RD Sharma Solutions Class 12 Maths Chapter 19 Ex 19.14

Indefinite Integrals Ex 19.14 Q1

Let $I = \int \frac{1}{a^2 - b^2x^2}dx$

$$= \frac{1}{b^2}\int \frac{1}{\frac{a^2}{b^2} - x^2}dx$$

$$= \frac{1}{b^2}\int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2}dx$$

$$I = \frac{1}{b^2} \times \frac{1}{2 \times \left(\frac{a}{b}\right)} \log\left|\frac{\frac{a}{b} + x}{\frac{a}{b} - x}\right| + c \qquad \left[\text{Since } \int \frac{1}{a^2 - x^2}dx = \frac{1}{2a}\log\left|\frac{x+a}{x-a}\right| + c\right]$$

$$I = \frac{1}{2ab}\log\left|\frac{a+bx}{a-bx}\right| + c$$

Indefinite Integrals Ex 19.14 Q2

Let $I = \int \frac{1}{a^2x^2 - b^2}dx$

$$= \frac{1}{a^2}\int \frac{1}{x^2 - \frac{b^2}{a^2}}dx$$

$$= \frac{1}{a^2}\int \frac{1}{x^2 - \left(\frac{b}{a}\right)^2}dx$$

$$I = \frac{1}{a^2} \times \frac{1}{2 \times \left(\frac{b}{a}\right)} \times \log\left|\frac{x - \frac{b}{a}}{x + \frac{b}{a}}\right| + c \qquad \left[\text{Since } \int \frac{1}{x^2 - a^2}dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c\right]$$

$$I = \frac{1}{2ab}\log\left|\frac{ax-b}{ax+b}\right| + c$$

Indefinite Integrals Ex 19.14 Q3

Let $I = \int \frac{1}{a^2x^2 + b^2} dx$

$$= \frac{1}{a^2} \int \frac{1}{x^2 + \frac{b^2}{a^2}} dx$$

$$= \frac{1}{a^2} \int \frac{1}{x^2 + \left(\frac{b}{a}\right)^2} dx$$

$$I = \frac{1}{a^2} \times \frac{1}{\left(\frac{b}{a}\right)} \tan^{-1}\left(\frac{x}{\frac{b}{a}}\right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c\right]$$

$$I = \frac{1}{ab} \tan^{-1}\left(\frac{ax}{b}\right) + c$$

Indefinite Integrals Ex 19.14 Q4

Let $I = \int \frac{x^2 - 1}{x^2 - 4} dx$

Let $I = \int \frac{x^2 - 1}{x^2 + 4} dx$

$$= \int \frac{\left(x^2 + 4\right) - 4 - 1}{x^2 + 4} dx$$

$$= \int \frac{x^2 + 4}{x^2 + 4} dx - \int \frac{5}{x^2 + 4} dx$$

$$= \int dx - 5 \int \frac{1}{x^2 + (2)^2} dx$$

$$I = x - 5 \times \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + c \qquad \left[\text{Since } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$I = x - \frac{5}{2} \tan^{-1}\left(\frac{x}{2}\right) + c$$

Indefinite Integrals Ex 19.14 Q5

Let $2x = t$

$\Rightarrow 2dx = dt$

$$\Rightarrow \int \frac{1}{\sqrt{1 + 4x^2}} dx = \frac{1}{2} \int \frac{dt}{\sqrt{1 + t^2}}$$

$$= \frac{1}{2}\left[\log\left|t + \sqrt{t^2 + 1}\right|\right] + C \qquad \left[\int \frac{1}{\sqrt{x^2 + a^2}} dt = \log\left|x + \sqrt{x^2 + a^2}\right|\right]$$

$$= \frac{1}{2} \log\left|2x + \sqrt{4x^2 + 1}\right| + C$$

Indefinite Integrals Ex 19.14 Q6

Let $I = \int \frac{1}{\sqrt{a^2 + b^2x^2}} dx$

Let $bx = t$

$\Rightarrow \quad bdx = dt$

$dx = \frac{dt}{b}$

$I = \frac{1}{b} \int \frac{1}{\sqrt{a^2 + t^2}} dt$

$I = \frac{1}{b} \log\left|t + \sqrt{a^2 + t^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log\left|x + \sqrt{a^2 + x^2}\right| + c\right]$

$I = \frac{1}{b} \log\left|bx + \sqrt{a^2 + b^2x^2}\right| + c \qquad [\text{since } t = bx]$

Indefinite Integrals Ex 19.14 Q7

Let $I = \int \frac{1}{\sqrt{a^2 - b^2x^2}} dx$

Let $bx = t$

$\Rightarrow \quad bdx = dt$

$dx = \frac{dt}{b}$

so, $\quad I = \frac{1}{b} \int \frac{1}{\sqrt{a^2 - t^2}} dt$

$I = \frac{1}{b} \sin^{-1}\left(\frac{t}{a}\right) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c\right]$

$I = \frac{1}{b} \sin^{-1}\left(\frac{bx}{a}\right) + c \qquad [\text{since } bx = t]$

Indefinite Integrals Ex 19.14 Q8

Let $I = \int \frac{1}{\sqrt{(2 - x)^2 + 1}} dx$

Let $\quad 2 - x = t$

$\Rightarrow \quad -dx = dt$

$dx = -dt$

so, $\quad I = -\int \frac{1}{\sqrt{t^2 + (1)^2}} dt$

$I = -\log\left|t + \sqrt{t^2 + 1}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c\right]$

$I = -\log\left|(2 - x) + \sqrt{(2 - x)^2 + 1}\right| + c \qquad [\text{since } t = (2 - x)]$

Indefinite Integrals Ex 19.14 Q9

Let $I = \int \frac{1}{\sqrt{(2-x)^2 - 1}} dx$

Let $2 - x = t$

$\Rightarrow \quad -dx = dt$

$dx = -dt$

so, $\quad I = -\int \frac{1}{\sqrt{t^2 - (1)^2}} dt$

$I = -\log\left|t + \sqrt{t^2 - (1)^2}\right| + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c\right]$

$I = -\log\left|(2-x) + \sqrt{(2-x)^2 - 1}\right| + c \qquad [\text{since } t = (2-x)]$

Indefinite Integrals Ex 19.14 Q10

Let $I = \int \frac{x^4 + 1}{x^2 + 1} dx$

$I = \int \frac{(x^2+1)^2 - 2x^2}{x^2+1} dx \qquad \left[a^2 + b^2 = (a+b)^2 - 2ab\right]$

$I = \int \frac{(x^2+1)^2}{x^2+1} dx - \int \frac{2x^2}{(x^2+1)} dx$

$I = \int (x^2+1) dx - \int \frac{2x^2 + 2 - 2}{(x^2+1)} dx$

$I = \int (x^2+1) dx - \int \frac{2(x^2+1)}{(x^2+1)} dx + 2\int \frac{1}{x^2+1} dx$

$I = \int (x^2+1) dx - \int 2dx + 2\int \frac{1}{x^2+1} dx$

$I = \frac{x^3}{3} + x - 2x + 2 \times \tan^{-1}(x) + c \qquad \left[\text{Since } \int \frac{1}{\sqrt{x^2+1}} dx = \tan^{-1}(x) + c\right]$

$I = \frac{x^3}{3} - x + 2\tan^{-1}(x) + c$