RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.2

Indefinite Integrals Ex 19.2 Q1

$$\int \left(3x\sqrt{5} + 4\sqrt{x} + 5\right) dx$$

$$= \int 3x\sqrt{5}dx + \int 4\sqrt{x}dx + \int 5dx$$

$$= \int 3x^{\frac{3}{2}}dx + 4\int x^{\frac{1}{2}}dx + 5\int dx$$

$$= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{4x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 5x + c$$

$$= \frac{6}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 5x + c$$

Indefinite Integrals Ex 19.2 Q2

$$\int \left(2^x + \frac{5}{x} - \frac{1}{x^{\frac{1}{3}}}\right) dx$$

$$= \int 2^x dx + 5\int \frac{1}{x} dx - \int \frac{1}{x^{\frac{1}{3}}} dx$$

$$= \frac{2^x}{\log 2} + 5\log x - \frac{3}{2}x^{\frac{2}{3}} + c$$

Indefinite Integrals Ex 19.2 Q3

$$\int \left\{\sqrt{x}\left(ax^2 + bx + c\right)\right\} dx$$

$$= \int \sqrt{x} \times ax^2dx + \int \sqrt{x} \times bxdx + \int c\sqrt{x}dx$$

$$= \int ax^{\frac{5}{2}}dx + \int bx^{\frac{3}{2}}dx + \int cx^{\frac{1}{2}}dx$$

$$= \frac{ax^{\frac{5}{2}+1}}{\frac{5}{2}+1} + \frac{bx^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{cx^{\frac{1}{2}+1}}{\frac{1}{2}+1} + d$$

$$= \frac{2ax^{\frac{7}{2}}}{7} + \frac{2bx^{\frac{5}{2}}}{5} + \frac{2cx^{\frac{3}{2}}}{3} + d$$

Indefinite Integrals Ex 19.2 Q4

$$\int (2 - 3x)(3 + 2x)(1 - 2x)\ dx$$

$$= \int \left(6 + 4x - 9x - 6x^2\right)(1 - 2x)dx$$

$$= \int \left(-6x^2 - 5x + 6\right)(1 - 2x)dx$$

$$= \int \left(-6x^2 + 12x^3 - 5x + 10x^2 + 6 - 12x\right) dx$$

$$= \int \left(4x^2 + 12x^3 - 17x + 6\right)dx$$

$$= \int \left(12x^3 + 4x^2 - 17x + 6\right)dx$$

$$= \frac{12}{4}x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c$$

$$= 3x^4 + \frac{4}{3}x^3 - \frac{17}{2}x^2 + 6x + c$$

Indefinite Integrals Ex 19.2 Q5

$$\int\left(\frac{m}{x}+\frac{x}{m}+m^{x}+x^{m}+mx\right)dx$$

$$= m\int\frac{1}{x}dx+\frac{1}{m}\int xdx+\int m^{x}dx\int x^{m}dx+m\int xdx$$

$$= m\log|x|+\frac{x^{2}}{2m}+\frac{m^{x}}{\log m}+\frac{x^{m+1}}{m+1}+\frac{mx^{2}}{2}+c$$

Indefinite Integrals Ex 19.2 Q6

$$\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^{2}dx$$

$$=\int\left(x+\frac{1}{x}-2\right)dx$$

$$=\int xdx+\int\frac{1}{x}dx-2\int 1.dx$$

$$=\frac{x^{2}}{2}+\log|x|-2x+C$$

Indefinite Integrals Ex 19.2 Q7

$$\int\frac{(1+x)^{3}}{\sqrt{x}}dx$$

$$=\int\frac{1+x^{3}+3x^{2}+3x}{5x}dx$$

$$=\int\frac{1}{\sqrt{x}}dx+\int\frac{x^{3}}{\sqrt{x}}dx+\int\frac{3x^{2}}{\sqrt{x}}dx+\int\frac{3x}{\sqrt{x}}dx$$

$$=\int x^{\frac{-1}{2}}dx+\int x^{\frac{5}{2}}dx+3\int x^{\frac{3}{2}}dx+3\int x^{\frac{1}{2}}dx$$

$$=\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+\frac{3x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+c$$

$$=\frac{x^{\frac{1}{2}}}{\frac{1}{2}}+\frac{x^{\frac{7}{2}}}{\frac{7}{2}}+\frac{3x^{\frac{5}{2}}}{\frac{5}{2}}+3\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+c$$

$$=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\frac{6}{5}x^{\frac{5}{2}}+\frac{6}{3}x^{\frac{3}{2}}+c$$

$$=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\frac{6}{5}x^{\frac{5}{2}}+2x^{\frac{3}{2}}+c$$

$$\therefore\quad\int\frac{(1+x)^{3}}{\sqrt{x}}dx=2x^{\frac{1}{2}}+\frac{2}{7}x^{\frac{7}{2}}+\cdot\frac{6}{5}\;x^{\frac{5}{2}}+2x^{\frac{3}{2}}+c$$

