

RD Sharma
Solutions
Class 12 Maths
Chapter 19
Ex 19.1

Indefinite Integrals Ex 19.1 Q1

(i)

$$\begin{aligned}\int x^4 dx &= \frac{x^{4+1}}{4+1} + c \\ &= \frac{x^5}{5} + c\end{aligned}$$

(ii)

$$\begin{aligned}\int x^{\frac{5}{4}} dx &= \frac{x^{\frac{5}{4}+1}}{\frac{5}{4}+1} + c \\ &= \frac{x^{\frac{5+4}{4}}}{\frac{5+4}{4}} + c \\ &= \frac{4x^{\frac{9}{4}}}{9} + c\end{aligned}$$

(iii)

$$\begin{aligned}\int \frac{1}{x^5} dx &= \int x^{-5} dx \\ &= \frac{x^{-5+1}}{-5+1} + c \\ &= \frac{x^{-4}}{-4} + c \\ &= \frac{-1}{4x^4} + c\end{aligned}$$

(iv)

$$\begin{aligned}\int \frac{1}{x^{\frac{3}{2}}} dx &= \int x^{-\frac{3}{2}} dx \\ &= \int x^{\frac{-3}{2}} dx \\ &= \frac{x^{\frac{-3}{2}+1}}{\frac{-3}{2}+1} + c \\ &= \frac{x^{-\frac{1}{2}}}{\frac{-1}{2}} + c \\ &= -2 \times \frac{1}{\sqrt{x}} + c \\ &= \frac{-2}{\sqrt{x}} + c\end{aligned}$$

(v)

$$\int 3^x dx = \frac{3^x}{\log 3} + c$$

$$\left[\because \int a^x dx = \frac{a^x}{\log a} + c \right]$$

(vi)

$$\begin{aligned}\int \frac{1}{\sqrt[3]{x^2}} dx &= \int \frac{1}{x^{\frac{2}{3}}} dx \\ &= \int x^{-\frac{2}{3}} dx \\ &= \frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C \\ &= \frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C \\ &= 3\sqrt[3]{x} + C\end{aligned}$$

(vii)

$$\begin{aligned}\int 3^{2\log_3 x} dx &= \int 3^{\log_3 x^2} dx \\ &= \int x^2 dx && [\because a^{\log_a x} = x] \\ &= \frac{x^3}{3} + C\end{aligned}$$

(viii)

$$\begin{aligned}\int \log_x x dx &= \int 1 dx \\ &= x + C.\end{aligned}$$

Indefinite Integrals Ex 19.1 Q2

(i)

$$\begin{aligned}\int \sqrt{\frac{1 + \cos 2x}{2}} dx &= \int \sqrt{\frac{2 \cos^2 x}{2}} dx && [\because \cos 2x = 2 \cos^2 x - 1] \\ &= \int \cos x dx \\ &= \sin x + C\end{aligned}$$

(ii)

$$\begin{aligned}\int \sqrt{\frac{1 - \cos 2x}{2}} dx &= \int \sqrt{\frac{2 \sin^2 x}{2}} dx \\ &= \int \sin x dx \\ &= -\cos x + C\end{aligned}$$

Indefinite Integrals Ex 19.1 Q3

Evaluate the integral as follows

$$\begin{aligned}\int \frac{e^{6 \log_e x} - e^{5 \log_e x}}{e^{4 \log_e x} - e^{3 \log_e x}} dx &= \int \frac{x^6 - x^5}{x^4 - x^3} dx \\ &= \int \frac{x^5(x-1)}{x^3(x-1)} dx \\ &= \int x^2 dx \\ &= \frac{x^3}{3} + C\end{aligned}$$

Indefinite Integrals Ex 19.1 Q4

$$\begin{aligned}
\int \frac{1}{a^x b^x} dx &= \int a^{-x} b^{-x} dx \\
&= \int (ab)^{-x} dx \\
&= \frac{(ab)^{-x}}{\log_e (ab)^{-1}} + c \\
&= \frac{(ab)^{-x}}{-\log_e (ab)} + c \\
&= \frac{a^{-x} b^{-x}}{-\log_e (ab)} + c
\end{aligned}$$

Indefinite Integrals Ex 19.1 Q5

$$\begin{aligned}
\text{(i)} \quad \int \frac{\cos 2x + 2 \sin^2 x}{\sin^2 x} dx &= \int \frac{\cos 2x + 2 \sin^2 x}{\sin^2 x} dx \\
&= \int \frac{1 - 2 \sin^2 x + 2 \sin^2 x}{\sin^2 x} dx \\
&= \int \frac{1}{\sin^2 x} dx \\
&= \int \operatorname{cosec}^2 x dx \\
&= -\cot x + c
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad \int \frac{2 \cos^2 x - \cos 2x}{\cos^2 x} dx &= \int \frac{2 \cos^2 x - (2 \cos^2 x - 1)}{\cos^2 x} dx \\
&= \int \frac{2 \cos^2 x - 2 \cos^2 x + 1}{\cos^2 x} dx \\
&= \int \frac{1}{\cos^2 x} dx \\
&= \int \sec^2 x dx \\
&= \tan x + c
\end{aligned}$$

Indefinite Integrals Ex 19.1 Q6

$$\begin{aligned}
\int \frac{e^{\log \sqrt{x}}}{x} dx &= \int \frac{\sqrt{x}}{x} dx \\
&= \int x^{\frac{1}{2}} \times x^{-1} dx \\
&= \int x^{\frac{1}{2}-1} dx \\
&= \int x^{-\frac{1}{2}} dx \\
&= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c \\
&= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \\
&= 2\sqrt{x} + c
\end{aligned}$$