

**RD Sharma**  
**Solutions Class**  
**12 Maths**  
**Chapter 19**  
**Ex 19.25**

### Indefinite Integrals Ex 19.25 Q1

$$\text{Let } I = \int x \cos x \, dx$$

Using integration by parts,

$$\begin{aligned} I &= x \int \cos x \, dx - \int (1 \times \int \cos x \, dx) \, dx + c \\ &= x \sin x - \int \sin x \, dx + c \end{aligned}$$

$$I = x \sin x + \cos x + c$$

### Indefinite Integrals Ex 19.25 Q2

$$\begin{aligned} \text{Let } I &= \int \log(x+1) \, dx \\ &= \int 1 \times \log(x+1) \, dx \end{aligned}$$

Using integration by parts,

$$\begin{aligned} I &= \log(x+1) \int 1 \, dx - \int \left( \frac{1}{x+1} \times \int 1 \, dx \right) \, dx + c \\ &= x \log(x+1) - \int \left( \frac{x}{x+1} \right) \, dx + c \\ &= x \log(x+1) - \int \left( 1 - \frac{1}{x+1} \right) \, dx + c \end{aligned}$$

$$I = x \log(x+1) - x + \log(x+1) + c$$

### Indefinite Integrals Ex 19.25 Q3

$$\text{Let } I = \int x^3 \log x \, dx$$

Using integration by parts,

$$\begin{aligned} I &= \log x \int x^3 \, dx - \int \left( \frac{1}{x} \times \int x^3 \, dx \right) dx + c \\ &= \frac{x^4}{4} \log x - \int \frac{x^4}{4x} \, dx + c \\ &= \frac{x^4}{4} \log x - \frac{1}{4} \int x^3 \, dx + c \\ &= \frac{x^4}{4} \log x - \frac{1}{4} \int \frac{x^4}{4} \, dx + c \end{aligned}$$

$$I = \frac{x^4}{4} \log x - \frac{1}{16} x^4 + c$$

### Indefinite Integrals Ex 19.25 Q4

Take first function as  $x$  and second function as  $e^x$ . The integral of the second function is  $e^x$ .  
Therefore,  $\int x e^x \, dx = x e^x - \int 1 \cdot e^x \, dx = x e^x - e^x + C$ .

### Indefinite Integrals Ex 19.25 Q5

$$\text{Let } I = \int x e^{2x} \, dx$$

Using integration by parts,

$$\begin{aligned} I &= x \int e^{2x} \, dx - \int (1 \times \int e^{2x} \, dx) dx + c \\ &= \frac{x e^{2x}}{2} - \int \left( \frac{e^{2x}}{2} \right) dx + c \\ &= \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + c \end{aligned}$$

$$I = \left( \frac{x}{2} - \frac{1}{4} \right) e^{2x} + c$$

### Indefinite Integrals Ex 19.25 Q6

$$\text{Let } I = \int x^2 e^{-x} dx$$

Using integration by parts,

$$\begin{aligned} I &= x^2 \int e^{-x} dx - \int (2x) e^{-x} dx \\ &= -x^2 e^{-x} - \int (2x) (-e^{-x}) \\ &= -x^2 e^{-x} + 2 \int x e^{-x} dx \\ &= -x^2 e^{-x} + 2 \left[ x \int e^{-x} dx - \int (1 \times \int e^{-x} dx) dx \right] \\ &= -x^2 e^{-x} + 2 \left[ x (-e^{-x}) - \int (-e^{-x}) dx \right] \\ &= -x^2 e^{-x} - 2x e^{-x} + 2 \int e^{-x} dx \\ I &= -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} + c \\ I &= -e^{-x} (x^2 + 2x + 2) + c \end{aligned}$$

### Indefinite Integrals Ex 19.25 Q7

$$\text{Let } I = \int x^2 \cos x dx$$

Using integration by parts,

$$\begin{aligned} I &= x^2 \int \cos x dx - \int (2x) (\cos x) dx \\ &= x^2 \sin x - 2 \int (x) (\sin x) dx \\ &= x^2 \sin x - 2 \left[ x \int \sin x dx - \int (1 \times \int \sin x dx) dx \right] \\ &= x^2 \sin x - 2 \left[ x (-\cos x) - \int (-\cos x) dx \right] \\ &= x^2 \sin x + 2x \cos x - 2 \int (\cos x) dx \\ I &= x^2 \sin x + 2x \cos x - 2 \sin x + c \end{aligned}$$

### Indefinite Integrals Ex 19.25 Q8

$$\text{Let } I = \int x^2 \cos 2x \, dx$$

Using integration by parts,

$$\begin{aligned} I &= x^2 \int \cos 2x \, dx - \int (2x) \cos 2x \, dx \\ &= x^2 \frac{\sin 2x}{2} - 2 \int x \left( \frac{\sin 2x}{2} \right) dx \\ &= \frac{1}{2} x^2 \sin 2x - \int x \sin 2x \, dx \\ &= \frac{1}{2} x^2 \sin 2x - \left[ x \int \sin 2x \, dx - \int (1) \sin 2x \, dx \right] \\ &= \frac{1}{2} x^2 \sin 2x - \left[ x \left( -\frac{\cos 2x}{2} \right) - \int \left( -\frac{\cos 2x}{2} \right) dx \right] \\ &= \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{2} \int (\cos 2x) \, dx \\ \\ I &= \frac{1}{2} x^2 \sin 2x + \frac{x}{2} \cos 2x - \frac{1}{4} \sin 2x + c \end{aligned}$$

### Indefinite Integrals Ex 19.25 Q9

$$\text{Let } I = \int x \sin 2x \, dx$$

Using integration by parts,

$$\begin{aligned} I &= x \int \sin 2x \, dx - \int (1) \sin 2x \, dx \\ &= x \left( -\frac{\cos 2x}{2} \right) - \int \left( -\frac{\cos 2x}{2} \right) dx \\ &= -\frac{x}{2} \cos 2x + \frac{1}{2} \int \cos 2x \, dx \\ &= -\frac{x}{2} \cos 2x + \frac{1}{2} \frac{\sin 2x}{2} + c \\ I &= -\frac{x}{2} \cos 2x + \frac{1}{4} \sin 2x + c \end{aligned}$$

### Indefinite Integrals Ex 19.25 Q10

$$\begin{aligned}\text{Let } I &= \int \frac{\log(\log x)}{x} dx \\ &= \int \left(\frac{1}{x}\right) (\log(\log x)) dx\end{aligned}$$