Indefinite Integrals Ex 19.2 Q8

$$\int \left\{x^2 + e^{\log x} + \left(\frac{e}{2}\right)^x\right\}dx$$

$$= \int x^2 dx + \int e^{\log x} dx + \int \left(\frac{e}{2}\right)^x dx$$

$$= \frac{x^3}{3} + \int x dx + \int \left(\frac{e}{2}\right) dx$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + \frac{1}{\log\left(\frac{e}{2}\right)} \times \left(\frac{e}{2}\right)^x + c$$

Indefinite Integrals Ex 19.2 Q9

$$\int \left(x^e + e^x + e^e\right)dx$$

$$= \int x^e dx + \int e^x dx + \int e^e dx$$

$$= \frac{x^{e+1}}{e+1} + e^x + e^e x + c \qquad [\because e \text{ is constant}]$$

$$\therefore \qquad \int \left(x^e + e^x + e^e\right)dx = \frac{x^{e+1}}{e+1} + e^x + e^e x + c$$

Indefinite Integrals Ex 19.2 Q10

$$\int \sqrt{x}\left(x^3 - \frac{2}{x}\right)dx = \int x^{\frac{7}{2}} dx - 2\int x^{-\frac{1}{2}} dx$$

$$= \frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} - 2\,\frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + c$$

$$= \frac{x^{\frac{9}{2}}}{\frac{9}{2}} - \frac{2x^{\frac{-1}{2}}}{\frac{-1}{2}} + c$$

$$= \frac{2}{9}x^{\frac{9}{2}} - 4x^{\frac{-1}{2}} + c$$

$$\therefore \qquad \int \sqrt{x}\left(x^3 - \frac{2}{x}\right) = \frac{2}{9}x^{\frac{9}{2}} - 4\sqrt{x} + c$$

Indefinite Integrals Ex 19.2 Q11

$$\int \frac{1}{\sqrt{x}}\left(1+\frac{1}{x}\right)dx$$

$$= \int\left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{x}\times x}\right)dx$$

$$= \int x^{\frac{-1}{2}} + \int x^{\frac{-3}{2}}dx$$

$$= 2x^{\frac{1}{2}} - 2x^{\frac{-1}{2}} + c$$

$$= 2\sqrt{x} - \frac{2}{\sqrt{x}} + c$$

$$\therefore \quad \int \frac{1}{\sqrt{x}}\left(1+\frac{1}{x}\right)dx = 2\sqrt{x} - \frac{2}{\sqrt{x}} + c$$

Indefinite Integrals Ex 19.2 Q12

$$\int \frac{x^6+1}{x^2+1}dx$$

$$= \int \frac{\left(x^2\right)^3 + (1)^3}{x^2+1}dx$$

$$= \int \frac{\left(x^2+1\right)\left(x^4+1-x^2\right)}{x^2+1}dx$$

$$= \int \left(x^4 - x^2 + 1\right)dx$$

$$= \int x^4dx - \int x^2dx + \int 1dx$$

$$= \frac{x^5}{5} - \frac{x^3}{3} + x + c$$

Indefinite Integrals Ex 19.2 Q13

$$\int \frac{x^{\frac{-1}{3}} + \sqrt{x} + 2}{\sqrt[3]{x}}dx$$

$$= \int \frac{x^{\frac{-1}{3}}dx}{x^{\frac{1}{3}}} + \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}}dx + \int \frac{2}{x^{\frac{1}{3}}}dx$$

$$= \int x^{\frac{-2}{3}}dx + \int x^{\frac{1}{6}}dx + 2\int x^{\frac{-1}{3}}dx$$

$$= 3x^{\frac{1}{3}} + \frac{6}{7}x^{\frac{7}{6}} + 3x^{\frac{2}{3}} + c$$

Indefinite Integrals Ex 19.2 Q14

$$\int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}dx$$

$$= \int \frac{1+x+2\sqrt{x}}{x^{\frac{1}{2}}}dx$$

$$= \int x^{\frac{-1}{2}} + \int x^{\frac{1}{2}}dx + 2\int dx$$

$$= 2\sqrt{x} + \frac{2}{3}x^{\frac{3}{2}} + 2x + c$$

$$\therefore \quad \int \frac{\left(1+\sqrt{x}\right)^2}{\sqrt{x}}dx = 2\sqrt{x} + 2x + \frac{2}{3}x^{\frac{3}{2}} + c$$

