RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.6

Indefinite Integrals Ex 19.16 Q1

Let $I = \int \frac{\sec^2 x}{1 - \tan^2 x} dx$

Let $\tan x = t$

$\Rightarrow \quad \sec^2 x \; dx = dt$

$I = \int \frac{dt}{(1)^2 - t^2}$

$= \frac{1}{2(1)} \log\left|\frac{1+t}{1-t}\right| + c \quad \left[\text{Since, } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c\right]$

$I = \frac{1}{2} \log\left|\frac{1 + \tan x}{1 - \tan x}\right| + c$

Indefinite Integrals Ex 19.16 Q2

Let $I = \int \frac{e^x}{1 + e^{2x}} dx$

Let $\tan e^x = t$

$\Rightarrow \quad e^x \; dx = dt$

so, $\quad I = \int \frac{dt}{1 + t^2}$

$= \tan^{-1}(t) + c \quad \left[\text{Since, } \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c\right]$

$I = \tan^{-1}\left(e^x\right) + c$

Indefinite Integrals Ex 19.16 Q3

Let $I = \int \frac{\cos x}{\sin^2 x + 4\sin x + 5} dx$

Let $\sin x = t$

$\Rightarrow \quad \cos x\, dx = dt$

so, $\quad I = \int \frac{dx}{t^2 + 4t + 5}$

$= \int \frac{dt}{t^2 + 2t(2) + (2)^2 - (2)^2 + 5}$

$= \int \frac{dt}{(t+2)^2 + 1}$

Again, Let $(t+2) = u$

$dt = du$

$I = \int \frac{dt}{u^2 + 1}$

$= \tan^{-1}(u) + c \qquad \left[\text{Since, } \int \frac{1}{x^2+1} dx = \tan^{-1} x + c\right]$

$I = \tan^{-1}(t+2) + c$

$I = \tan^{-1}(\sin x + 2) + c$

Indefinite Integrals Ex 19.16 Q4

Let $I = \int \frac{e^x}{e^{2x} + 5e^x + 6} dx$

Let $e^x = t$

$\Rightarrow \quad e^x\, dx = dt$

so, $\quad I = \int \frac{dt}{t^2 + 5t + 6}$

$$= \int \frac{dt}{t^2 + 2t\left(\frac{5}{2}\right) + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 + 6}$$

$$= \int \frac{dt}{\left(t + \frac{5}{2}\right)^2 - \frac{1}{4}}$$

Put $\left(t + \frac{5}{2}\right) = u$

$\Rightarrow \quad dt = du$

$$I = \int \frac{dt}{u^2 - \left(\frac{1}{2}\right)^2}$$

$$= \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{u - \frac{1}{2}}{u + \frac{1}{2}}\right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\left|\frac{x - a}{x + a}\right| + c\right]$$

$$I = \log\left|\frac{2u - 1}{2u + 1}\right| + c$$

$$I = \log\left|\frac{2\left(t + \frac{5}{2}\right) - 1}{2\left(t + \frac{5}{2}\right) + 1}\right| + c$$

$$I = \log\left|\frac{e^x + 2}{e^x + 3}\right| + c$$

Indefinite Integrals Ex 19.16 Q5

$$\text{Let } I = \int \frac{e^{3x}}{4e^{6x} - 9} dx$$

$$\text{Let } e^{3x} = t$$

$$\Rightarrow \quad 3e^{3x}\, dx = dt$$

$$\Rightarrow \quad e^{3x} dx = \frac{dt}{3}$$

$$I = \frac{1}{3} \int \frac{dt}{4t^2 - 9}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \frac{9}{4}}$$

$$= \frac{1}{12} \int \frac{dt}{t^2 - \left(\frac{3}{2}\right)^2}$$

$$= \frac{1}{12} \times \frac{1}{2\left(\frac{3}{2}\right)} \log \left| \frac{t - \frac{3}{2}}{t + \frac{3}{2}} \right| + c \qquad \left[\text{Since, } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \left| \frac{x - a}{x + a} \right| + c \right]$$