Using integration by parts,

$$\begin{aligned}I &= \log \log x \int \frac{1}{x} dx - \int \left(\frac{1}{x \log x} \int \frac{1}{x} dx\right) dx \\ &= \log x \times \log(\log x) - \int \left(\frac{1}{x \log x} \log x\right) dx \\ &= \log x \times \log(\log x) - \int \frac{1}{x} dx \\ &= \log x \times \log(\log x) - \log x + c \\ \\ I &= \log x (\log \log x - 1) + c\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q11

$$\text{Let } I = \int x^2 \cos x dx$$

Using integration by parts,

$$\begin{aligned}I &= x^2 \int \cos x dx - \int (2x \int \cos x dx) dx \\ &= x^2 \sin x - 2 \int x \sin x dx \\ &= x^2 \sin x - 2 \left[ x \int \sin x dx - \int (1 \int \sin x dx) dx \right] \\ &= x^2 \sin x - 2 \left[ x (-\cos x) - \int (-\cos x) dx \right] \\ &= x^2 \sin x + 2x \cos x - 2 \int (\cos x) dx \\ \\ I &= x^2 \sin x + 2x \cos x - 2 \sin x + c\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q12

$$\text{Let } I = \int x \operatorname{cosec}^2 x dx$$

Using integration by parts,

$$\begin{aligned}I &= x \int \operatorname{cosec}^2 x dx - \int (\int \operatorname{cosec}^2 x dx) dx \\ &= -x \cot x + \int \cot x dx \\ \\ &= -x \cot x + \log |\sin x| + c\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q13

$$\text{Let } I = \int x \cos^2 x \, dx$$

Using integration by parts,

$$\begin{aligned} I &= x \int \cos^2 x \, dx - \int (1) \left( \int \cos^2 x \, dx \right) dx \\ &= x \int \left( \frac{\cos 2x + 1}{2} \right) dx - \int \left( \int \left( \frac{1 + \cos 2x}{2} \right) dx \right) dx \\ &= \frac{x}{2} \left[ \frac{\sin 2x}{2} + x \right] - \frac{1}{2} \int \left( x + \frac{\sin 2x}{2} \right) dx \\ &= \frac{x}{4} \sin 2x + \frac{x^2}{2} - \frac{1}{2} \times \frac{x^2}{2} - \frac{1}{4} \left( -\frac{\cos 2x}{2} \right) + c \end{aligned}$$

$$I = \frac{x}{4} \sin 2x + \frac{x^2}{4} + \frac{1}{8} \cos 2x + c$$

### Indefinite Integrals Ex 19.25 Q14

$$\text{Let } I = \int x^n \log x \, dx$$

Using integration by parts,

$$\begin{aligned} I &= \log x \int x^2 \, dx - \int \left( \frac{1}{x} \int x^2 \, dx \right) dx \\ &= \frac{x^{n+1}}{n+1} \log x - \int \left( \frac{1}{x} \times \frac{x^{n+1}}{n+1} \right) dx \\ &= \frac{x^{n+1}}{n+1} \log x - \int \left( \frac{x^n}{n+1} \right) dx \end{aligned}$$

$$I = \frac{x^{n+1}}{n+1} \log x - \frac{1}{(n+1)^2} x^{n+1} + c$$

### Indefinite Integrals Ex 19.25 Q15

$$\int \frac{\log x}{x^n} \, dx = \int (\log x) \left( \frac{1}{x^n} \right) dx$$

by integration by parts

$$\begin{aligned} \int (\log x) \left( \frac{1}{x^n} \right) dx &= \log x \int \left( \frac{1}{x^n} \right) dx - \int \left( \frac{d(\log x)}{dx} \right) \left( \int \left( \frac{1}{x^n} \right) dx \right) dx \\ &= \log x \left( \frac{x^{1-n}}{1-n} \right) - \int \frac{1}{x} \left( \frac{x^{1-n}}{1-n} \right) dx = \log x \left( \frac{x^{1-n}}{1-n} \right) - \int \left( \frac{x^{-n}}{1-n} \right) dx \\ &= \log x \left( \frac{x^{1-n}}{1-n} \right) - \left( \frac{1}{1-n} \right) \left( \frac{x^{1-n}}{1-n} \right) = \log x \left( \frac{x^{1-n}}{1-n} \right) - \left( \frac{x^{1-n}}{[1-n]^2} \right) + C \end{aligned}$$

### Indefinite Integrals Ex 19.25 Q16

$$\begin{aligned}
\text{Let } I &= \int x^2 \sin^2 x \, dx \\
&= \int x^2 \left( \frac{1 - \cos 2x}{2} \right) dx \\
&= \int \frac{x^2}{2} dx - \int \left( \frac{x^2 \cos 2x}{2} \right) dx \\
&= \frac{x^3}{6} - \frac{1}{2} \left[ \int x^2 \cos 2x \, dx \right] \\
&= \frac{x^3}{6} - \frac{1}{2} \left[ x^2 \int \cos 2x \, dx - \int (2x) \cos 2x \, dx \right] \\
&= \frac{x^3}{6} - \frac{1}{2} \left( x^2 \frac{\sin 2x}{2} \right) + \frac{1}{2} \times 2 \int \left( x \frac{\sin 2x}{2} \right) dx \\
&= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2} \left[ x \int \sin 2x \, dx - \int (1) \sin 2x \, dx \right] \\
&= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x + \frac{1}{2} \left[ x \left( -\frac{\cos 2x}{2} \right) - \int \left( -\frac{\cos 2x}{2} \right) dx \right] \\
&= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{4} \frac{\sin 2x}{2} + c \\
\\
I &= \frac{x^3}{6} - \frac{1}{4} x^2 \sin 2x - \frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c
\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q17

$$\text{Let } I = \int 2x^3 e^{x^2} x \, dx$$

$$\text{Let } x^2 = t$$

$$2x \, dx = dt$$

$$I = \int t \times e^t dt$$

Using integration by parts,

$$= t \int e^t dt - \int (1 \times \int e^t dt) dt$$

$$= te^t - \int e^t dt$$

$$= te^t - e^t + c$$

$$= e^t (t - 1) + c$$

$$I = e^{x^2} (x^2 - 1) + c$$

### Indefinite Integrals Ex 19.25 Q18



$$\text{Let } I = \int x^3 \cos x^2 dx$$

$$\text{Let } x^2 = t$$

$$2x dx = dt$$

$$I = \frac{1}{2} \int t \cos t dt$$

Using integration by parts,

$$= \frac{1}{2} [t \int \cos t dt - \int (1 \times \int \cos t dt) dt]$$

$$= \frac{1}{2} [t \times \sin t - \int \sin t dt]$$

$$= \frac{1}{2} [t \sin t + \cos t] + c$$

$$I = \frac{1}{2} [x^2 \sin x^2 + \cos x^2] + c$$

### Indefinite Integrals Ex 19.25 Q19

$$\text{Let } I = \int x \sin x \cos x dx$$

$$= \int \frac{x}{2} (2 \sin x \cos x) dx$$

$$= \frac{1}{2} \int x \sin 2x dx$$

Using integration by parts,

$$= \frac{1}{2} [x \int \sin 2x dx - \int (1 \times \int \sin 2x dx) dx]$$

$$= \frac{1}{2} \left[ x \left( \frac{-\cos 2x}{2} \right) - \int \left( \frac{-\cos 2x}{2} \right) dx \right]$$

