RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.23

Indefinite Integrals Ex 19.23 Q1

Let $I = \int \frac{1}{5 + 4\cos x} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{5 + 4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{5\left(1 + \tan^2 \frac{x}{2}\right) + 4\left(1 - \tan^2 \frac{x}{2}\right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{5 + 5\tan^2 \frac{x}{2} + 4 - 4\tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2}\sec^2 \frac{x}{2} dx = dt$$

$$= \int \frac{2dt}{(3)^2 + t^2}$$

$$= 2 \times \frac{1}{3} \tan^{-1}(t) + c$$

$$I = \frac{2}{3} \tan^{-1}\left(\frac{\tan \frac{x}{2}}{3}\right) + c$$

Indefinite Integrals Ex 19.23 Q2

Let $I = \int \frac{1}{5-4\sin x}dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$= \int \frac{1}{5-4\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)-4\left(2\tan\frac{x}{2}\right)}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{5+5\tan^2\frac{x}{2}-8\tan\frac{x}{2}}dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$= \int \frac{2dt}{5t^2-8t+5}$$

$$= \frac{2}{5}\int \frac{dt}{t^2-\frac{8}{5}t+1}$$

$$= \frac{2}{5}\int \frac{dt}{t^2-2t\left(\frac{4}{5}\right)+\left(\frac{4}{5}\right)^2-\left(\frac{4}{5}\right)^2+1}$$

$$I = \frac{2}{5}\int \frac{dt}{\left(t-\frac{4}{5}\right)^2+\left(\frac{3}{5}\right)^2}$$

$$= \frac{2}{5}\times\frac{1}{\frac{3}{5}}\tan^{-1}\left(\frac{t-\frac{4}{5}}{\frac{3}{5}}\right)+c$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{5t-4}{3}\right)+c$$

$$I = \frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2}-4}{3}\right)+c$$

Indefinite Integrals Ex 19.23 Q3

Let $I = \int \frac{1}{1 - 2\sin x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{1 - 2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{1 + \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2} - 4\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{\tan^2\frac{x}{2} - 4\tan\frac{x}{2} + 1} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{t^2 - 4t + 1}$$

$$= \int \frac{2dt}{t^2 - 2t(2) + (2)^2 - (2)^2 + 1}$$

$$I = 2\int \frac{dt}{(t-2)^2 + 3}$$

$$= 2\int \frac{dt}{(t-2)^2 + \left(\sqrt{3}\right)^2}$$

$$= 2 \times \frac{1}{2\sqrt{3}} \log\left|\frac{t - 2 - \sqrt{3}}{t - 2 + \sqrt{3}}\right| + c$$

$$I = \frac{1}{\sqrt{3}} \log\left|\frac{\tan\frac{x}{2} - 2 - \sqrt{3}}{\tan\frac{x}{2} - 2 + \sqrt{3}}\right| + c$$

Indefinite Integrals Ex 19.23 Q4

Let $I = \int \frac{1}{4\cos x - 1} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) - 1} dx$$

$$= \int \frac{1 + \tan^2 \frac{x}{2}}{4\left(1 - \tan^2 \frac{x}{2}\right) - \left(1 + \tan^2 \frac{x}{2}\right)} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{4 - 4\tan^2 \frac{x}{2} - 1 - \tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{3 - 5\tan^2 \frac{x}{2}} dx$$

Let $\sqrt{5} \tan \frac{x}{2} = t$

$$\frac{\sqrt{5}}{2} \sec^2 \frac{x}{2} dt = dt$$

$$I = \int \frac{dt}{\left(\sqrt{3}\right)^2 - t^2}$$

$$I = \frac{1}{\sqrt{15}} \log \left| \frac{\sqrt{3} + \sqrt{5} \tan \frac{x}{2}}{\sqrt{3} - \sqrt{5} \tan \frac{x}{2}} \right| + c$$

Indefinite Integrals Ex 19.23 Q5

Let $I = \int \frac{1}{1 - \sin x + \cos x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{1 - \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}} + \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}} dx$$

$$= \int \frac{1 + \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2} - 2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2 - 2\tan\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$= \frac{2}{2}\int \frac{dt}{1 - t}$$

$$= -\log\left|1 - t\right| + c$$

$$I = -\log\left|1 - \tan\frac{x}{2}\right| + c$$

Indefinite Integrals Ex 19.23 Q6

Let $I = \int \frac{1}{3 + 2\sin x + \cos x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$, $\cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{3 + 2\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + \left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{3\left(1 + \tan^2\frac{x}{2}\right) + 4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{3 + 3\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan^2\frac{x}{2} + 4\tan\frac{x}{2} + 4} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \frac{2}{2}\int \frac{dt}{t^2 + 2t + 2}$$