$$I = \frac{1}{36} \log \left| \frac{2t - 3}{2t + 3} \right| + c$$

$$I = \frac{1}{36} \log \left| \frac{2e^{3x} - 3}{2e^{3x} + 3} \right| + c$$

Indefinite Integrals Ex 19.16 Q6

$$\text{Let } I = \int \frac{dx}{e^x + e^{-x}}$$

$$= \int \frac{dx}{e^x + \frac{1}{e^x}}$$

$$= \int \frac{e^x dx}{\left(e^x\right)^2 + 1}$$

$$\text{Let } e^x = t$$

$$\Rightarrow \quad e^x dx = dt$$

$$I = \int \frac{dt}{t^2 + 1}$$

$$I = \tan^{-1} t + c \qquad \left[\text{Since } \int \frac{1}{1 + x^2} dx = \tan^{-1} x + c\right]$$

$$I = \tan^{-1}\left(e^x\right) + c$$

Indefinite Integrals Ex 19.16 Q7

Let $I = \int \frac{x}{x^4 + 2x^2 + 3} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$$I = \frac{1}{2} \int \frac{dt}{t^2 + 2t + 3}$$

$$= \frac{1}{2} \int \frac{dt}{t^2 + 2t + 1 - 1 + 3}$$

$$= \frac{1}{2} \int \frac{dt}{(t+1)^2 + 2}$$

put $t + 1 = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{2} \int \frac{du}{u^2 + \left(\sqrt{2}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{t+1}{\sqrt{2}}\right) + c$$

$$I = \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right) + c$$

Indefinite Integrals Ex 19.16 Q8

Let $I = \int \frac{3x^5}{1 + x^{12}} dx$

$$= \int \frac{3x^5}{1 + \left(x^6\right)^2} dx$$

Let $x^6 = t$

$\Rightarrow \quad 6x^5\,dx = dt$

$\Rightarrow \quad x^5\,dx = \frac{dt}{6}$

$$I = \frac{3}{6} \int \frac{dt}{1 + t^2}$$

$$= \frac{1}{2} \tan^{-1}(t) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + 1} dx = \tan^{-1} x + c\right]$$

$$I = \frac{1}{2} \tan^{-1}\left(x^6\right) + c$$

Indefinite Integrals Ex 19.16 Q9

Let $I = \int \frac{x^2}{x^6 - a^6} dx$

$$= \int \frac{x^2}{\left(x^3\right)^2 - \left(a^3\right)^2} dx$$

Let $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\Rightarrow \quad x^2 dx = \frac{dt}{3}$

so, $\quad I = \frac{1}{3} \int \frac{dt}{t^2 - \left(a^3\right)^2}$

$$= \frac{1}{3} \times \frac{1}{2a^3} \log\left|\frac{t - a^3}{t + a^3}\right| \qquad \left[\text{Since} \quad \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log\left|\frac{x - a}{x + a}\right| + c\right]$$

$$I = \frac{1}{6a^3} \log\left|\frac{x^3 - a^3}{x^3 + a^3}\right| + c$$

Indefinite Integrals Ex 19.16 Q10

Let $I = \int \frac{x^2}{x^6 + \left(a^3\right)^2} dx$

$$= \int \frac{x^2}{\left(x^3\right)^2 + \left(a^3\right)^2} dx$$

Let $x^3 = t$

$\Rightarrow \quad 3x^2 dx = dt$

$\Rightarrow \quad x^2 dx = \frac{dt}{3}$

so, $\quad I = \frac{1}{3} \int \frac{dt}{t^2 + \left(a^3\right)^2}$

$$= \frac{1}{3} \times \frac{1}{\left(a^3\right)} \tan^{-1}\left(\frac{t}{a^3}\right) + c \qquad \left[\text{Since} \quad \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{3a^3} \tan^{-1}\left(\frac{x^3}{a^3}\right) + c$$

Indefinite Integrals Ex 19.16 Q11

Let $I = \int \frac{1}{x\left(x^6+1\right)} dx$

$= \int \frac{x^5}{x^6\left(x^6+1\right)} dx$

Let $x^6 = t$

$\Rightarrow \quad 6x^5 dx = dt$

$\Rightarrow \quad x^5 dx = \frac{dt}{6}$

$I = \frac{1}{6} \int \frac{dt}{t(t+1)}$

$= \frac{1}{6} \int \frac{dt}{t^2+t}$

$= \frac{1}{6} \int \frac{dt}{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$