$$= -\frac{1}{4} x \cos 2x + \frac{1}{4} \int \cos 2x dx$$

$$I = -\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + c$$

### Indefinite Integrals Ex 19.25 Q20

$$\text{Let } I = \int \sin x (\log \cos x) dx$$

$$\text{Let } \cos x = t$$

$$-\sin x dx = dt$$

$$I = -\int \log t dt$$

$$= -\int 1 \times \log t dt$$

Using integration by parts,

$$= -\left[ \log t \int dt - \int \left( \frac{1}{t} \times \int dt \right) dt \right]$$

$$= -\left[ t \log t - \int \frac{1}{t} \times t dt \right]$$

$$= -[t \log t - \int dt]$$

$$= -[t \log t - t + c_1]$$

$$= t(1 - t \log t) + c$$

$$I = \cos x (1 - \log \cos x) + c$$

### Indefinite Integrals Ex 19.25 Q21

$$\text{Let } I = \int (\log x)^2 x dx$$

Using integration by parts,

$$= (\log x)^2 \int x dx - \int \left( 2(\log x) \left( \frac{1}{x} \right) \int x dx \right) dx$$

$$= \frac{x^2}{2} (\log x)^2 - 2 \int (\log x) \left( \frac{1}{x} \right) \left( \frac{x^2}{2} \right) dx$$

$$= \frac{x^2}{2} (\log x)^2 - \int x (\log x) dx$$

$$= \frac{x^2}{2} (\log x)^2 - \left[ \log x \int x dx - \int \left( \frac{1}{x} \int x dx \right) dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \left[ \frac{x^2}{2} \log x - \int \left( \frac{1}{x} \times \frac{x^2}{2} \right) dx \right]$$

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{2} \int x dx$$

$$= \frac{x^2}{2} (\log x)^2 - \frac{x^2}{2} \log x + \frac{1}{4} x^2 + c$$

$$I = \frac{x^2}{2} \left[ (\log x)^2 - \log x + \frac{1}{2} \right] + c$$

### Indefinite Integrals Ex 19.25 Q22

$$\text{Let } I = \int e^{\sqrt{x}} dx$$

$$\text{Let } \sqrt{x} = t$$

$$x = t^2$$

$$dx = 2t dt$$

$$I = 2 \int e^t t dt$$

$$I = 2 \left[ t \int e^t dt - \int (1) e^t dt \right]$$

$$I = 2 \left[ t e^t - \int e^t dt \right]$$

$$= 2 \left[ t e^t - e^t \right] + c$$

$$= 2e^t (t - 1) + c$$

$$I = 2e^{\sqrt{x}} (\sqrt{x} - 1) + c$$

### Indefinite Integrals Ex 19.25 Q23

$$\text{Let } I = \int \frac{\log(x+2)}{(x+2)^2} dx$$

$$\text{Let } \frac{1}{x+2} = t$$

$$-\frac{1}{(x+2)^2} dx = dt$$

$$I = -\int \log\left(\frac{1}{t}\right) dt$$

$$= -\int \log t^{-1} dt$$

$$= -\int 1 \times \log t dt$$

Using integration by parts,

$$I = \log t \int dt - \int \left(\frac{1}{t}\right) \int dt dt$$

$$= t \log t - \int \left(\frac{1}{t} \times t\right) dt$$

$$= t \log t - \int dt$$

$$= t \log t - t + c$$

$$= \frac{1}{x+2} (\log(x+2)^{-1} - 1) + c$$

$$I = \frac{-1}{x+2} - \frac{\log(x+2)}{x+2} + c$$

### Indefinite Integrals Ex 19.25 Q24

$$\begin{aligned}
 \text{Let } I &= \int \frac{x + \sin x}{1 + \cos x} dx \\
 &= \int \frac{x}{2 \cos^2 \frac{x}{2}} dx + \int \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{2 \cos^2 \frac{x}{2}} dx \\
 &= \frac{1}{2} \int x \sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx
 \end{aligned}$$

Using integration by parts,

$$\begin{aligned}
 &= \frac{1}{2} \left[ x \int \sec^2 \frac{x}{2} dx - \int \left( 1 \int \sec^2 \frac{x}{2} dx \right) dx \right] + \int \tan \frac{x}{2} dx \\
 &= \frac{1}{2} \left[ 2x \tan \frac{x}{2} - 2 \int \tan \frac{x}{2} dx \right] + \int \tan \frac{x}{2} dx + c \\
 &= x \tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx + c
 \end{aligned}$$

$$I = x \tan \frac{x}{2} + c$$

### Indefinite Integrals Ex 19.25 Q25

$$\begin{aligned}
 \text{Let } I &= \int \log_{10} x dx \\
 &= \int \frac{\log x}{\log 10} dx \\
 &= \frac{1}{\log 10} \int 1 \times \log x dx
 \end{aligned}$$

Using integration by parts,

$$\begin{aligned}
 &= \frac{1}{\log 10} \left[ \log x \int dx - \int \left( \frac{1}{x} \int dx \right) dx \right] \\
 &= \frac{1}{\log 10} \left[ x \log x - \int \left( \frac{x}{x} \right) dx \right] \\
 &= \frac{1}{\log 10} [x \log x - x]
 \end{aligned}$$

$$I = \frac{x}{\log 10} (\log x - 1)$$

### Indefinite Integrals Ex 19.25 Q26

$$\begin{aligned}
\text{Let } I &= \int \cos \sqrt{x} \, dx \\
\sqrt{x} &= t \\
x &= t^2 \\
dx &= 2t \, dt \\
&= \int 2t \cos t \, dt \\
I &= 2 \int t \cos t \, dt \\
I &= 2 \left[ t \int \cos t \, dt - \int (1) \cos t \, dt \right] \\
&= 2 \left[ t \sin t - \int \sin t \, dt \right] \\
&= 2 \left[ t \sin t + \cos t \right] + c \\
\\
I &= 2 \left[ \sqrt{x} \sin \sqrt{x} + \cos \sqrt{x} \right] + c
\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q27

$$\text{Let } I = \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$$

Let us substitute,  $t = \cos^{-1} x$

$$\Rightarrow dt = \frac{-1}{\sqrt{1-x^2}} dx$$

Also,  $\cos t = x$

Thus,

$$I = - \int t \cos t \, dt$$

Now let us solve this by the 'by parts' method.

