$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$$

$$\lambda + \mu = 2 \quad \Rightarrow \quad \mu = 2 - \lambda = \frac{3}{2}$$

$$\therefore \quad \mu = \frac{3}{2}$$

$$\begin{aligned}\therefore \quad I &= \int \left(\frac{1}{2}(2x+1) + \frac{3}{2} \right) \sqrt{x^2+x+1} dx \\ &= \frac{1}{2} \int (2x+1)\sqrt{x^2+x+1} + \frac{3}{2} \int \sqrt{x^2+x+1} dx\end{aligned}$$

$$\begin{aligned}\text{Let } x^2+x+1 &= t \\ (2x+1)dx &= dt\end{aligned}$$

$$\therefore \quad I = \frac{1}{2} \int \sqrt{t} dt + \frac{3}{2} \int \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$\Rightarrow \quad I = \frac{1}{2} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{3}{2} \left\{ \frac{\left(x + \frac{1}{2}\right)}{2} \sqrt{x^2+x+1} + \frac{3}{8} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + c \right.$$

Hence,

$$I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{3}{8} (2x+1) \sqrt{x^2+x+1} + \frac{9}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + c$$

$$\text{Let } I = \int (4x + 1) \sqrt{x^2 - x - 2} dx$$

$$\begin{aligned} \text{Let } 4x + 1 &= \lambda \frac{d}{dx} (x^2 - x - 2) + \mu \\ &= \lambda (2x - 1) + \mu \end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned} 2\lambda &= 4 & \Rightarrow & \lambda = 2 \\ -\lambda + \mu &= 1 & \Rightarrow & \mu = 3 \end{aligned}$$

So,

$$\begin{aligned} I &= \int (2(2x - 1) + 3) \sqrt{x^2 - x - 2} dx \\ &= 2 \int (2x - 1) \sqrt{x^2 - x - 2} dx + 3 \int \sqrt{x^2 - x - 2} dx \end{aligned}$$

$$\begin{aligned} \text{Let } x^2 - x - 2 &= t \\ (2x - 1) dx &= dt \end{aligned}$$

$$\begin{aligned} \therefore I &= 2 \int \sqrt{t} dt + 3 \int \sqrt{\left(x - \frac{1}{2}\right)^2 - \left(\frac{3}{2}\right)^2} dx \\ \Rightarrow I &= 2 \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + 3 \left\{ \frac{\left(x - \frac{1}{2}\right)}{2} \sqrt{x^2 - x - 2} - \frac{9}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2} \right| \right\} + c \end{aligned}$$

Hence,

$$I = \frac{4}{3} (x^2 - x - 2)^{\frac{3}{2}} + \frac{3}{4} (2x - 1) \sqrt{x^2 - x - 2} - \frac{27}{8} \log \left| \left(x - \frac{1}{2}\right) + \sqrt{x^2 - x - 2} \right| + c$$

$$\text{Let } I = \int (x-2)\sqrt{2x^2-6x+5} dx$$

$$\begin{aligned}\text{Let } x-2 &= \lambda \frac{d}{dx}(2x^2-6x+5) + \mu \\ &= \lambda(4x-6) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}4\lambda &= 1 & \Rightarrow & \lambda = \frac{1}{4} \\ -6\lambda + \mu &= -2 & \Rightarrow & \mu = -2 + 6\lambda = -\frac{2}{4} = -\frac{1}{2} \\ \therefore \mu &= -\frac{1}{2}\end{aligned}$$

So,

$$\begin{aligned}I &= \int \left(\frac{1}{4}(4x-6) + \left(-\frac{1}{2}\right) \right) \sqrt{2x^2-6x+5} dx \\ &= \frac{1}{4} \int (4x-6)\sqrt{2x^2-6x+5} - \frac{1}{2} \int \sqrt{2x^2-6x+5} dx\end{aligned}$$

$$\begin{aligned}\text{Let } 2x^2-6x+5 &= t \\ (4x-6)dx &= dt\end{aligned}$$

$$\begin{aligned}\therefore I &= \frac{1}{4} \int \sqrt{t} dt - \frac{\sqrt{2}}{2} \int \sqrt{x^2-3x+\frac{5}{2}} dx \\ \Rightarrow I &= \frac{1}{4} \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{\sqrt{2}} \int \sqrt{\left(x-\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} dx \\ &= \frac{1}{6} (2x^2-6x+5)^{\frac{3}{2}} - \frac{1}{\sqrt{2}} \left\{ \frac{\left(x-\frac{3}{2}\right)}{2} \sqrt{x^2-3x+\frac{5}{2}} + \frac{1}{8} \log \left| \left(x-\frac{3}{2}\right) + \sqrt{x^2-3x+\frac{5}{2}} \right| + c\right.\end{aligned}$$