Indefinite Integrals Ex 19.2 Q15

$$\int \sqrt{x}\left(3-5x\right)dx$$

$$= 3\int \sqrt{x}dx - 5\int x^{\frac{3}{2}}dx$$

$$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - 5\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$$

$$= 2x^{\frac{3}{2}} - 2x^{\frac{5}{2}} + c$$

Indefinite Integrals Ex 19.2 Q16

$$\int \frac{(x+1)(x-2)}{\sqrt{x}}dx$$

$$= \int \frac{x^2-2x+x-2}{x^{\frac{1}{2}}}dx$$

$$= \int \frac{x^2-x-2}{x^{\frac{1}{2}}}dx$$

$$= \int \frac{x^2}{x^{\frac{1}{2}}}dx - \int x^{\frac{1}{2}}dx - 2\int x^{\frac{-1}{2}}dx$$

$$= \frac{2x^{\frac{5}{2}}}{5} - \frac{2x^{\frac{3}{2}}}{3} - 4x^{\frac{1}{2}} + c$$

$$\therefore \quad \int \frac{(x+1)(x-2)}{\sqrt{x}}dx = \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + c$$

$$= \frac{2}{5}x^{\frac{5}{2}} - \frac{2x^{\frac{3}{2}}}{3} - 4\sqrt{x} + c$$

Indefinite Integrals Ex 19.2 Q17

$$\int \frac{x^5 + x^{-2} + 2}{x^2} dx$$

$$= \int \left(\frac{x^5}{x^2} + \frac{x^{-2}}{x^2} + \frac{2}{x^2} \right) dx$$

$$= \int x^3 dx + \int x^{-4} + 2\int x^{-2} dx$$

$$= \frac{x^4}{4} + \frac{x^{-3}}{-3} + \frac{2x^{-1}}{-1} + c$$

$$= \frac{x^4}{4} - \frac{x^{-3}}{3} - \frac{2}{x} + c$$

Indefinite Integrals Ex 19.2 Q18

$$\int (3x + 4)^2 dx$$

$$= \int \left(9x^2 + 16 + 24x\right) dx$$

$$= 9\int x^2 dx + 16\int dx + 24\int x dx$$

$$= 9\frac{x^3}{3} + 16x + 24\frac{x^2}{2} + c$$

$$= 3x^3 + 16x + 12x^2 + c$$

$$\therefore \quad \int (3x + 4)^2 = 3x^3 + 12x^2 + 16x + c$$

Indefinite Integrals Ex 19.2 Q19

$$\int \frac{2x^4 + 7x^3 + 6x^2}{x^2 + 2x} dx$$

$$= \int \frac{x\left(2x^3 + 7x^2 + 6x\right)}{x(x + 2)} dx$$

$$= \int \frac{2x^3 + 7x^2 + 6c}{x + 2} dx$$

$$= \int \frac{2x^3 + 4x^2 + 3x^2 + 6x}{(x + 2)} dx$$

$$= \int \frac{2x^2(x + 2) + 3x(x + 2)}{(x + 2)} dx$$

$$= \int \frac{(x + 2)\left(2x^2 + 3x\right)}{x + 2} dx$$

$$= \int \left(2x^2 + 3x\right) dx$$

$$= \int 2x^2 dx + \int 3x dx$$

$$= \frac{2}{3}x^3 + \frac{3}{2}x^2 + c$$

Indefinite Integrals Ex 19.2 Q20

$$\int \frac{5x^4 + 12x^3 + 7x^2}{x^2 + x} dx$$

$$= \int \frac{5x^4 + 7x^3 + 5x^3 + 7x^2}{x^2 + x} dx$$

$$= \int \frac{5x^3 + 7x^2 + 5x^2 + 7x}{x+1} dx$$

$$= \int \frac{5x^2 (x+1) + 7x(x+1)}{x+1} dx$$

$$= \int \left(5x^2 + 7x \right) dx$$

$$= \frac{5x^3}{3} + \frac{7x^2}{2} + C$$

Indefinite Integrals Ex 19.2 Q21

$$\int \frac{\sin^2 x}{1 + \cos x} dx$$

$$= \int \frac{1 - \cos^2 x}{1 + \cos x} dx$$

$$= \int \frac{(1 - \cos x)(1 + \cos x)}{(1 + \cos x)} dx$$

$$= \int (1 - \cos x) dx$$

$$= x - \sin x + c$$

Indefinite Integrals Ex 19.2 Q22

$$\int \left(\sec^2 x + \operatorname{cosec}^2 x \right) dx$$

$$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$$

$$= \tan x - \cot x + c$$

$$\therefore \quad \int \left(\sec^2 x + \operatorname{cosec}^2 x \right) dx = \tan x - \cot x + c$$