Let  $u = t$ ;  $du = dt$

$$\int \cos t \, dt = \int dv$$

$$\Rightarrow \sin t = v$$

$$\text{Thus, } I = - \left[ t \sin t - \int \sin t \, dt \right]$$

$$\Rightarrow I = - \left[ t \sin t + \cos t \right] + C$$

Substituting the value  $t = \cos^{-1} x$ , we have,

$$I = - \left[ \cos^{-1} x \sin t + x \right] + C$$

$$\Rightarrow I = - \left[ \cos^{-1} x \sqrt{1-x^2} + x \right] + C$$

### Indefinite Integrals Ex 19.25 Q29

$$\begin{aligned}\text{Let } I &= \int \operatorname{cosec}^3 x \, dx \\ &= \int \operatorname{cosec} x - \operatorname{cosec}^2 x \, dx\end{aligned}$$

Using integration by parts,

$$\begin{aligned}&= \operatorname{cosec} x \times \int \operatorname{cosec}^2 x \, dx + \int (\operatorname{cosec} x \cot x) \operatorname{cosec}^2 x \, dx \\ &= \operatorname{cosec} x \times (-\cot x) + \int \operatorname{cosec} x \cot x (-\cot x) \, dx \\ &= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x \cot^2 x \, dx \\ &= -\operatorname{cosec} x \cot x - \int \operatorname{cosec} x (\operatorname{cosec}^2 x - 1) \, dx \\ &= -\operatorname{cosec} x \cot x - \int \operatorname{cosec}^3 x \, dx + \int \operatorname{cosec} x \, dx \\ I &= -\operatorname{cosec} x \cot x - I + \log \left| \tan \frac{x}{2} \right| + c_1 \\ 2I &= -\operatorname{cosec} x \cot x + \log \left| \tan \frac{x}{2} \right| + c_1 \\ I &= -\frac{1}{2} \operatorname{cosec} x \cot x + \frac{1}{2} \log \left| \tan \frac{x}{2} \right| + c\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q30

$$\text{Let } I = \int \sec^{-1} \sqrt{x} \, dx$$

$$\text{Let } \sqrt{x} = t$$

$$x = t^2$$

$$dx = 2t \, dt$$

$$I = \int 2t \sec^{-1} t \, dt$$

$$= 2 \left[ \sec^{-1} t \int t \, dt - \int \left( \frac{1}{t\sqrt{t^2-1}} \int t \, dt \right) dt \right]$$

$$= 2 \left[ \frac{t^2}{2} \sec^{-1} t - \int \left( \frac{t}{2t\sqrt{t^2-1}} \right) dt \right]$$

$$= t^2 \sec^{-1} t - \int \frac{t}{\sqrt{t^2-1}} \, dt$$

$$= t^2 \sec^{-1} t - \frac{1}{2} \int \frac{2t}{\sqrt{t^2-1}} \, dt$$

$$= t^2 \sec^{-1} t - \frac{1}{2} \times 2\sqrt{t^2-1} + c$$

$$I = x \sec^{-1} \sqrt{x} - \sqrt{x-1} + c$$

### Indefinite Integrals Ex 19.25 Q31

$$\int \sin^{-1} \sqrt{x} \, dx =$$

$$\text{let } x = t^2 \rightarrow dx = 2t \, dt$$

$$\int \sin^{-1} \sqrt{x} \, dx = \int \sin^{-1} \sqrt{t^2} \, 2t \, dt = \int \sin^{-1} t \, 2t \, dt$$

$$= \sin^{-1} t \int 2t \, dt - \left( \int \frac{d \sin^{-1} t}{dt} (\int 2t \, dt) \, dt \right)$$

$$= \sin^{-1} t (t^2) - \int \frac{1}{\sqrt{1-t^2}} (t^2) \, dt$$

$$\text{Let's solve } \int \frac{1}{\sqrt{1-t^2}} (t^2) \, dt$$

$$\int \frac{1}{\sqrt{1-t^2}} (t^2) \, dt = \int \frac{t^2 - 1 + 1}{\sqrt{1-t^2}} \, dt = \int \frac{t^2 - 1}{\sqrt{1-t^2}} \, dt + \int \frac{1}{\sqrt{1-t^2}} \, dt$$

$$\text{we know that, value of } \int \frac{1}{\sqrt{1-t^2}} \, dt = \sin^{-1} t$$

$$\text{Remaining integral to evaluate is } \int \frac{t^2 - 1}{\sqrt{1-t^2}} \, dt = \int -\sqrt{1-t^2} \, dt$$

$$\text{sub } t = \sin u, \, dt = \cos u \, du$$

$$\int -\sqrt{1-t^2} \, dt = \int -\cos^2 u \, du = -\int \left[ \frac{1 + \cos 2u}{2} \right] du$$

$$= -\frac{u}{2} - \frac{\sin 2u}{4}$$

$$\text{Substitute back } u = \sin^{-1} t \text{ and } t = \sqrt{x}$$

$$= -\frac{\sin^{-1} \sqrt{x}}{2} - \frac{\sin(2 \sin^{-1} \sqrt{x})}{4}$$

$$\int \sin^{-1} \sqrt{x} \, dx = x \sin^{-1} \sqrt{x} - \frac{\sin^{-1} \sqrt{x}}{2} - \frac{\sin(2 \sin^{-1} \sqrt{x})}{4}$$

$$\sin(2 \sin^{-1} \sqrt{x}) = 2\sqrt{x} \sqrt{1-x}$$

$$\int \sin^{-1} \sqrt{x} \, dx = x \sin^{-1} \sqrt{x} - \frac{\sin^{-1} \sqrt{x}}{2} - \frac{\sqrt{x(1-x)}}{2}$$

### Indefinite Integrals Ex 19.25 Q32

$$\text{Let } I = \int x \tan^2 x \, dx$$

$$= \int x (\sec^2 x - 1) \, dx$$

$$= \int x \sec^2 x \, dx - \int x \, dx$$

$$= \left[ x \int \sec^2 x \, dx - \int (1) \sec^2 x \, dx \right] - \frac{x^2}{2}$$

$$= x \tan x - \int \tan x \, dx - \frac{x^2}{2}$$

$$I = x \tan x - \log \sec x - \frac{x^2}{2} + c$$

### Indefinite Integrals Ex 19.25 Q33

$$\begin{aligned}
\text{Let } I &= \int x \left( \frac{\sec 2x - 1}{\sec 2x + 1} \right) dx \\
&= \int x \left( \frac{1 - \cos 2x}{1 + \cos 2x} \right) dx \\
&= \int x \left( \frac{\sec^2 x}{\cos^2 x} \right) dx \\
&= \int x \tan^2 x dx \\
&= \int x (\sec^2 x - 1) dx \\
&= \int x \sec^2 x dx - \int dx \\
&= \left[ x \int \sec^2 x dx - \int (1 \int \sec^2 x dx) dx \right] - \frac{x^2}{2} \\
&= x \tan x - \int \tan x dx - \frac{x^2}{2} \\
I &= x \tan x - \log \sec x - \frac{x^2}{2} + c
\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q34

