

RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.13

Indefinite Integrals Ex 19.13 Q1

$$\begin{aligned}\text{Let } I &= \int \frac{x}{\sqrt{x^4 + a^4}} dx \\ &= \int \frac{x}{\sqrt{(x^2)^2 + (a^2)^2}} dx\end{aligned}$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (a^2)^2}}$$

$$= \frac{1}{2} \log \left| t + \sqrt{t^2 + (a^2)^2} \right| + c$$

$$I = \frac{1}{2} \log \left| x^2 + \sqrt{(x^2)^2 + (a^2)^2} \right| + c$$

$$\left[\text{Since } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c \right]$$

$$I = \frac{1}{2} \log \left| x^2 + \sqrt{x^4 + a^4} \right| + c$$

Indefinite Integrals Ex 19.13 Q2

$$\text{Let } \tan x = t$$

$$\Rightarrow \sec^2 x dx = dt$$

$$\Rightarrow \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

$$= \log \left| t + \sqrt{t^2 + 4} \right| + C$$

$$= \log \left| \tan x + \sqrt{\tan^2 x + 4} \right| + C$$

Indefinite Integrals Ex 19.13 Q3

$$\text{Let } I = \int \frac{e^x}{\sqrt{16 - e^{2x}}} dx$$

$$\text{Let } e^x = t$$

$$\Rightarrow e^x dx = dt$$

$$I = \int \frac{dt}{\sqrt{(4)^2 - t^2}}$$

$$= \sin^{-1}\left(\frac{t}{4}\right) + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c \right]$$

$$I = \sin^{-1}\left(\frac{e^x}{4}\right) + c$$

Indefinite Integrals Ex 19.13 Q4

$$\text{Let } I = \int \frac{\cos x}{\sqrt{4 + \sin^2 x}} dx$$

$$\text{Let } \sin x = t$$

$$\Rightarrow \cos x dx = dt$$

$$I = \int \frac{dt}{\sqrt{(2)^2 + t^2}}$$

$$= \log \left| t + \sqrt{(2)^2 + t^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{a^2 + x^2} \right| + c \right]$$

$$I = \log \left| \sin x + \sqrt{4 + \sin^2 x} \right| + c$$

Indefinite Integrals Ex 19.13 Q5

$$\text{Let } I = \int \frac{\sin x}{\sqrt{4 \cos^2 x - 1}} dx$$

$$\text{Let } 2 \cos x = t$$

$$\Rightarrow -2 \sin x dx = dt$$

$$\Rightarrow \sin x dx = -\frac{dt}{2}$$

$$I = -\frac{1}{2} \int \frac{dt}{\sqrt{t^2 - 1}}$$

$$= -\frac{1}{2} \log \left| t + \sqrt{t^2 - 1} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c \right]$$

$$I = -\frac{1}{2} \log \left| 2 \cos x + \sqrt{4 \cos^2 x - 1} \right| + c$$

Indefinite Integrals Ex 19.13 Q6

$$\text{Let } I = \int \frac{x}{\sqrt{4-x^4}} dx$$

$$\text{Let } x^2 = t$$

$$\Rightarrow 2x dx = dt$$

$$\Rightarrow x dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \frac{1}{2} \sin^{-1} \left(\frac{t}{2} \right) + c$$

$$\left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = \frac{1}{2} \sin^{-1} \left(\frac{x^2}{2} \right) + c$$

Indefinite Integrals Ex 19.13 Q7

$$\text{Let } I = \int \frac{1}{x \sqrt{4-9(\log x)^2}} dx$$

$$\text{Let } 3 \log x = t$$

$$\Rightarrow \frac{3}{x} dx = dt$$

$$\Rightarrow \frac{1}{x} dx = \frac{dt}{3}$$

$$I = \frac{1}{3} \int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \frac{1}{3} \sin^{-1} \left(\frac{t}{2} \right) + c$$

$$\left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \left(\frac{x}{a} \right) + c \right]$$

$$I = \frac{1}{3} \sin^{-1} \left(\frac{3 \log x}{2} \right) + c$$

Indefinite Integrals Ex 19.13 Q8

$$\text{Let } I = \int \frac{\sin 8x}{\sqrt{9 + (\sin 4x)^4}} dx$$

$$\text{Let } \sin^2 4x = t$$

$$\Rightarrow 2 \sin 4x \cdot \cos 4x (4) dx = dt$$

$$\Rightarrow 4 \sin 8x dx = dt$$

$$\Rightarrow \sin 8x dx = \frac{dt}{4}$$

$$I = \frac{1}{4} \int \frac{dt}{\sqrt{(3)^2 + t^2}}$$

$$= \frac{1}{4} \log \left| t + \sqrt{(3)^2 + t^2} \right| + c$$

$$\left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{a^2 + x^2} \right| + c \right]$$