Indefinite Integrals Ex 19.2 Q23

Evaluate the integral as follows

$$\int \frac{\sin^3 x - \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin^3 x}{\sin x^2 \cos^2 x} - \frac{\cos^3 x}{\sin x^2 \cos^2 x} \right) dx$$

$$= \int \left(\sin x \sec^2 x - \cos x \operatorname{cosec}^2 x \right) dx$$

$$= \int \left(\tan x \sec x - \cot x \operatorname{cosec} x \right) dx$$

$$= \sec x + \operatorname{cosec} x + C$$

Indefinite Integrals Ex 19.2 Q24

$$I \int \frac{5\cos^3 x + 6\sin^3 x}{2\sin^2 x\cos^2 x}dx$$

Now,

$$I = \int \frac{5\cos^3 x + \sin^3 x}{2\sin^2 x\cos^2 x}dx$$
$$= \int \frac{5\cos^3 x}{2\sin^2 x\cos^2 x}dx + \int \frac{6\sin^3 x}{2\sin^2 x\cos^2 x}dx$$
$$= \frac{5}{2}\int \frac{\cos x}{\sin^2 x}dx + 3\int \frac{\sin x}{\cos^2 x}dx$$
$$= \frac{5}{2}\int \cot x\ \mathrm{cosec}\, x dx + 3\int \sec x \tan x dx$$
$$= + \frac{-5}{2}\mathrm{cosec}\, x + 3\sec x + c$$

$$\therefore \quad I = \frac{-5}{2}\cos\sec x + 3\sec x + c$$

Indefinite Integrals Ex 19.2 Q25

$$\int (\tan x + \cot x)^2 dx$$

$$= \int \left(\tan^2 x + \cot^2 x + 2\tan x \cot x\right)dx$$
$$= \int \left(\sec^2 x - 1 + \mathrm{cosec}^2 x - 1 + \frac{2\times 1}{\cot x}\cot x\right)dx$$
$$= \int \left(\sec^2 x + \mathrm{cosec}^2 x\right)dx$$
$$= \int \sec^2 x dx + \int \mathrm{cosec}^2 x$$
$$= \tan x - \cot x + c$$

$$\therefore \quad \int (\tan x + \cot x)^2 = \tan x - \cot x + c$$

Indefinite Integrals Ex 19.2 Q26

$$\int \frac{1-\cos 2x}{1+\cos 2x}dx$$

$$= \int \frac{2\sin^2 x}{2\cos^2 x}dx$$
$$= \int \tan^2 x dx$$
$$= \int \left(\sec^2 x - 1\right)dx$$
$$= \int \sec^2 x dx - \int dx$$
$$= \tan x - x + c$$

$$\therefore \quad \int \frac{1-\cos 2x}{1+\cos 2x}dx = \tan x - x + c$$

Indefinite Integrals Ex 19.2 Q27

$$\int \frac{\cos x}{1-\cos x}dx$$

$$= \int \frac{\cos x\left(1+\cos x\right)}{\left(1-\cos x\right)\left(1+\cos x\right)}dx$$

$$= \int \frac{\cos x + \cos^2 x}{1-\cos^2 x}dx$$

$$= \int \frac{\cos x + \cos^2 x}{\sin^2 x}dx$$

$$= \int \frac{\cos x}{\sin^2 x}dx + \int \frac{\cos^2 x}{\sin^2 x}dx$$

$$= \int \cot x \times \operatorname{cosec} x dx + \int \left(\operatorname{cosec}^2 x - 1\right)dx$$

$$= -\operatorname{cosec} x - \cot x - x + c$$

$$\therefore \quad \int \frac{\cos x}{1-\cos x} \times dx = -\operatorname{cosec} x - \cot x - x + c$$

Indefinite Integrals Ex 19.2 Q28

$$\int \frac{\cos^2 x - \sin^2 x}{\sqrt{1+\cos 4x}}dx$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\sqrt{2\cos^2 2x}}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{\cos^2 x - \sin^2 x}{\cos 2x}dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin^2 x}dx \qquad \left[\because \cos 2x = \cos^2 x - \sin^2 x\right]$$

$$= \frac{1}{\sqrt{2}}\int 1 \times dx$$

$$= \frac{x}{\sqrt{2}} + c$$

$$\therefore \quad \int \frac{\cos^2 x - \sin^2 x}{\sqrt{1+\cos 4x}} \times dx = \frac{x}{\sqrt{2}} + c$$