Hence,

$$I = \frac{1}{6} (2x^2-6x+5)^{\frac{3}{2}} - \frac{1}{8} (2x-3)\sqrt{2x^2-6x+5} - \frac{1}{8\sqrt{2}} \log \left| \left(x-\frac{3}{2}\right) + \sqrt{x^2-3x+\frac{5}{2}} \right| + c$$

$$\text{Let } I = \int (x+1) \sqrt{x^2+x+1} dx$$

$$\begin{aligned}\text{Let } x+1 &= \lambda \frac{d}{dx} (x^2+x+1) + \mu \\ &= \lambda (2x+1) + \mu\end{aligned}$$

Equating similar terms, we get,

$$2\lambda = 1 \Rightarrow \lambda = \frac{1}{2}$$

$$\lambda + \mu = 1 \Rightarrow \mu = \frac{1}{2}$$

So,

$$\begin{aligned}I &= \int \left(\frac{1}{2}(2x+1) + \frac{1}{2} \right) \sqrt{x^2+x+1} dx \\ &= \frac{1}{2} \int (2x+1) \sqrt{x^2+x+1} dx + \frac{1}{2} \int \sqrt{x^2+x+1} dx\end{aligned}$$

$$\text{Let } x^2+x+1 = t$$

$$\Rightarrow (2x+1) dx = dt$$

$$= \frac{1}{2} \int \sqrt{t} dt + \frac{1}{2} \int \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$= \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{1}{2} \left\{ \frac{\left(x + \frac{1}{2}\right) \sqrt{x^2+x+1}}{2} + \frac{3}{8} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| \right\} + c$$

$$\Rightarrow I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{1}{8} (2x+1) \sqrt{x^2+x+1} + \frac{3}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + c$$

Hence,

$$I = \frac{1}{3} (x^2+x+1)^{\frac{3}{2}} + \frac{1}{8} (2x+1) \sqrt{x^2+x+1} + \frac{3}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x+1} \right| + c$$

$$\text{Let } I = \int (2x + 3) \sqrt{x^2 + 4x + 3} dx$$

$$\begin{aligned}\text{Let } (2x + 3) &= \lambda \frac{d}{dx}(x^2 + 4x + 3) + \mu \\ &= \lambda(2x + 4) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}\lambda &= 1 \text{ and } 4\lambda + \mu = 3 \\ \Rightarrow \quad \mu &= -1\end{aligned}$$

So,

$$\begin{aligned}I &= \int \left((2x + 4) + (-1) \right) \sqrt{x^2 + 4x + 3} dx \\ &= \int (2x + 4) \sqrt{x^2 + 4x + 3} dx - \int \sqrt{x^2 + 4x + 3} dx\end{aligned}$$

$$\begin{aligned}\text{Let } x^2 + 4x + 3 &= t \\ \Rightarrow \quad (2x + 4) dx &= dt\end{aligned}$$

$$\begin{aligned}\therefore \quad I &= \int \sqrt{t} dt - \int \sqrt{(x+2)^2 - 1} dx \\ &= \frac{3}{2} t^{\frac{3}{2}} - \frac{(x+2)}{2} \sqrt{x^2 + 4x + 3} + \frac{1}{2} \log \left| (x+2) + \sqrt{x^2 + 4x + 3} \right| + c\end{aligned}$$

Hence,

$$I = \frac{2}{3} (x^2 + 4x + 3)^{\frac{3}{2}} - \left(\frac{x+2}{2} \right) \sqrt{x^2 + 4x + 3} + \frac{1}{2} \log \left| (x+2) + \sqrt{x^2 + 4x + 3} \right| + c$$

$$\text{Let } I = \int (2x - 5) \sqrt{x^2 - 4x + 3} dx$$

$$\begin{aligned}\text{Let } (2x - 5) &= \lambda \frac{d}{dx} (x^2 - 4x + 3) + \mu \\ &= \lambda (2x - 4) + \mu\end{aligned}$$

Equating similar terms, we get,

$$\begin{aligned}\lambda &= 1 \text{ and } -4\lambda + \mu = -5 \\ \Rightarrow \quad \mu &= -1\end{aligned}$$