$$I = \frac{1}{4} \log \left| \sin^2 4x + \sqrt{9 + \sin^4 4x} \right| + c$$

Indefinite Integrals Ex 19.13 Q9

$$\text{Let } I = \int \frac{\cos 2x}{\sqrt{\sin^2 2x + 8}} dx$$

$$\text{Let } \sin 2x = t$$

$$\Rightarrow 2 \cos 2x dx = dt$$

$$\Rightarrow \cos 2x dx = \frac{dt}{2}$$

$$I = \frac{1}{2} \int \frac{dt}{\sqrt{t^2 + (2\sqrt{2})^2}}$$

$$= \frac{1}{2} \log \left| t + \sqrt{t^2 + (2\sqrt{2})^2} \right| + c$$

$$\left[\text{Since } \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log \left| x + \sqrt{x^2 + a^2} \right| + c \right]$$

$$I = \frac{1}{2} \log \left| \sin 2x + \sqrt{\sin^2 2x + 8} \right| + c$$

Indefinite Integrals Ex 19.13 Q10

$$\text{Let } I = \int \frac{\sin 2x}{\sqrt{\sin^4 x + 4 \sin^2 x - 2}} dx$$

$$\text{Let } \sin^2 x = t$$

$$\Rightarrow 2 \sin x \cos x dx = dt$$

$$\Rightarrow \sin 2x dx = dt$$

$$\begin{aligned} I &= \int \frac{dt}{\sqrt{t^2 + 4t - 2}} \\ &= \int \frac{dt}{\sqrt{t^2 + 2t(2) + (2)^2 - (2)^2 - 2}} dx \\ &= \int \frac{dt}{\sqrt{(t+2)^2 - 6}} \end{aligned}$$

$$\text{Let } t+2 = u$$

$$dt = du$$

$$= \int \frac{du}{\sqrt{u^2 - (\sqrt{6})^2}}$$

$$= \log \left| u + \sqrt{u^2 - 6} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right]$$

$$= \log \left| t + 2 + \sqrt{(t+2)^2 - 6} \right| + c$$

$$I = \log \left| \sin^2 x + 2 + \sqrt{\sin^4 x + 4 \sin^2 x - 2} \right| + c$$

Indefinite Integrals Ex 19.13 Q11

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx =$$

$$\text{let } t = \cos^2 x \rightarrow -dt = 2 \cos x \sin x dx$$

$$\int \frac{\sin 2x}{\sqrt{\cos^4 x - \sin^2 x + 2}} dx = \int \frac{-1}{\sqrt{t^2 - (1-t) + 2}} dt$$

$$= \int \frac{-1}{\sqrt{t^2 + t + 1}} dt = \int \frac{-1}{\sqrt{t^2 + t + \frac{1}{4} + \frac{3}{4}}} dt$$

$$= \int \frac{-1}{\sqrt{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}} dt = -\log \left| \left(t + \frac{1}{2}\right) + \sqrt{t^2 + t + 1} \right|$$

$$= -\log \left| \left(\cos^2 x + \frac{1}{2}\right) + \sqrt{\cos^4 x + \cos^2 x + 1} \right| + C$$

Indefinite Integrals Ex 19.13 Q12

$$\text{Let } I = \int \frac{\cos x}{\sqrt{4 - \sin^2 x}} dx$$

$$\text{Let } \sin x = t$$

$$\Rightarrow \cos x dx = dt$$

$$= \int \frac{dt}{\sqrt{(2)^2 - t^2}}$$

$$= \sin^{-1}\left(\frac{t}{2}\right) + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c \right]$$

$$I = \sin^{-1}\left(\frac{\sin x}{2}\right) + c$$

Indefinite Integrals Ex 19.13 Q13

$$\text{Let } I = \int \frac{1}{x^{\frac{2}{3}} \sqrt{x^{\frac{2}{3}} - 4}} dx$$

$$\text{Let } x^{\frac{1}{3}} = t$$

$$\Rightarrow \frac{1}{3} x^{\frac{1}{3}-1} dx = dt$$

$$\Rightarrow \frac{1}{3} x^{-\frac{2}{3}} dx = dt$$

$$\Rightarrow \frac{dx}{x^{\frac{2}{3}}} = 3dt$$

$$I = 3 \int \frac{dt}{\sqrt{t^2 - (2)^2}}$$

$$= 3 \log \left| t + \sqrt{t^2 - 4} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right]$$