Indefinite Integrals Ex 19.2 Q29

$$\int \frac{1}{1-\cos x}dx$$

$$= \int \frac{1}{1-\cos x} \times \frac{1+\cos x}{1+\cos x} \times dx$$
$$= \int \frac{1+\cos x}{1-\cos^2 x} \times dx$$
$$= \int \frac{1+\cos x}{\sin^2 x} \times dx$$
$$= \int \frac{1}{\sin^2 x}dx + \int \frac{\cos x}{\sin^2 x}dx$$
$$= \int \operatorname{cosec}^2 x dx + \int \cot x \times \operatorname{cosec} x dx$$
$$= -\cot x - \operatorname{cosec} x + c$$

$$\therefore \quad \int \frac{1}{1-\cos x}dx = -\cot x - \operatorname{cosec} x + c.$$

Indefinite Integrals Ex 19.2 Q30

$$\int \frac{1}{1-\sin x}dx$$

$$= \int \frac{1}{1-\sin x} \times \frac{1+\sin x}{1+\sin x} \times dx$$
$$= \int \frac{1+\sin x}{1-\sin^2 x} \times dx$$
$$= \int \frac{1+\sin x}{\cos^2 x} \times dx$$
$$= \int \left(\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x}\right) \times dx$$
$$=$$
$$= \int \frac{1}{\cos^2 x}dx + \int \frac{\sin x}{\cos^2 x} \times dx$$
$$= \int \sec^2 x dx + \int \tan x \sec x dx$$
$$= \tan x + \sec x + c$$

$$\therefore \quad \int \frac{1}{1-\sin x} \times dx = \tan x + \sec x + c.$$

Indefinite Integrals Ex 19.2 Q31

$$\int \frac{\tan x}{\sec x + \tan x} \times dx$$

$$= \int \frac{\tan x}{\sec x + \tan x} \times \frac{\sec x - \tan x}{\sec x - \tan x} \times dx$$
$$= \int \frac{\tan x\left(\sec x - \tan x\right)}{\sec^2 x - \tan^2 x} \times dx$$
$$= \int \left(\tan x \sec x - \tan^2 x\right) dx$$
$$= \int \sec \tan x dx - \int \left(\sec^2 x - 1\right) dx$$
$$= \int \sec x \tan x dx - \int \sec^2 x dx + \int dx$$
$$= \sec x - \tan x + x + c$$

$$\therefore \quad \int \frac{\tan x}{\sec x + \tan x} \times dx = \sec x - \tan x + x + c.$$

Indefinite Integrals Ex 19.2 Q32

$$\int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times dx$$

$$= \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times \frac{\operatorname{co\,sec} x + \cot x}{\operatorname{co\,sec} x + \cot x} \times dx$$
$$= \int \frac{\operatorname{cosec} x\left(\operatorname{cosec} x + \cot x\right)}{\operatorname{cosec}^2 x - \cot^{2x}} \times dx$$
$$= \int \left(\operatorname{cosec}^2 x + \operatorname{cosec} x \cot x\right) dx$$
$$= \int \operatorname{cosec}^2 x dx + \int \operatorname{cosec} x dx$$
$$= -\cot x - \operatorname{cosec} x + c$$

$$\therefore \quad \int \frac{\operatorname{cosec} x}{\operatorname{cosec} x - \cot x} \times dx = -\cot x - \operatorname{cosec} x + c.$$

Indefinite Integrals Ex 19.2 Q33

$$\int \frac{1}{1 + \cos 2x} \times dx$$

$$= \int \frac{1}{2\cos^2 x} \times dx$$
$$= \frac{1}{2} \int \sec^2 x \times dx$$
$$= \frac{1}{2} \times \tan x + c$$
$$= \frac{\tan x}{2} + c$$

$$\therefore \quad \int \frac{1}{1 + \cos 2x} \times dx = \frac{1}{2} \tan x + c.$$