So,

$$\begin{aligned}I &= \int ((2x - 4) - 1) \sqrt{x^2 - 4x + 3} dx \\ &= \int (2x - 4) \sqrt{x^2 - 4x + 3} dx - \int \sqrt{x^2 - 4x + 3} dx\end{aligned}$$

$$\begin{aligned}\text{Let } x^2 - 4x + 3 &= t \\ \Rightarrow \quad 2x - 4 dx &= dt\end{aligned}$$

$$\begin{aligned}I &= \int \sqrt{t} dt - \int \sqrt{(x-2)^2 - 1} dx \\ &= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{(x-2)}{2} \sqrt{x^2 - 4x + 3} + \frac{1}{2} \log \left| (x-2) + \sqrt{x^2 - 4x + 3} \right| + c\end{aligned}$$

Thus,

$$I = \frac{2}{3} (x^2 - 4x + 3)^{\frac{3}{2}} - \frac{1}{2} (x - 2) \sqrt{x^2 - 4x + 3} + \frac{1}{2} \log \left| (x - 2) + \sqrt{x^2 - 4x + 3} \right| + c$$

$$\text{Let } I = \int x\sqrt{x^2+x} dx$$

$$\begin{aligned}\text{Let } x &= \lambda \frac{d}{dx}(x^2+x) + \mu \\ &= \lambda(2x+1) + \mu\end{aligned}$$

Equating similar terms, we get,

$$2\lambda = 1 \quad \Rightarrow \quad \lambda = \frac{1}{2}$$

$$\lambda + \mu = 0 \quad \Rightarrow \quad \mu = -\frac{1}{2}$$

So,

$$\begin{aligned}I &= \int \left(\frac{1}{2}(2x+1) - \frac{1}{2} \right) \sqrt{x^2+x} dx \\ &= \frac{1}{2} \int (2x+1) \sqrt{x^2+x} - \frac{1}{2} \int \sqrt{x^2+x} dx\end{aligned}$$

$$\text{Let } x^2+x = t$$

$$\Rightarrow (2x+1) dx = dt$$

So,

$$\begin{aligned}I &= \frac{1}{2} \int \sqrt{t} dt - \frac{1}{2} \int \sqrt{\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \\ I &= \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2} \left\{ \frac{\left(x + \frac{1}{2}\right)}{2} \sqrt{x^2+x} - \frac{1}{8} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x} \right| + c \right.\end{aligned}$$

Hence,

$$I = \frac{1}{3} (x^2+x)^{\frac{3}{2}} - \frac{1}{8} \left(x + \frac{1}{2}\right) \sqrt{x^2+x} + \frac{1}{16} \log \left| \left(x + \frac{1}{2}\right) + \sqrt{x^2+x} \right| + c$$

Consider the integral $I = \int (x - 3)\sqrt{x^2 + 3x - 18} dx$

Let us express $x - 3 = \lambda \frac{d}{dx} [x^2 + 3x - 18] + \mu$

$$\Rightarrow x - 3 = \lambda[2x + 3] + \mu$$

$$\Rightarrow x - 3 = 2\lambda x + 3\lambda + \mu$$

Comparing the coefficients, we have,

$$2\lambda = 1 \text{ and } 3\lambda + \mu = -3$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } 3 \times \frac{1}{2} + \mu = -3$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } \mu = -3 - \frac{3}{2}$$

$$\Rightarrow \lambda = \frac{1}{2} \text{ and } \mu = -\frac{9}{2}$$

Then

$$x - 3 = \lambda[2x + 3] + \mu$$

Now the integral $I = \int (x - 3)\sqrt{x^2 + 3x - 18} dx$

$$= \int \left[\frac{1}{2}[2x + 3] - \frac{9}{2} \right] \sqrt{x^2 + 3x - 18} dx$$

$$I = \frac{1}{2} \int [2x + 3] \sqrt{x^2 + 3x - 18} dx - \frac{9}{2} \int \sqrt{x^2 + 3x - 18} dx$$

$$\Rightarrow I = I_1 + I_2$$

where, $I_1 = \frac{1}{2} \int [2x + 3] \sqrt{x^2 + 3x - 18} dx$ and

$$I_2 = -\frac{9}{2} \int \sqrt{x^2 + 3x - 18} dx$$

Let us consider the integral, I_1 :

$$I_1 = \frac{1}{2} \int [2x + 3] \sqrt{x^2 + 3x - 18} dx$$

Substituting, $x^2 + 3x - 18 = t$

$$\Rightarrow (2x + 3) dx = dt$$

Thus,

$$I_1 = \frac{1}{2} \int \sqrt{t} dt$$

$$= \frac{1}{2} \times \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{1}{2} \times \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{1}{2} \times \frac{2}{3} \times t^{\frac{3}{2}} + C$$

$$= \frac{1}{3} \times t^{\frac{3}{2}} + C$$

$$= \frac{1}{3} \times (x^2 + 3x - 18)^{\frac{3}{2}} + C$$

Now consider the integral

$$I_2 = -\frac{9}{2} \int \sqrt{x^2 + 3x - 18} \, dx$$

$$= -\frac{9}{2} \int \sqrt{x^2 + 2 \times \frac{3}{2}x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 18} \, dx$$

$$= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \frac{9}{4} - 18} \, dx$$

$$= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{4} + 18\right)} \, dx$$

$$= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9+72}{4}\right)} \, dx$$

$$= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{81}{4}\right)} \, dx$$

$$= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} \, dx$$

$$= -\frac{9}{2} \int \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} \, dx$$