$$\text{Let } I = \int (x + 1) e^x \log(xe^x) dx$$

$$\text{Let } xe^x = t$$

$$(1 \times e^x + xe^x) dx = dt$$

$$(x + 1) e^x dx = dt$$

$$I = \int \log t dt$$

$$= \int 1 \times \log t dt$$

$$= \log t \int dt - \int \left( \frac{1}{t} \int dt \right) dt$$

$$= t \log t - \int \left( \frac{1}{t} t \right) dt$$

$$= t \log t - \int dt$$

$$= t \log t - t + c$$

$$= t (\log t - 1) + c$$

$$I = xe^x (\log xe^x - 1) + c$$

### Indefinite Integrals Ex 19.25 Q35



$$\text{Let } I = \int \sin^{-1}(3x - 4x^3) dx$$

$$\text{Let } x = \sin \theta$$

$$dx = \cos \theta d\theta$$

$$= \int \sin^{-1}(3 \sin \theta - 4 \sin^3 \theta) \cos \theta d\theta$$

$$= \int \sin^{-1}(\sin 3\theta) \cos \theta d\theta$$

$$= \int 3\theta \cos \theta d\theta$$

$$= 3 \left[ \theta \int \cos \theta d\theta - \int (1 \int \cos \theta d\theta) d\theta \right]$$

$$= 3 \left[ \theta \sin \theta - \int \sin \theta d\theta \right]$$

$$= 3 \left[ \theta \sin \theta + \cos \theta \right] + c$$

$$I = 3 \left[ x \sin^{-1} x + \sqrt{1-x^2} \right] + c$$

### Indefinite Integrals Ex 19.25 Q36

$$\text{Let } x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$$

$$\therefore \sin^{-1} \left( \frac{2x}{1+x^2} \right) = \sin^{-1} \left( \frac{2 \tan \theta}{1 + \tan^2 \theta} \right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$$\Rightarrow \int \sin^{-1} \left( \frac{2x}{1+x^2} \right) dx = \int 2\theta \cdot \sec^2 \theta d\theta = 2 \int \theta \cdot \sec^2 \theta d\theta$$

Integrating by parts, we obtain

$$2 \left[ \theta \cdot \int \sec^2 \theta d\theta - \int \left( \frac{d}{d\theta} \theta \right) \int \sec^2 \theta d\theta \right] d\theta$$

$$= 2 \left[ \theta \cdot \tan \theta - \int \tan \theta d\theta \right]$$

$$= 2 \left[ \theta \tan \theta + \log |\cos \theta| \right] + C$$

$$= 2 \left[ x \tan^{-1} x + \log \left| \frac{1}{\sqrt{1+x^2}} \right| \right] + C$$

$$= 2x \tan^{-1} x + 2 \log(1+x^2)^{-\frac{1}{2}} + C$$

$$= 2x \tan^{-1} x + 2 \left[ -\frac{1}{2} \log(1+x^2) \right] + C$$

$$= 2x \tan^{-1} x - \log(1+x^2) + C$$

### Indefinite Integrals Ex 19.25 Q37

$$\text{Let } I = \int \tan^{-1} \left( \frac{3x - x^3}{1 - 3x^2} \right) dx$$

$$\text{Let } x = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$I = \int \tan^{-1} \left( \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) \sec^2 \theta d\theta$$

$$= \int \tan^{-1} (\tan 3\theta) \sec^2 \theta d\theta$$

$$= \int 3\theta \sec^2 \theta d\theta$$

$$= 3 \left[ \theta \int \sec^2 \theta d\theta - \int (1 \int \sec^2 \theta d\theta) d\theta \right]$$

$$= 3 \left[ \theta \tan \theta - \int \tan \theta d\theta \right]$$

$$= 3 \left[ \theta \tan \theta + \log \sec \theta \right] + c$$

$$= 3 \left[ x \tan^{-1} x - \log \sqrt{1 + x^2} \right] + c$$

$$I = 3x \tan^{-1} x - \frac{3}{2} \log |1 + x^2| + c$$

### Indefinite Integrals Ex 19.25 Q38

$$\text{Let } I = \int x^2 \sin^{-1} x dx$$

$$I = \sin^{-1} x \int x^2 dx - \int \left( \frac{1}{\sqrt{1-x^2}} \int x^2 dx \right) dx$$

$$= \frac{x^3}{3} \sin^{-1} x - \int \frac{x^3}{3\sqrt{1-x^2}} dx$$

$$I = \frac{x^3}{3} \sin^{-1} x - \frac{1}{3} I_1 + c_1 \text{ ----- (1)}$$

$$I_1 = \int \frac{x^3}{\sqrt{1-x^2}} dx$$

$$\text{Let } 1 - x^2 = t^2$$

$$-2x dx = 2t dt$$

$$-x dx = t dt$$

$$I_1 = - \int \frac{(1-t^2) t dt}{t}$$

$$= \int (t^2 - 1) dt$$

$$= \frac{t^3}{3} - t + c_2$$

$$= \frac{(1-x^2)^{\frac{3}{2}}}{3} - (1-x^2)^{\frac{1}{2}} + c_2$$

Now,

$$I = \frac{x^3}{3} \sin^{-1} x - \frac{1}{9} (1-x^2)^{\frac{3}{2}} + \frac{1}{3} (1-x^2)^{\frac{1}{2}} + c$$

## Indefinite Integrals Ex 19.25 Q39

$$\begin{aligned}
 \text{Let } I &= \int \frac{\sin^{-1} x}{x^2} dx \\
 &= \int \left( \frac{1}{x^2} \right) (\sin^{-1} x) dx \\
 I &= \left[ \sin^{-1} x \int \frac{1}{x^2} dx - \int \left( \frac{1}{\sqrt{1-x^2}} \int \frac{1}{x^2} dx \right) dx \right] \\
 &= \sin^{-1} x \left( -\frac{1}{x} \right) - \int \frac{1}{\sqrt{1-x^2}} \left( -\frac{1}{x} \right) dx \\
 I &= -\frac{1}{x} \sin^{-1} x + \int \frac{1}{x\sqrt{1-x^2}} dx \\
 I &= -\frac{1}{x} \sin^{-1} x + I_1 \text{ ----- (1)}
 \end{aligned}$$