Indefinite Integrals Ex 19.2 Q34

$$\int \frac{1}{1-\cos 2x} dx$$

$$= \int \frac{1}{2\sin^2 x} \times dx$$

$$= \frac{1}{2} \int cosec^2 x \times dx$$

$$= \frac{-1}{2} \times \cot x + c$$

$$= \frac{-1 \cot x}{2} + c$$

$$\therefore \quad \int \frac{1}{1-\cos 2x} = \frac{-1}{2} \cot x + c.$$

Indefinite Integrals Ex 19.2 Q35

$$\int \tan^{-1}\left[\frac{\sin 2x}{1+\cos 2x}\right] dx$$

$$= \int \tan^{-1}\left[\frac{2\sin x \cos x}{2\cos^2 x}\right] dx$$

$$= \int \tan^{-1}\left[\frac{\sin x}{\cos x}\right] dx$$

$$= \int \tan^{-1}(\tan x)\, dx$$

$$= \int x dx$$

$$= \frac{x^2}{2} + c$$

$$\therefore \quad \int \tan^{-1}\left[\frac{\sin 2x}{1+\cos 2x}\right] dx = \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q36

$$\int \cos^{-1}(\sin x) dx$$

$$= \int \cos^{-1}\left[\cos\left(\frac{\pi}{2} - x\right)\right] dx$$

$$= \int \left(\frac{\pi}{2} - x\right) dx$$

$$= \frac{\pi}{2} \int dx - \int x dx$$

$$= \frac{\pi}{2} \times x - \frac{x^2}{2} + c$$

$$\therefore \quad \int \cos^{-1}(\sin x) dx = \frac{\pi}{2} \times x - \frac{x^2}{2} + c.$$

Indefinite Integrals Ex 19.2 Q37

$\int \cos^{-1}(\sin x)dx$

$= \int \cot^{-1}\left[\frac{\sin 2x}{1-\cos 2x}\right]dx$

$= \int \cot^{-1}\left(\frac{\cos x}{\sin x}\right)dx$

$= \int \cot^{-1}(\cot x)dx$

$= \int x dx$

$= \frac{x^2}{2} + c$

$\therefore \quad \int \cot^{-1}\left[\frac{\sin 2x}{1-\cos 2x}\right]dx = \frac{x^2}{2} + c.$

Indefinite Integrals Ex 19.2 Q38

$\int \sin^{-1}\left(\frac{2\tan x}{1+\tan^2 x}\right)dx$

$= \int \sin^{-1}(\sin 2x)dx \qquad \left[\because \sin 2x = \frac{2\tan x}{1+\tan^2 x}\right]$

$= \int 2x dx$

$= 2\int x\,dx$

$= \frac{2x^2}{2} + c$

$= x^2 + c$

$\therefore \quad \int \sin^{-1}\left(\frac{2\tan x}{1+\tan^2 x}\right) = x^2 + c.$

Indefinite Integrals Ex 19.2 Q39

$\int \frac{(x^3+8)(x-1)}{x^2-2x+4}dx$

$= \int \frac{(x+2)(x^2-2x+4)(x-1)}{x^2-2x+4}dx$

$= \int (x+2)(x-1)\,dx$

$= \int (x^2 - x + 2x - 2)\,dx$

$= \int (x^2 + x - 2)\,dx$

$= \frac{x^3}{3} + \frac{x^2}{2} - 2x + c$

$\therefore \quad \int \frac{(x^3+8)(x-1)}{x^2-2x+4}dx = \frac{x^3}{3} + \frac{x^2}{2} - 2x + c.$

Indefinite Integrals Ex 19.2 Q40

$\int (a\tan x + b\cot x)^2 dx$

$$= \int \left(a^2\tan^2 x + b^2\cot^2 x + 2ab\tan x\cot x\right)dx$$
$$= \int \left[a^2\left(\sec^2 x - 1\right) + b^2\left(\operatorname{cosec}^2 x - 1\right) + 2ab\right]dx$$
$$= \int \left[a^2\sec^2 x - a^2 + b^2\operatorname{cosec}^2 x - b^2 + 2ab\right]dx$$
$$= a^2\tan x - a^2 x - b^2\cot x - b^2 x + 2abx + c$$
$$= a^2\tan x + -b^2\cot x - \left(a^2 + b^2 - 2ab\right)x + c$$

$\therefore \quad \int (a\tan x + b\cot x)^2 dx = a^2\tan x - b^2\cot x - \left(a^2 + b^2 - 2ab\right)x + c.$

Indefinite Integrals Ex 19.2 Q41

$\int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2}dx$

$$= \frac{1}{2}\int \frac{x^3}{x^2}dx - \frac{3}{2}\int \frac{x^2}{x^2}dx + \frac{5}{2}\int x\frac{x}{x^2}dx - \frac{7}{2}\int x^{-2}dx + \frac{1}{2}\int \frac{x^2 a^x}{x^2}dx$$
$$= \frac{1}{2}\times\frac{x^2}{2} - \frac{3}{2}x + \frac{5}{2}\log x - \frac{7}{2}x^{-1} + \frac{1}{2}\frac{a^x}{\log a} + c$$
$$= \frac{1}{2}\left[\frac{x^2}{2} - 3x + 5\log x + \frac{7}{x} + \frac{a^x}{\log a}\right] + c$$