$$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$$

$$I = \int \frac{dt}{(t + 1)^2 + (1)^2}$$

$$= \tan^{-1}(t + 1) + c$$

$$I = \tan^{-1}\left(\tan\frac{x}{2} + 1\right) + c$$

Indefinite Integrals Ex 19.23 Q7

Let $I = \int \frac{1}{13 + 3\cos x + 4\sin x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}$

$$I = \int \frac{1}{13 + 3\left(\frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right) + 4\left(\frac{2\tan\frac{x}{2}}{1 + \tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2\frac{x}{2}\right)}{13\left(1 + \tan^2\frac{x}{2}\right) + 3 - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{16 + 13\tan^2\frac{x}{2} - 3\tan^2\frac{x}{2} + 8\tan\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{16 + 10t^2 + 8t}$$

$$= \frac{2}{10}\int \frac{dt}{t^2 + \frac{4}{5}t + \frac{8}{5}}$$

$$I = \frac{1}{5}\int \frac{dt}{t^2 + 2t\left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^2 - \left(\frac{2}{5}\right)^2 + \frac{8}{5}}$$

$$= \frac{1}{5}\int \frac{dt}{\left(t + \frac{2}{5}\right)^2 + \left(\frac{6}{5}\right)^2}$$

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$$= \frac{1}{5} \times \frac{1}{\left(\frac{6}{5}\right)} \tan^{-1}\left(\frac{t + \frac{2}{5}}{\frac{6}{5}}\right) + c$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{5t + 2}{6}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{5\tan\frac{x}{2} + 2}{6}\right) + c$$

Indefinite Integrals Ex 19.23 Q8

Let $I = \int \frac{1}{\cos x - \sin x} dx$

Put $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) - \left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{1-\tan^2\frac{x}{2} - 2\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{1-\tan^2\frac{x}{2} - 2\tan\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{1-t^2-2t}$$

$$= -\int \frac{2dt}{t^2+2t-1}$$

$$I = -\int \frac{2dt}{t^2+2t+1-1-1}$$

$$I = -\int \frac{2dt}{(t+1)^2 - \left(\sqrt{2}\right)^2}$$

$$= \int \frac{2dt}{\left(\sqrt{2}\right)^2 - (t+1)^2}$$

$$= \frac{2}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+t+1}{\sqrt{2}-t-1}\right| + c$$

$$I = \frac{1}{\sqrt{2}} \log\left|\frac{\sqrt{2}+\tan\frac{x}{2}+1}{\sqrt{2}-\tan\frac{x}{2}-1}\right| + c$$

Indefinite Integrals Ex 19.23 Q9

Let $I = \int \frac{1}{\sin x + \cos x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}} dx$$

$$= \int \frac{1+\tan^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{2t+1-t^2}$$

$$= -2\int \frac{dt}{t^2-2t-1}$$

$$I = -2\int \frac{dt}{t^2-2t+1-1-1}$$

$$I = -2\int \frac{dt}{(t-1)^2-(\sqrt{2})^2}$$

$$= 2\int \frac{2dt}{(\sqrt{2})^2-(t-1)^2}$$

$$= \frac{2}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+t-1}{\sqrt{2}-t+1}\right| + c$$

$$I = \frac{1}{\sqrt{2}} \log\left|\frac{\sqrt{2}+\tan\frac{x}{2}-1}{\sqrt{2}-\tan\frac{x}{2}+1}\right| + c$$

Indefinite Integrals Ex 19.23 Q10

Let $I = \int \frac{1}{5 - 4\cos x} dx$

Put $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$I = \int \frac{1}{5 - 4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1 + \tan^2 \frac{x}{2}\right)}{5 + 5\tan^2 \frac{x}{2} - 4 + 4\tan^2 \frac{x}{2}} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{9\tan^2 \frac{x}{2} + 1} dx$$

Let $3\tan \frac{x}{2} = t$

$$\frac{3}{2}\sec^2 \frac{x}{2} dx = dt$$

$$I = \frac{2}{3}\int \frac{dt}{t^2 + 1}$$

$$= \frac{2}{3}\tan^{-1}(t) + c$$

$$I = \frac{2}{3}\tan^{-1}\left(3\tan \frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.23 Q11

Let $I = \int \frac{1}{2+\sin x+\cos x}dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{2+\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{2+2\tan^2\frac{x}{2}+2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{\tan^2\frac{x}{2}+2\tan\frac{x}{2}+3}dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}dx = dt$$

$$I = \int \frac{2dt}{t^2+2t+3}$$

$$= 2\int \frac{dt}{t^2+2t+1-1+3}$$

$$I = 2\int \frac{dt}{(t+1)^2+\left(\sqrt{2}\right)^2}$$

$$= \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right)+c$$

$$I = \sqrt{2}\tan^{-1}\left(\frac{\tan\frac{x}{2}+1}{\sqrt{2}}\right)+c$$

Indefinite Integrals Ex 19.23 Q12

Let $\quad I = \int \frac{1}{\sin x + \sqrt{3}\cos x}\,dx$

Put $\quad \sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$I = \int \frac{1}{\left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right) + \sqrt{3}\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}\,dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{2\tan\frac{x}{2} + \sqrt{3} - \sqrt{3}\tan^2\frac{x}{2}}\,dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2} + \sqrt{3} - \sqrt{3}\tan^2\frac{x}{2}}\,dx$$

