RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.22

Indefinite Integrals Ex 19.22 Q1

Let $I = \int \frac{1}{4\cos^2 x + 9\sin^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$= \int \frac{\frac{1}{\cos^2 x}}{4 + 9\tan^2 x} dx$$

$$I = \int \frac{\sec^2 x}{4 + 9\tan^2 x} dx$$

Let $\tan x = t$

$$\sec^2 x\, dx = dt$$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4 + (3t)^2}$$

Let $3t = u$

$$3dt = du$$

$$I = \frac{1}{3}\int \frac{du}{(2)^2 + (u)^2}$$

$$= \frac{1}{3} \times \frac{1}{2} \times \tan^{-1}\left(\frac{u}{2}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{3t}{2}\right) + c$$

$$I = \frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right) + c$$

Indefinite Integrals Ex 19.22 Q2

Let $\quad I = \int \frac{1}{4\sin^2 x + 5\cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{4\tan^2 x + 5} dx$$

$$= \int \frac{\sec^2 x}{4\tan^2 x + 5} dx$$

Let $\quad \tan x = t$

$$\sec^2 x\, dx = dt$$

$$I = \int \frac{dt}{4 + 9(t)^2}$$

$$= \int \frac{dt}{4t^2 + 5}$$

Let $\quad 2t = u$

$$2dt = du$$

$$I = \frac{1}{2}\int \frac{du}{(4)^2 + (\sqrt{5})^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{5}} \times \tan^{-1}\left(\frac{u}{\sqrt{5}}\right) + c$$

$$= \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{2t}{\sqrt{5}}\right) + c$$

$$I = \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c$$

Indefinite Integrals Ex 19.22 Q3

Let $I = \int \frac{2}{2+\sin 2x} dx$

$= \int \frac{2}{2+2\sin x\cos x} dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{\sin x \cos x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + \tan x} dx$$

$$I = \int \frac{\sec^2 x}{1+\tan^2 x + \tan x} dx$$

Let $\tan x = t$

$\sec^2 x\, dx = dt$

$$I = \int \frac{dt}{t^2+t+1}$$

$$= \int \frac{dt}{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$I = \int \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\,t+1}{\sqrt{3}}\right) + c$$

$$I = \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\tan x + 1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.22 Q4

Let $I = \int \frac{\cos x}{\cos 3x} dx$

$$= \int \frac{\cos x}{4\cos^3 x - 3\cos x} dx$$

Diving numerator and denominator by $\cos^3 x$

$$I = \int \frac{\frac{\cos x}{\cos^3 x}}{\frac{4\cos^3 x}{\cos^3 x} + \frac{3\cos x}{\cos^3 x}} dx$$

$$= \int \frac{\sec^2 x}{4 - 3\sec^2 x} dx$$

$$= \int \frac{\sec^2 x}{4 - 3\left(1 + \tan^2 x\right)} dx$$

$$= \int \frac{\sec^2 x}{4 - 3 - 3\tan^2 x} dx$$

$$= \int \frac{\sec^2 x}{1 - 3\tan^2 x} dx$$

Let $\tan x = t$

$\sec^2 x \, dx = dt$

$$I = \int \frac{dt}{1 - 3t^2}$$

$$= \int \frac{dt}{1 - \left(\sqrt{3}t\right)^2}$$

Let $\sqrt{3}t = u$

$\sqrt{3}dt = du$

$$= \int \frac{du}{(1)^2 - (4)^2}$$

$$= \frac{1}{2\sqrt{3}} \log\left|\frac{u+1}{1-u}\right| + c$$

$$= \frac{1}{2\sqrt{3}} \log\left|\frac{\sqrt{3}t+1}{1-\sqrt{3}t}\right| + c$$

$$I = \frac{1}{2\sqrt{3}} \log\left|\frac{1+\sqrt{3}\tan x}{1-\sqrt{3}\tan x}\right| + c$$

Indefinite Integrals Ex 19.22 Q5

Let $\quad I = \int \frac{1}{1+3\sin^2 x}\,dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{3\sin^2 x}{\cos^2 x}}\,dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x}\,dx$$

$$= \int \frac{\sec^2 x}{1+\tan^2 x + 3\tan^2 x}\,dx$$

$$= \int \frac{\sec^2 x}{1+4\tan^2 x}\,dx$$

$$= \int \frac{\sec^2 x}{1+(2\tan x)^2}\,dx$$

Let $\quad 2\tan x = t$

$$2\sec^2 x\,dx = dt$$

$$I = \frac{1}{2}\int \frac{dt}{1+t^2}$$

$$= \frac{1}{2}\tan^{-1} t + c$$

$$I = \frac{1}{2}\tan^{-1}\left(2\tan x\right) + c$$

Indefinite Integrals Ex 19.22 Q6

Let $I = \int \frac{1}{3 + 2\cos^2 x} dx$

Diving numerator and denominator by $\cos^2 x$

$$I = \int \frac{\frac{1}{\cos^2 x}}{\frac{3}{\cos^2 x} + \frac{2\cos^2 x}{\cos^2 x}} dx$$

$$= \int \frac{\sec^2 x}{3\sec^2 x + 2} dx$$

$$= \int \frac{\sec^2 x}{3\left(1 + \tan^2 x\right) + 2} dx$$

$$= \int \frac{\sec^2 x}{3 + 3\tan^2 x + 2} dx$$

$$= \int \frac{\sec^2 x}{5 + 3\tan^2 x} dx$$

Let $\sqrt{3}\tan x = t$

$\sqrt{3}\sec^2 x\, dx = dt$

$$I = \frac{1}{\sqrt{3}} \int \frac{dt}{\left(\sqrt{5}\right)^2 + t^2}$$

$$= \frac{1}{\sqrt{3} \times \sqrt{5}} \tan^{-1}\left(\frac{t}{\sqrt{5}}\right) + c$$

$$I = \frac{1}{\sqrt{15}} \tan^{-1}\left(\frac{\sqrt{3}\tan x}{\sqrt{5}}\right) + c$$