$\therefore \quad \int \frac{x^3 - 3x^2 + 5x - 7 + x^2 a^x}{2x^2}dx = \frac{1}{2}\left[\frac{x^2}{2} - 3x + 5\log x + \frac{7}{x} + \frac{a^x}{\log a}\right] + c$

Indefinite Integrals Ex 19.2 Q42

$$\frac{\cos x}{1+\cos x} = \frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} \text{ and } \cos x = 2\cos^2\frac{x}{2} - 1\right]$$
$$= \frac{1}{2}\left[1 - \tan^2\frac{x}{2}\right]$$

$$\therefore \int \frac{\cos x}{1+\cos x}dx = \frac{1}{2}\int\left(1 - \tan^2\frac{x}{2}\right)dx$$
$$= \frac{1}{2}\int\left(1 - \sec^2\frac{x}{2} + 1\right)dx$$
$$= \frac{1}{2}\int\left(2 - \sec^2\frac{x}{2}\right)dx$$
$$= \frac{1}{2}\left[2x - \frac{\tan\frac{x}{2}}{\frac{1}{2}}\right] + C$$
$$= x - \tan\frac{x}{2} + C$$

Indefinite Integrals Ex 19.2 Q43

$$\frac{1-\cos x}{1+\cos x} = \frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} \qquad \left[2\sin^2\frac{x}{2} = 1-\cos x \text{ and } 2\cos^2\frac{x}{2} = 1+\cos x\right]$$

$$= \tan^2\frac{x}{2}$$

$$= \left(\sec^2\frac{x}{2} - 1\right)$$

$$\therefore \int \frac{1-\cos x}{1+\cos x}dx = \int\left(\sec^2\frac{x}{2} - 1\right)dx$$

$$= \left[\frac{\tan\frac{x}{2}}{\frac{1}{2}} - x\right] + C$$

$$= 2\tan\frac{x}{2} - x + C$$

Indefinite Integrals Ex 19.2 Q44

$$\int\left\{3\sin x - 4\cos x + \frac{5}{\cos^2 x} - \frac{6}{\sin^2 x} + \tan^2 x - \cot^2 x\right\}dx$$

$$= 3\int \sin x dx - 4\int \cos x dx + 5\int \sec^2 dx - 6\int \operatorname{cosec}^2 x + \int \tan^2 x dx - \int \cot^2 x dx$$

$$= 3\int \sin x dx - 4\int \cos x dx + 5\int \sec^2 x dx - 6\int \operatorname{cosec}^2 x + \int\left(\sec^2 x - 1\right)dx - \int\left(\operatorname{cosec}^2 x - 1\right)dx$$

$$= 3\int \sin x dx - 4\int \cos x dx + 6\int \sec^2 x dx - 7\int \operatorname{cosec}^2 x dx$$

$$= -3\cos x - 4\sin x + 6\tan x + 7\cot x + c$$

Indefinite Integrals Ex 19.2 Q45

It is given that $f'(x) = x - \frac{1}{x^2}$

$$\therefore \quad \int f'(x) = \int \left(x - \frac{1}{x^2}\right)dx$$

$$\Rightarrow \quad f(x) = \int x\,dx - \int \frac{1}{x^2}dx$$

$$= \frac{x^2}{2} + x^{-1} + c$$

$$= \frac{x^2}{2} + \frac{1}{x} + c$$

$$\Rightarrow \quad f(x) = \frac{x^2}{2} + \frac{1}{x} + c \qquad \text{---(i)}$$