We know that $\int \sqrt{x^2 - a^2} \, dx = \frac{1}{2}x\sqrt{x^2 - a^2} - \frac{1}{2}a^2 \log |x + \sqrt{x^2 - a^2}| + C$

$$\therefore I_2 = -\frac{9}{2} \left\{ \frac{1}{2} \left(x + \frac{3}{2}\right) \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} - \frac{1}{2} \left(\frac{9}{2}\right)^2 \log \left| \left(x + \frac{3}{2}\right) + \sqrt{\left(x + \frac{3}{2}\right)^2 - \left(\frac{9}{2}\right)^2} \right| \right\} + C$$

$$= -\frac{9}{4} \left\{ \left(\frac{2x+3}{2}\right) \sqrt{x^2 + 3x - 18} - \left(\frac{729}{4}\right) \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18} \right| \right\} + C$$

$$= -\frac{9}{8} (2x+3) \sqrt{x^2 + 3x - 18} + \frac{729}{16} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18} \right| + C$$

$$\text{Thus, } I = \frac{1}{3} (x^2 + 3x - 18)^{\frac{3}{2}} - \frac{9}{8} (2x+3) \sqrt{x^2 + 3x - 18} + \frac{729}{16} \log \left| \left(x + \frac{3}{2}\right) + \sqrt{x^2 + 3x - 18} \right| + C$$

$$\int (x+3)\sqrt{3-4x-x^2} dx$$

$$\text{Let } x+3 = A \frac{d}{dx}(3-4x-x^2) + B$$

$$x+3 = A(-4-2x) + B$$

$$x+3 = -2Ax + B - 4A$$

$$-2A = 1, B - 4A = 3$$

$$A = -\frac{1}{2},$$

$$B = 4x \left(-\frac{1}{2}\right) + 3 = 1$$

$$\int (x+3)\sqrt{3-4x-x^2} dx$$

$$x+3 = -\frac{1}{2}(-4-2x) + 1$$

$$\int \left[-\frac{1}{2}(-4-2x) + 1\right] \sqrt{3-4x-x^2} dx$$

$$= -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} dx + \int \sqrt{3-4x-x^2} dx$$

$$= I_1 + I_2 \dots \dots (i)$$

$$I_1 = -\frac{1}{2} \int (-4-2x)\sqrt{3-4x-x^2} dx$$

$$\text{Let } z = 3-4x-x^2$$

$$dz = -4-2x$$

$$I_1 = -\frac{1}{2} \int \sqrt{z} dz$$

$$= -\frac{1}{2} \left[\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]$$

$$= -\frac{1}{2} \left[\frac{z^{\frac{3}{2}}}{\frac{3}{2}} \right]$$

$$= -\left[\frac{(3-4x-x^2)^{\frac{3}{2}}}{3} \right]$$

$$I_2 = \int \sqrt{3-4x-x^2} dx$$

$$= \int \sqrt{3-(x^2+4x+4)+4} dx$$

$$\begin{aligned}
&= \int \sqrt{7 - (x^2 + 4x + 4)} dx \\
&= \int \sqrt{(\sqrt{7})^2 - (x + 2)^2} dx \\
&= \frac{(x + 2)\sqrt{(\sqrt{7})^2 - (x + 2)^2}}{2} + \frac{1}{2}(\sqrt{7})^2 \tan^{-1}\left(\frac{x + 2}{\sqrt{7}}\right) + C \\
&= \frac{(x + 2)\sqrt{3 - 4x - x^2}}{2} + \frac{7}{2} \tan^{-1}\left(\frac{x + 2}{\sqrt{7}}\right)
\end{aligned}$$

From (i),

$$\begin{aligned}
&= I_1 + I_2 \\
&= -\frac{1}{3}(3 - 4x - x^2)^{\frac{3}{2}} + \frac{1}{2}(x + 2)\sqrt{3 - 4x - x^2} + \frac{7}{2} \tan^{-1}\left(\frac{x + 2}{\sqrt{7}}\right) + C
\end{aligned}$$