Where,

$$I_1 = \int \frac{1}{x\sqrt{1-x^2}} dx$$

$$\begin{aligned}
 \text{Let } 1-x^2 &= t^2 \\
 -2x dx &= 2t dt \\
 I_1 &= \int \frac{x}{x^2\sqrt{1-x^2}} dx \\
 &= -\int \frac{t dt}{(1-t^2)\sqrt{t}} \\
 &= -\int \frac{dt}{(1-t^2)} \\
 &= \int \frac{1}{t^2-1} dt \\
 &= \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| \\
 &= \frac{1}{2} \log \left| \frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1} \right| + c_1
 \end{aligned}$$

Now,

$$\begin{aligned}I &= -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log \left| \left( \frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1} \right) \left( \frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}-1} \right) \right| + c \\&= -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log \left| \frac{(\sqrt{1-x^2}-1)^2}{1-x^2-1} \right| + c \\&= -\frac{\sin^{-1} x}{x} + \frac{1}{2} \log \left| \frac{(\sqrt{1-x^2}-1)^2}{-x^2} \right| + c \\&= -\frac{\sin^{-1} x}{x} + \log \left| \frac{\sqrt{1-x^2}-1}{-x} \right| + c \\I &= -\frac{\sin^{-1} x}{x} + \log \left| \frac{1-\sqrt{1-x^2}}{x} \right| + c\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q40

Let  $I = \int \frac{x^2 \tan^{-1} x}{1+x^2} dx$

Let  $\tan^{-1} x = t$

$$x = \tan t$$

$$\frac{1}{1+x^2} dx = dt$$

$$I = \int t \tan^2 t dt$$

$$= \int t (\sec^2 t - 1) dt$$

$$= \int (t \sec^2 t - t) dt$$

$$= \int t \sec^2 t dt - \int t dt$$

$$= \left[ \int t \sec^2 t dt - \int (1) \sec^2 t dt \right] - \frac{t^2}{2}$$

$$= \left[ t \tan t - \int \tan t dt \right] - \frac{t^2}{2}$$

$$= t \tan t - \log |\sec t| - \frac{t^2}{2} + c$$

$$= x \tan^{-1} x - \log \sqrt{1+x^2} - \frac{\tan^2 x}{2} + c$$

$$I = x \tan^{-1} x - \frac{1}{2} \log |1+x^2| - \frac{\tan^2 x}{2} + c$$

### Indefinite Integrals Ex 19.25 Q41

$$\text{Let } I = \int \cos^{-1}(4x^3 - 3x) dx$$

$$\text{Let } x = \cos \theta$$

$$dx = -\sin \theta d\theta$$

$$I = -\int \cos^{-1}(4\cos^3 \theta - 3\cos \theta) \sin \theta d\theta$$

$$= -\int \cos^{-1}(\cos 3\theta) \sin \theta d\theta$$

$$= -\int 3\theta \sin \theta d\theta$$

$$= -3 \left[ \theta \int \sin \theta d\theta - \int (1) \sin \theta d\theta \right]$$

$$= -3 \left[ -\theta \cos \theta + \int \cos \theta d\theta \right]$$

$$= 3\theta \cos \theta - 3 \sin \theta + c$$

$$I = 3x \cos^{-1} x - 3\sqrt{1-x^2} + c$$

### Indefinite Integrals Ex 19.25 Q42

$$\text{Let } I = \int \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right) dx$$

$$\text{Let } x = \tan t$$

$$dx = \sec^2 t dt$$

$$I = \int \cos^{-1} \left( \frac{1-\tan^2 t}{1+\tan^2 t} \right) \sec^2 t dt$$

$$= \int \cos^{-1}(\cos 2t) \sec^2 t dt$$

$$= \int 2t \sec^2 t dt$$

$$= 2 \left[ t \int \sec^2 t dt - \int (1) \sec^2 t dt \right]$$

$$= 2 \left[ t \tan^2 t - \int \tan t dt \right]$$

$$= 2 \left[ t \tan^2 t - \log |\sec t| \right] + c$$

$$= 2 \left[ x \tan^{-1} x - \log \sqrt{1+x^2} \right] + c$$

$$I = 2x \tan^{-1} x - \log |1+x^2| + c$$

### Indefinite Integrals Ex 19.25 Q43

$$\text{Let } I = \int \tan^{-1}\left(\frac{2x}{1-x^2}\right) dx$$

$$\text{Let } x = \tan \theta$$

$$dx = \sec^2 \theta d\theta$$

$$I = \int \tan^{-1}\left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right) \sec^2 \theta d\theta$$

$$= \int \tan^{-1}(\tan 2\theta) \sec^2 \theta d\theta$$

$$= \int 2\theta \sec^2 \theta d\theta$$

$$= 2 \left[ \theta \int \sec^2 \theta d\theta - \int (1) \sec^2 \theta d\theta \right] d\theta$$

$$= 2 \left[ \theta \tan \theta - \int \tan \theta d\theta \right]$$

$$= 2 \left[ \theta \tan \theta - \log \sec \theta \right] + c$$

$$= 2 \left[ x \tan^{-1} x - \log \sqrt{1+x^2} \right] + c$$

$$I = 2x \tan^{-1} x - \log |1+x^2| + c$$

### Indefinite Integrals Ex 19.25 Q44

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$$\text{Let } I = \int (x+1) \log x dx$$

$$= \log x \int (x+1) dx - \int \left( \frac{1}{x} \int (x+1) dx \right) dx$$

$$= \left( \frac{x^2}{2} + x \right) \log x - \int \frac{1}{x} \left( \frac{x^2}{2} + x \right) dx$$

$$= \left( \frac{x^2}{2} + x \right) \log x - \frac{1}{2} \int x dx - \int dx$$

$$= \left( x + \frac{x^2}{2} \right) \log x - \frac{1}{2} x \frac{x^2}{2} - x + c$$

$$I = \left( x + \frac{x^2}{2} \right) \log x - \left( \frac{x^2}{4} + x \right) + c$$

### Indefinite Integrals Ex 19.25 Q45

$$\begin{aligned}
\text{Let } I &= \int x^2 \tan^{-1} x \, dx \\
&= \tan^{-1} x \int x^2 \, dx - \int \left( \frac{1}{1+x^2} \int x^2 \, dx \right) \\
&= \tan^{-1} x \left( \frac{x^3}{3} \right) - \frac{1}{3} \int \frac{x^3}{1+x^2} \, dx \\
&= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{3} \int \left( x - \frac{x}{1+x^2} \right) dx \\
&= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{3} x \frac{x^2}{2} + \frac{1}{3} \int \frac{x}{1+x^2} \, dx \\
I &= \frac{1}{3} x^3 \tan^{-1} x - \frac{1}{6} x^2 + \frac{1}{6} \log|1+x^2| + c
\end{aligned}$$