Now,

$$f(1) = \frac{1}{2} \qquad [\text{given}]$$

$$\Rightarrow \quad \frac{1^2}{2} + \frac{1}{1} + c = \frac{1}{2}$$

$$\Rightarrow \quad c = -1$$

Putting $c = -1$ in (i), we get

$$f(x) = \frac{x^2}{2} + \frac{1}{x} - 1.$$

Indefinite Integrals Ex 19.2 Q46

It is given that $f'(x) = x + b$

$$\therefore \quad \int f'(x) = \int (x + b)dx$$

$$\Rightarrow \quad f(x) = \frac{x^2}{2} + bx + c \qquad \text{---(i)}$$

Since,

$$f(1) = 5$$

$$\therefore \quad \frac{1^2}{2} + b \times 1 + c = 5$$

$$\Rightarrow \quad \frac{1}{2} + b + c = 5$$

$$\Rightarrow \quad b + c = \frac{9}{2} \qquad \text{---(ii)}$$

and, $f(2) = 13$

$$\Rightarrow \quad \frac{(2)^2}{2} + b \times 2 + c = 13$$

$$\Rightarrow \quad 2 + 2b + c = 13$$

$$\Rightarrow \quad 2b + c = 11 \qquad \text{---(iii)}$$

Subtracting equation (ii) from equatio n(iii), we get

$$b = 11 - \frac{9}{2}$$

$$\Rightarrow \quad b = \frac{13}{2}$$

Putting $b = \frac{13}{2}$ in equation (ii), we get

$$\frac{13}{2} + c = \frac{9}{2}$$

$$\Rightarrow \quad c = \frac{9}{2} - \frac{13}{2}$$

$$\Rightarrow \quad c = \frac{9 - 13}{2} = \frac{-4}{2} = -2$$

Putting $b = \frac{13}{2}$ and $c = -2$ in equation (i), we get

$$f(x) = \frac{x^2}{x} + \frac{13}{2}x - 2$$

$$\therefore \quad f(x) = \frac{x^2}{2} + \frac{13}{2}x - 2$$

Indefinite Integrals Ex 19.2 Q47

We have,

$$f'(x) = 8x^3 - 2x$$

$$\Rightarrow \quad f(x) = \int f'(x)dx = \int \left(8x^3 - 2x\right)dx$$

$$\Rightarrow \quad f(x) = \int \left(8x^3 - 2x\right)dx$$

$$= \int 8x^3 dx - \int 2x dx$$

$$= \frac{8x^4}{4} - \frac{2x^2}{2} + c$$

$$= 2x^4 - x^2 + c$$

$$\Rightarrow \quad f(x) = 2x^4 - x^2 + c \qquad \text{---(i)}$$

Since, $f(2) = 8$

$$\therefore \quad f(2) = 2(2)^4 - (2)^2 + c = 8$$

$$\Rightarrow \quad 32 - 4 + c = 8$$

$$\Rightarrow \quad 28 + c = 8$$

$$\Rightarrow \quad c = -20$$

Putting $c = -20$ in equation (i), we get

$$f(x) = 2x^4 - x^2 - 20$$

Hence, $f(x) = 2x^4 - x^2 - 20$.

Indefinite Integrals Ex 19.2 Q48

We have,

$$f(x) = \int f'(x)\,dx$$

$$\Rightarrow \quad f(x) = \int (a\sin x + b\cos x)\,dx$$

$$= -a\cos x + b\sin x + c$$

$$\therefore \quad f(x) = -a\cos x + b\sin x + c \qquad \text{---(i)}$$

Since,

$$f'(0) = 4$$

$$\therefore \quad f'(0) = a\sin 0 + b\cos 0 = 4$$

$$\Rightarrow \quad a \times 0 + b \times 1 = 4$$

$$\Rightarrow \quad b = 4$$

Now,

$$f(0) = 3$$

$$\therefore \quad f(0) = -a\cos 0 + b\sin 0 + c = 3$$

$$\Rightarrow \quad -a + 0 + c = 3$$

$$\Rightarrow \quad c - a = 3 \qquad \text{---(ii)}$$

and, $f\left(\frac{\pi}{2}\right) = 5$

$$\therefore \quad f\left(\frac{\pi}{2}\right) = -a\cos\left(\frac{\pi}{2}\right) + b\sin\left(\frac{\pi}{2}\right) + c = 5$$

$$\Rightarrow \quad -a \times 0 + b \times 1 + c = 5$$

$$\Rightarrow \quad b + c = 5$$

$$\Rightarrow \quad 4 + c = 5 \qquad [\because b = 4]$$

$$\Rightarrow \quad c = 5 - 4$$

$$\Rightarrow \quad c = 1$$

Putting $c = 1$ in equation (ii), we get

$$1 - a = 3$$

$$\Rightarrow \quad -a = 3 - 1$$

$$\Rightarrow \quad -a = 2$$

$$\Rightarrow \quad a = -2$$

Putting $a = -2,\ b = 4$ and $c = 1$ in equatio (i), we get

$$f(x) = -(-2)\cos x + 4\sin x + 1$$

$$\Rightarrow \quad f(x) = 2\cos x + 4\sin x + 1$$

Hence, $f(x) = 2\cos x + 4\sin x + 1$

Indefinite Integrals Ex 19.2 Q49

We have,

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$$

$$\Rightarrow \quad \int f(x) = \int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$$

$$= \int x^{\frac{1}{2}} dx + \int x^{\frac{-1}{2}} dx$$

$$= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$$

Hence, the primitive or anti-derivative of $f(x) = \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$.