$$I = 3 \log \left| x^{\frac{1}{3}} + \sqrt{x^{\frac{2}{3}} - 4} \right| + c$$

Indefinite Integrals Ex 19.13 Q14

$$\text{Let } I = \int \frac{1}{\sqrt{(1-x^2)[9+(\sin^{-1}x)^2]}} dx$$

$$\text{Let } \sin^{-1}x = t$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} dx = dt$$

$$I = \int \frac{dt}{\sqrt{(3)^2 + t^2}}$$

$$= \log \left| t + \sqrt{9+t^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{a^2+x^2}} dx = \log \left| x + \sqrt{a^2+x^2} \right| + c \right]$$

$$I = \log \left| \sin^{-1}x + \sqrt{9+(\sin^{-1}x)^2} \right| + c$$

Indefinite Integrals Ex 19.13 Q15

$$\text{Let } I = \int \frac{\cos x}{\sqrt{\sin^2 x - 2 \sin x - 3}} dx$$

$$\text{Let } \sin x = t$$

$$\Rightarrow \cos x dx = dt$$

$$= \int \frac{dt}{\sqrt{t^2 - 2t - 3}}$$

$$= \int \frac{dt}{\sqrt{t^2 - 2t + (1)^2 - (1)^2 - 3}}$$

$$= \int \frac{dt}{\sqrt{(t-1)^2 - (2)^2}}$$

$$\text{Let } t-1 = u$$

$$\Rightarrow dt = du$$

$$I = \int \frac{du}{\sqrt{u^2 - (2)^2}}$$

$$= \log \left| u + \sqrt{u^2 - 4} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right]$$

$$= \log \left| t - 1 + \sqrt{(t-1)^2 - 4} \right| + c$$

$$I = \log \left| \sin x - 1 + \sqrt{\sin^2 x - 2 \sin x - 3} \right| + c$$

Indefinite Integrals Ex 19.13 Q16

$$\begin{aligned}
 \text{Let } I &= \int \sqrt{\operatorname{cosec} x - 1} dx \\
 &= \int \sqrt{\frac{1 - \sin x}{\sin x}} dx \\
 &= \int \sqrt{\frac{(1 - \sin x) + (1 + \sin x)}{\sin x (1 + \sin x)}} dx \\
 &= \int \sqrt{\frac{\cos^2 x}{\sin^2 x + \sin x}} dx \\
 &= \int \frac{\cos x}{\sqrt{\sin^2 x + \sin x}} dx
 \end{aligned}$$

$$\begin{aligned}
 \text{Let } \sin x &= t \\
 \Rightarrow \cos x dx &= dt \\
 &= \int \frac{dt}{\sqrt{t^2 + t}} \\
 &= \int \frac{dt}{\sqrt{t^2 + 2t\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} \\
 &= \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Let, } t + \frac{1}{2} &= u \\
 \Rightarrow dt &= du \\
 &= \int \frac{du}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}} \\
 &= \log \left| u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2} \right| + c \quad \left[\text{Since } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log \left| x + \sqrt{x^2 - a^2} \right| + c \right] \\
 &= \log \left| \left(t + \frac{1}{2}\right) + \sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} \right| + c
 \end{aligned}$$

$$I = \log \left| \sin x + \frac{1}{2} + \sqrt{\sin^2 x + \sin x} \right| + c$$

Indefinite Integrals Ex 19.13 Q17

To evaluate the following integral follow the steps:

$$\int \frac{\sin x - \cos x}{\sqrt{\sin 2x}} dx = \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx$$

Let $\sin x + \cos x = t$ therefore $(\cos x - \sin x) dx = dt$

Now

$$\begin{aligned}
 \int \frac{\sin x - \cos x}{\sqrt{(\sin x + \cos x)^2 - 1}} dx &= -\int \frac{dt}{\sqrt{t^2 - 1}} \\
 &= -\ln \left| t + \sqrt{t^2 - 1} \right| + c \\
 &= -\ln \left| \sin x + \cos x + \sqrt{\sin 2x} \right| + c
 \end{aligned}$$