RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.17

Indefinite Integrals Ex 19.17 Q1

Let $I = \int \frac{1}{\sqrt{2x - x^2}} dx$

$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\right]}} dx$

$= \int \frac{1}{\sqrt{-\left[x^2 - 2x(1) + 1^2 - 1^2\right]}} dx$

$= \int \frac{1}{\sqrt{-\left[(x-1)^2 - 1\right]}} dx$

$= \int \frac{1}{\sqrt{1 - (x-1)^2}} dx$

Let $(x - 1) = t$

$\Rightarrow \quad dx = dt$

so, $\quad I = \int \frac{1}{\sqrt{1 - t^2}} dt$

$= \sin^{-1} t + c \quad \left[\text{Since } \int \frac{1}{\sqrt{1 - x^2}} dx = \sin^{-1} x + c\right]$

$I = \sin^{-1}(x - 1) + c$

Indefinite Integrals Ex 19.17 Q2

$8+3x-x^2$ can be written as $8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$.

Therefore,

$$8-\left(x^2-3x+\frac{9}{4}-\frac{9}{4}\right)$$

$$=\frac{41}{4}-\left(x-\frac{3}{2}\right)^2$$

$$\Rightarrow \int \frac{1}{\sqrt{8+3x-x^2}}dx=\int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx$$

Let $x-\frac{3}{2}=t$

$\therefore\ dx=dt$

$$\Rightarrow \int \frac{1}{\sqrt{\frac{41}{4}-\left(x-\frac{3}{2}\right)^2}}dx=\int \frac{1}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-t^2}}dt$$

$$=\sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right)+C$$

$$=\sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right)+C$$

Indefinite Integrals Ex 19.17 Q3

Let $I = \int \frac{1}{\sqrt{5 - 4x - 2x^2}} dx$

$$= \int \frac{1}{\sqrt{-2\left[x^2 + 2x - \frac{5}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[x^2 + 2x + (1)^2 - (1)^2 - \frac{5}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[(x+1)^2 - \frac{7}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\frac{7}{2} - (x+1)^2}} dx$$

Let $(x + 1) = t$

$\Rightarrow \quad dx = dt$

so, $\quad I = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\sqrt{\frac{7}{2}}\right)^2 - t^2}} dt$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{t}{\sqrt{\frac{7}{2}}}\right) + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\sqrt{\frac{2}{7}} \times (x+1)\right) + c$$

Indefinite Integrals Ex 19.17 Q4

Let $I = \int \frac{1}{\sqrt{3x^2 + 5x + 7}} dx$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \frac{5}{3}x + \frac{7}{3}}} dx$$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + 2x\left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)^2 - \left(\frac{5}{6}\right)^2 + \frac{7}{3}}} dx$$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{\left(x + \frac{5}{6}\right)^2 - \frac{59}{36}}} dx$$

Let $\left(x + \frac{5}{6}\right) = t$

$\Rightarrow \quad dx = dt$

$$I = \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{t^2 - \left(\frac{\sqrt{59}}{6}\right)^2}} dt$$

$$= \frac{1}{\sqrt{3}} \log\left|t + \sqrt{t^2 - \left(\frac{\sqrt{59}}{6}\right)}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$$

$$I = \frac{1}{\sqrt{3}} \log\left|x + \frac{5}{6} + \sqrt{\left(x + \frac{5}{6}\right)^2 - \left(\frac{\sqrt{59}}{6}\right)^2}\right| + c$$

$$I = \frac{1}{\sqrt{3}} \log\left|x + \frac{5}{6} + \sqrt{x^2 + \frac{5x}{3} + \frac{7}{3}}\right| + c$$

Indefinite Integrals Ex 19.17 Q5

Let $I = \int \frac{1}{\sqrt{(x-\alpha)(\beta - x)}} dx, (\text{as} \quad \beta > \alpha)$

$$= \int \frac{1}{\sqrt{-x^2 - x(\alpha+\beta) - \alpha\beta}} dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 - 2x\left(\frac{\alpha+\beta}{2}\right) + \left(\frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2 + \alpha\beta\right]}} dx$$

$$= \int \frac{1}{\sqrt{-\left[\left(x - \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha+\beta}{2}\right)^2\right]}} dx$$

$$= \int \frac{1}{\sqrt{\left[\left(\frac{\beta-\alpha}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2\right]}} dx, \qquad \left[\because \qquad \beta > \alpha\right]$$