Let $\quad \tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$$

$$I = \int \frac{2dt}{2t + \sqrt{3} - \sqrt{3}t^2}$$

$$= -\frac{2}{\sqrt{3}}\int \frac{dt}{t^2 - \frac{2}{\sqrt{3}}t + \left(\frac{1}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 - 1}$$

$$I = -\frac{2}{\sqrt{3}}\int \frac{dt}{\left(t - \frac{1}{\sqrt{3}}\right)^2 - \left(\frac{2}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}}\int \frac{dt}{\left(\frac{2}{\sqrt{3}}\right)^2 - \left(t - \frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{2}{\sqrt{3}} \times \frac{1}{2\left(\frac{2}{\sqrt{3}}\right)}\log\left|\frac{\frac{2}{\sqrt{3}} + t + \frac{1}{\sqrt{3}}}{\frac{2}{\sqrt{3}} - t + \frac{1}{\sqrt{3}}}\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{\sqrt{3}t + 1}{3 - \sqrt{3}t}\right| + c$$

$$I = \frac{1}{2}\log\left|\frac{1 + \sqrt{3}\tan\frac{x}{2}}{3 - \sqrt{3}\tan\frac{x}{2}}\right| + c$$

Indefinite Integrals Ex 19.23 Q13

Let $I = \int \frac{1}{\sqrt{3}\sin x + \cos x} dx$

Let $\sqrt{3} = r\cos\theta,\ \text{and } 1 = r\sin\theta$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

$r = \sqrt{3+1} = 2$

$I = \int \frac{1}{r\cos\theta\sin x + r\sin\theta\cos x} dx$

$= \frac{1}{r}\int \frac{1}{\sin(x+\theta)} dx$

$= \frac{1}{2}\int \operatorname{cosec}(x+\theta)\, dx$

$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\theta}{2}\right)\right| + c$

$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right| + c$

Indefinite Integrals Ex 19.23 Q14

Let $I = \int \frac{1}{\sin x - \sqrt{3}\cos x} dx$

Let $1 = r\cos\theta,\ \text{and } \sqrt{3} = r\sin\theta$

$r = \sqrt{3+1} = 2$

$\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$I = \int \frac{1}{r\cos\theta\sin x - r\sin\theta\cos x} dx$

$= \frac{1}{2}\int \frac{1}{\sin(x-\theta)} dx$

$= \frac{1}{2}\int \operatorname{cosec}(x-\theta)\, dx$

$= \frac{1}{2}\log\left|\tan\left(\frac{x}{2}-\frac{\theta}{2}\right)\right| + c$

$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)\right| + c$

Indefinite Integrals Ex 19.23 Q15

Let $I = \int \frac{1}{5+7\cos x + \sin x} dx$

Put $\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}, \cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

Now,

$$I = \int \frac{1}{5 + \frac{7\left(1-\tan^2\frac{x}{2}\right)}{\left(1+\tan^2\frac{x}{2}\right)} + \left(\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)} dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{5\left(1+\tan^2\frac{x}{2}\right) + 7 - 7\tan^2\frac{x}{2} + 2\tan\frac{x}{2}} dx$$

$$= \int \frac{\sec^2\frac{x}{2}}{-2\tan^2\frac{x}{2} + 12 + 2\tan\frac{x}{2}} dx$$

Let $\tan\frac{x}{2} = t$

$$\frac{1}{2}\sec^2\frac{x}{2} dx = dt$$

$$I = \int \frac{2dt}{-2t^2 + 12 + 2t}$$

$$= -\int \frac{dt}{t^2 - t - 6}$$

$$= -\int \frac{dt}{t^2 - 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 6}$$

$$= -\int \frac{dt}{\left(t-\frac{1}{2}\right)^2 - \left(\frac{5}{2}\right)^2}$$

$$= -\frac{1}{2\left(\frac{5}{2}\right)} \log\left|\frac{t-\frac{1}{2}-\frac{5}{2}}{t-\frac{1}{2}+\frac{5}{2}}\right| + c$$

$$= -\frac{1}{5}\log\left|\frac{t-3}{t+2}\right| + c$$

$$I = \frac{1}{5}\log\left|\frac{\tan\frac{x}{2}+2}{\tan\frac{x}{2}-3}\right| + c$$