### Indefinite Integrals Ex 19.25 Q46

$$\begin{aligned}
\text{Let } I &= \int (e^{\log x} + \sin x) \cos x \, dx \\
&= \int (x + \sin x) \cos x \, dx \\
&= \int x \cos x \, dx + \int \sin x \cos x \, dx \\
&= \left[ x \int \cos x \, dx - \int (1) \int \cos x \, dx \right] dx + \frac{1}{2} \int \sin 2x \, dx \\
&= \left[ x \sin x - \int \sin x \, dx \right] + \frac{1}{2} \left( -\frac{\cos 2x}{2} \right) + c
\end{aligned}$$

$$I = x \sin x + \cos x - \frac{1}{4} \cos 2x + c$$

$$I = x \sin x + \cos x - \frac{1}{4} [1 - 2 \sin^2 x] + c$$

$$I = x \sin x + \cos x - \frac{1}{4} + \frac{1}{2} \sin^2 x + c$$

$$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + c - \frac{1}{4}$$

$$I = x \sin x + \cos x + \frac{1}{2} \sin^2 x + k, \text{ where } k = c - \frac{1}{4}$$

### Indefinite Integrals Ex 19.25 Q47

$$\text{Let } I = \int \frac{(x \tan^{-1} x)}{(1+x^2)^{\frac{3}{2}}} dx$$

$$\text{Let } \tan^{-1} x = t$$

$$\frac{1}{1+x^2} dx = dt$$

$$I = \int \frac{t \tan t}{\sqrt{1+\tan^2 t}} dt$$

$$= \int \frac{t \tan t}{\sec t} dt$$

$$= \int t \frac{\sin t}{\cos t} \cos t dt$$

$$= \int t \sin t dt$$

$$= [t \int \sin t dt - \int (1) \sin t dt] dt$$

$$= [-t \cos t + \int \cos t dt]$$

$$= [-t \cos t + \sin t] + c$$

$$I = -\frac{\tan^{-1} x}{\sqrt{1+x^2}} + \frac{x}{\sqrt{1+x^2}} + c$$

### Indefinite Integrals Ex 19.25 Q48

$$\text{Let } I = \int \tan^{-1}(\sqrt{x}) dx$$

$$\text{Let } x = t^2$$

$$dx = 2t dt$$

$$I = \int 2t \tan^{-1} t dt$$

$$= 2 \left[ \tan^{-1} t \int t dt - \int \left( \frac{1}{1+t^2} \right) \int t dt \right] dt$$

$$= 2 \left[ \frac{t^2}{2} \tan^{-1} t - \int \frac{t^2}{2(1+t^2)} dt \right]$$

$$= t^2 \tan^{-1} t - \int \frac{t^2 + 1 - 1}{1+t^2} dt$$

$$= t^2 \tan^{-1} t - \int \left( 1 - \frac{1}{1+t^2} \right) dt$$

$$= t^2 \tan^{-1} t - t + \tan^{-1} t + c$$

$$= (t^2 + 1) \tan^{-1} t - t + c$$

$$I = (x + 1) \tan^{-1} \sqrt{x} - \sqrt{x} + c$$

### Indefinite Integrals Ex 19.25 Q49



$$\int x^3 \tan^{-1} x \, dx =$$

$$\int x^3 \tan^{-1} x \, dx = \tan^{-1} x \int x^3 \, dx - \left( \int \frac{d \tan^{-1} x}{dx} \left( \int x^3 \, dx \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left( \int \frac{1}{1+x^2} \left( \frac{x^4}{4} \right) dx \right)$$

$$= \tan^{-1} x \frac{x^4}{4} - \left( \int \frac{1}{1+x^2} \left( \frac{x^4}{4} \right) dx \right)$$

$$\int \frac{1}{1+x^2} \left( \frac{x^4}{4} \right) dx = \frac{1}{4} \left[ \int \frac{1}{1+x^2} dx + (x^2 - 1) dx \right]$$

$$\int \frac{1}{1+x^2} \left( \frac{x^4}{4} \right) dx = \frac{1}{4} \left[ \tan^{-1} x + \frac{x^3}{3} - x \right]$$

$$\int x^3 \tan^{-1} x \, dx = \frac{x^4}{4} \tan^{-1} x - \frac{1}{4} \left[ \tan^{-1} x + \frac{x^3}{3} - x \right] + C$$

### Indefinite Integrals Ex 19.25 Q50

$$\text{Let } I = \int x \sin x \cos 2x \, dx$$

$$= \frac{1}{2} \int x (2 \sin x \cos 2x) \, dx$$

$$= \frac{1}{2} \int x (\sin(x + 2x) - \sin(2x - x)) \, dx$$

$$= \frac{1}{2} \int x (\sin 3x - \sin x) \, dx$$

$$= \frac{1}{2} \left[ x \int (\sin 3x - \sin x) \, dx - \int (1) (\sin 3x - \sin x) \, dx \right]$$

$$= \frac{1}{2} \left[ x \left( -\frac{\cos 3x}{3} + \cos x \right) - \int \left( -\frac{\cos 3x}{3} + \cos x \right) dx \right]$$

$$I = \frac{1}{2} \left[ -x \frac{\cos 3x}{3} + x \cos x + \frac{1}{9} \sin 3x - \sin x \right] + c$$

## Indefinite Integrals Ex 19.25 Q52

Let first function be  $\sin^{-1} x$  and second function be  $\frac{x}{\sqrt{1-x^2}}$ .

First we find the integral of the second function, i.e.,  $\int \frac{x dx}{\sqrt{1-x^2}}$ .

Put  $t = 1 - x^2$ . Then  $dt = -2x dx$

Therefore,  $\int \frac{x dx}{\sqrt{1-x^2}} = -\frac{1}{2} \int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{1-x^2}$

Hence, 
$$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx = (\sin^{-1} x) \left( -\sqrt{1-x^2} \right) - \int \frac{1}{\sqrt{1-x^2}} \left( -\sqrt{1-x^2} \right) dx$$
$$= -\sqrt{1-x^2} \sin^{-1} x + x + C = x - \sqrt{1-x^2} \sin^{-1} x + C$$

## Indefinite Integrals Ex 19.25 Q53

$$\begin{aligned}
 \text{Let } I &= \int \sin^3 \sqrt{x} \, dx \\
 \sqrt{x} &= t \\
 x &= t^2 \\
 dx &= 2t \, dt \\
 I &= 2 \int t \sin^3 t \, dt \\
 &= 2 \int t \left( \frac{3 \sin t - \sin 3t}{4} \right) dt \\
 &= \frac{1}{2} \int t (3 \sin t - \sin 3t) \, dt
 \end{aligned}$$