Let $\left(x - \frac{\alpha+\beta}{2}\right) = t$

$\Rightarrow \qquad dx = dt$

$$I = \int \frac{1}{\sqrt{\left(\frac{\beta-\alpha}{2}\right)^2 - t^2}} dt$$

$$= \sin^{-1}\left(\frac{t}{\frac{\beta-\alpha}{2}}\right) + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{2\left(x - \frac{\alpha+\beta}{2}\right)}{\beta - \alpha}\right) + c$$

$$I = \sin^{-1}\left(\frac{2x - \alpha - \beta}{\beta - \alpha}\right) + c$$

Indefinite Integrals Ex 19.17 Q6

$$\text{Let } I = \int \frac{1}{\sqrt{7 - 3x - 2x^2}} dx$$

$$= \int \frac{1}{\sqrt{-2\left[x^2 + \frac{3}{2}x - \frac{7}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[x^2 + 2x\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 - \frac{7}{2}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{-\left[\left(x - \frac{3}{4}\right)^2 - \frac{65}{16}\right]}} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2 - \left(x + \frac{3}{4}\right)^2}} dx$$

$$\text{Let } \left(x + \frac{3}{4}\right) = t$$

$$\Rightarrow \quad dx = dt$$

$$I = \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{\left(\frac{\sqrt{65}}{4}\right)^2 - t^2}} dt$$

$$= \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{t}{\frac{\sqrt{41}}{4}}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4\left(x + \frac{3}{4}\right)}{\sqrt{65}}\right) + c$$

$$I = \frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{4x + 3}{\sqrt{65}}\right) + c$$

Indefinite Integrals Ex 19.17 Q7

Let $I = \int \frac{1}{\sqrt{16 - 6x - x^2}} dx$

$$= \int \frac{1}{\sqrt{-\left[x^2 + 6x - 16\right]}} dx$$

$$= \int \frac{1}{\sqrt{-\left[x^2 + 2x(3) + (3)^2 - (3)^2 - 16\right]}} dx$$

$$= \int \frac{1}{\sqrt{-\left[(x+3)^2 - 25\right]}} dx$$

$$= \int \frac{1}{\sqrt{25 - (x+3)^2}} dx$$

Let $(x+3) = t$

$\Rightarrow \quad dx = dt$

$$I = \int \frac{1}{\sqrt{5^2 - t^2}} dt$$

$$= \sin^{-1}\left(\frac{t}{5}\right) + c \qquad \left[\text{Since} \quad \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \sin^{-1}\left(\frac{x+3}{5}\right) + c$$

Indefinite Integrals Ex 19.17 Q8

$7-6x-x^2$ can be written as $7-\left(x^2+6x+9-9\right)$.

Therefore,

$$7-\left(x^2+6x+9-9\right)$$
$$=16-\left(x^2+6x+9\right)$$
$$=16-(x+3)^2$$
$$=(4)^2-(x+3)^2$$

$$\therefore \int\frac{1}{\sqrt{7-6x-x^2}}dx=\int\frac{1}{\sqrt{(4)^2-(x+3)^2}}dx$$

Let $x+3=t$

$\Rightarrow dx=dt$

$$\Rightarrow \int\frac{1}{\sqrt{(4)^2-(x+3)^2}}dx=\int\frac{1}{\sqrt{(4)^2-(t)^2}}dt$$
$$=\sin^{-1}\left(\frac{t}{4}\right)+C$$
$$=\sin^{-1}\left(\frac{x+3}{4}\right)+C$$

Indefinite Integrals Ex 19.17 Q9

We have $\int\frac{dx}{\sqrt{5x^2-2x}}=\int\frac{dx}{\sqrt{5\left(x^2-\frac{2x}{5}\right)}}$

$$=\frac{1}{\sqrt{5}}\int\frac{dx}{\sqrt{\left(x-\frac{1}{5}\right)^2-\left(\frac{1}{5}\right)^2}} \quad \text{(completing the square)}$$

Put $x-\frac{1}{5}=t$. Then $dx=dt$.

Therefore, $\int\frac{dx}{\sqrt{5x^2-2x}}=\frac{1}{\sqrt{5}}\int\frac{dt}{\sqrt{t^2-\left(\frac{1}{5}\right)^2}}$

$$=\frac{1}{\sqrt{5}}\log\left|t+\sqrt{t^2-\left(\frac{1}{5}\right)^2}\right|+C \qquad \text{[by 7.4 (4)]}$$

$$=\frac{1}{\sqrt{5}}\log\left|x-\frac{1}{5}+\sqrt{x^2-\frac{2x}{5}}\right|+C$$