$= \frac{1}{6} \int \frac{dt}{\left(t+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}$

Let $t+\frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$I = \frac{1}{6} \int \frac{du}{u^2 - \left(\frac{1}{2}\right)^2}$

$= \frac{1}{6} \times \frac{1}{2\left(\frac{1}{2}\right)} \log\left|\frac{u-\frac{1}{2}}{u+\frac{1}{2}}\right| + c \qquad \left[\text{Since } \int \frac{1}{x^2-a^2} dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$

$I = \frac{1}{6}\log\left|\frac{t+\frac{1}{2}-\frac{1}{2}}{t+\frac{1}{2}+\frac{1}{2}}\right| + c$

$I = \frac{1}{6}\log\left|\frac{x^6}{x^6+1}\right| + c$

Indefinite Integrals Ex 19.16 Q12

Let $I = \int \frac{x}{x^4 - x^2 + 1} dx$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

so, $\quad I = \frac{1}{2} \int \frac{dt}{t^2 - t + 1}$

$$= \frac{1}{2} \int \frac{dt}{t^2 - 2t \times \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$= \frac{1}{2} \int \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)}$$

Let $\quad t - \frac{1}{2} = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{2} \int \frac{du}{u^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{1}{2} \times \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} \tan^{-1}\left(\frac{u}{\frac{\sqrt{3}}{2}}\right) + c \quad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t - \frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

$$I = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x^2 - 1}{\sqrt{3}}\right) + c$$

Indefinite Integrals Ex 19.16 Q13

Let $I = \int \frac{x}{3x^4 - 18x^2 + 11} dx$

$$= \frac{1}{3} \int \frac{x}{x^4 - 6x^2 + \frac{11}{3}} dx$$

Let $x^2 = t$

$\Rightarrow \quad 2x\,dx = dt$

$\Rightarrow \quad x\,dx = \frac{dt}{2}$

$$I = \frac{1}{3} \times \frac{1}{2} \int \frac{dt}{t^2 - 6t + \frac{11}{3}}$$

$$= \frac{1}{6} \int \frac{dt}{t^2 - 2t(3) + (3)^2 - (3)^2 + \frac{11}{3}}$$

$$= \frac{1}{6} \int \frac{dt}{(t-3)^2 - \left(\frac{16}{3}\right)}$$

Let $t-3 = u$

$\Rightarrow \quad dt = du$

$$I = \frac{1}{6} \int \frac{du}{u^2 - \left(\frac{4}{\sqrt{3}}\right)^2}$$

$$= \frac{1}{6} \times \frac{1}{2\left(\frac{4}{\sqrt{3}}\right)} \log \left| \frac{u - \frac{4}{\sqrt{3}}}{u + \frac{4}{\sqrt{3}}} \right| + c \quad \left[\text{Since} \quad \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c\right]$$

$$I = \frac{\sqrt{3}}{48} \log \left| \frac{t - 3 - \frac{4}{\sqrt{3}}}{t - 3 + \frac{4}{\sqrt{3}}} \right| + c$$

$$I = \frac{\sqrt{3}}{48} \log \left| \frac{x^2 - 3 - \frac{4}{\sqrt{3}}}{x^2 - 3 + \frac{4}{\sqrt{3}}} \right| + c$$

Indefinite Integrals Ex 19.16 Q14

To evaluate the following integral follow the steps:

Let $e^x = t$ therefore $e^x dx = dt$

Now

$$\int \frac{e^x}{(1+e^x)(2+e^x)}dx = \int \frac{dt}{(1+t)(2+t)}$$

$$= \int \frac{dt}{(1+t)} - \int \frac{dt}{(2+t)}$$

$$= \ln|1+t| - \ln|2+t| + c$$

$$= \ln\left|\frac{1+t}{2+t}\right| + c$$

$$= \ln\left|\frac{1+e^x}{2+e^x}\right| + c$$