Using integration by parts,

$$\begin{aligned}
 I &= \frac{1}{2} \left[ t \left( -3 \cos t + \frac{1}{3} \cos 3t \right) - \int \left( -3 \cos t + \frac{\cos 3t}{3} \right) dt \right] \\
 &= \frac{1}{2} \left[ \frac{-9t \cos t + t \cos 3t}{3} - \left\{ -3 \sin t + \frac{\sin 3t}{9} \right\} \right] + c \\
 &= \frac{1}{2} \left[ \frac{-9t \cos t + t \cos 3t}{3} + \frac{27 \sin t - 3 \sin 3t}{9} \right] + c \\
 &= \frac{1}{18} [-27t \cos t + 3t \cos 3t + 27 \sin t - 3 \sin 3t] + c
 \end{aligned}$$

$$I = \frac{1}{18} [3\sqrt{x} \cos 3\sqrt{x} + 27 \sin \sqrt{x} - 27\sqrt{x} \cos \sqrt{x} - 3 \sin 3\sqrt{x}] + c$$

### Indefinite Integrals Ex 19.25 Q54

$$\begin{aligned}
 \text{Let } I &= \int x \sin^3 x \, dx \\
 &= \int x \left( \frac{3 \sin x - \sin 3x}{4} \right) dx \\
 &= \frac{1}{4} \int x (3 \sin x - \sin 3x) \, dx
 \end{aligned}$$

Using integration by parts,

$$\begin{aligned}
 I &= \frac{1}{4} \left[ x \int (3 \sin x - \sin 3x) \, dx - \int \{1\} (3 \sin x - \sin 3x) \, dx \right] \\
 &= \frac{1}{4} \left[ x \left( -3 \cos x + \frac{\cos 3x}{3} \right) - \int \left( -3 \cos x + \frac{\cos 3x}{3} \right) dx \right] \\
 &= \frac{1}{4} \left[ -3x \cos x + \frac{x \cos 3x}{3} + 3 \sin x - \frac{\sin 3x}{9} \right] + c
 \end{aligned}$$

$$I = \frac{1}{36} [3x \cos 3x - 27x \cos x + 27 \sin x - \sin 3x] + c$$

## indefinite integrals Ex 19.25 Q56

$$\begin{aligned}\text{Let } I &= \int x \cos^3 x \, dx \\ &= \int x \left( \frac{3 \cos x + \cos 3x}{4} \right) dx \\ &= \frac{1}{4} \int x (3 \cos x + \cos 3x) \, dx\end{aligned}$$

Using integration by parts,

$$\begin{aligned}I &= \frac{1}{4} \left[ x \int (3 \cos x + \cos 3x) \, dx - \int (1 \int (3 \cos x + \cos 3x) \, dx) \, dx \right] \\ &= \frac{1}{4} \left[ x \left( 3 \sin x + \frac{\sin 3x}{3} \right) - \int \left( 3 \sin x + \frac{\sin 3x}{3} \right) dx \right] \\ &= \frac{1}{4} \left[ 3x \sin x + \frac{x \sin 3x}{3} + 3 \cos x + \frac{\cos 3x}{9} \right] + c \\ I &= \frac{3x \sin x}{4} + \frac{x \sin 3x}{12} + \frac{3 \cos x}{4} + \frac{\cos 3x}{36} + c\end{aligned}$$

~~indefinite integrals Ex 19.25 Q58~~

$$\text{Let } I = \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

$$\text{Let } x = a \tan^2 \theta$$

$$dx = 2a \tan \theta \sec^2 \theta d\theta$$

$$I = \int \left( \sin^{-1} \sqrt{\frac{a \tan^2 \theta}{a + a \tan^2 \theta}} \right) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= \int \left( \sin^{-1} \sqrt{\frac{\tan^2 \theta}{\sec^2 \theta}} \right) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= \int \sin^{-1} (\sin \theta) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= \int 2\theta a \tan \theta \sec^2 \theta d\theta$$

$$= 2a \int \theta (\tan \theta \sec^2 \theta) d\theta$$

$$= 2a \left[ \theta \int \tan \theta \sec^2 \theta d\theta - \int \left( \int \tan \theta \sec^2 \theta d\theta \right) d\theta \right]$$

$$= 2a \left[ \theta \frac{\tan^2 \theta}{2} - \int \frac{\tan^2 \theta}{2} d\theta \right]$$

$$= a\theta \tan^2 \theta - \frac{2a}{2} \int (\sec^2 \theta - 1) d\theta$$

$$= a\theta \tan^2 \theta - a \tan \theta + a\theta + c$$

$$= a \left( \tan^{-1} \sqrt{\frac{x}{a}} \right) \frac{x}{a} - a \sqrt{\frac{x}{a}} + a \tan^{-1} \sqrt{\frac{x}{a}} + c$$

$$I = x \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{ax} + a \tan^{-1} \sqrt{\frac{x}{a}} + c$$

$$\text{Let } I = \int \frac{x^3 \sin^{-1} x^2}{\sqrt{1-x^4}} dx$$

$$\text{Let } \sin^{-1} x^2 = t$$

$$\frac{1}{\sqrt{1-x^4}} (2x) dx = dt$$

$$I = \int \frac{x^2 \sin^{-1} x^2}{\sqrt{1-x^2}} x dx$$

$$= \int (\sin t) t \frac{dt}{2}$$

$$= \frac{1}{2} \int t \sin t dt$$

$$= \frac{1}{2} [t \int \sin t dt - \int (1 \int \sin t dt) dt]$$

$$= \frac{1}{2} [t(-\cos t) - \int (-\cos t) dt]$$

$$= \frac{1}{2} [-t \cos t + \sin t] + c$$

$$I = \frac{1}{2} [x^2 - \sqrt{1-x^4} \sin^{-1} x^2] + c$$

$$\text{Let } I = \int \frac{x^2 \sin^{-1} x}{(1-x^2)^{\frac{3}{2}}} dx$$

$$\text{Let } \sin^{-1} x = t$$

$$\frac{1}{\sqrt{1-x^2}} dx = dt$$

$$I = \int \frac{\sin^2 t \times t}{(1 - \sin^2 t)} dt$$

$$= \int \frac{t \sin^2 t}{\cos^2 t} dt$$

$$= \int t \tan^2 t dt$$

$$= \int t (\sec^2 t - 1) dt$$

$$= \int t \sec^2 t dt - \frac{t^2}{2}$$

$$= t \int \sec^2 t dt - \int (1 \int \sec^2 t dt) dt - \frac{t^2}{2}$$

$$= t \tan t - \int \tan t dt - \frac{t^2}{2}$$

$$= t \tan t - \log \sec t - \frac{t^2}{2} + c$$

$$I = \frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \log |1-x^2| - \frac{1}{2} (\sin^{-1} x)^2 + c$$