RD Sharma Solutions Class 12 Maths Chapter 18 Ex 18.4

Maxima and Minima 18.4 Q1(i)

The given function is $f(x) = 4x - \frac{1}{2}x^2$.

$\therefore f'(x) = 4 - \frac{1}{2}(2x) = 4 - x$

Now,

$f'(x) = 0 \Rightarrow x = 4$

Then, we evaluate the value of f at critical point $x = 4$ and at the end points of the interval $\left[-2, \frac{9}{2}\right]$.

$f(4) = 16 - \frac{1}{2}(16) = 16 - 8 = 8$

$f(-2) = -8 - \frac{1}{2}(4) = -8 - 2 = -10$

$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{1}{2}\left(\frac{9}{2}\right)^2 = 18 - \frac{81}{8} = 18 - 10.125 = 7.875$

Hence, we can conclude that the absolute maximum value of f on $\left[-2, \frac{9}{2}\right]$ is 8 occurring at $x = 4$ and the absolute minimum value of f on $\left[-2, \frac{9}{2}\right]$ is -10 occurring at $x = -2$.

Maxima and Minima 18.4 Q1(ii)

The given function is $f(x)=(x-1)^2+3$.

$\therefore f'(x)=2(x-1)$

Now,

$f'(x)=0 \Rightarrow 2(x-1)=0 \Rightarrow x=1$

Then, we evaluate the value of f at critical point $x = 1$ and at the end points of the interval [−3, 1].

$$f(1)=(1-1)^2+3=0+3=3$$
$$f(-3)=(-3-1)^2+3=16+3=19$$

Hence, we can conclude that the absolute maximum value of f on [−3, 1] is 19 occurring at $x = -3$ and the minimum value of f on [−3, 1] is 3 occurring at $x = 1$.

Maxima and Minima 18.4 Q1(iii)

Let $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$.

$$\therefore f'(x)=12x^3-24x^2+24x-48$$
$$=12(x^3-2x^2+2x-4)$$
$$=12[x^2(x-2)+2(x-2)]$$
$$=12(x-2)(x^2+2)$$

Now, $f'(x)=0$ gives $x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Now, we evaluate the value of f at critical point $x = 2$ and at the end points of the interval [0, 3].

$$f(2)=3(16)-8(8)+12(4)-48(2)+25$$
$$=48-64+48-96+25$$
$$=-39$$
$$f(0)=3(0)-8(0)+12(0)-48(0)+25$$
$$=25$$
$$f(3)=3(81)-8(27)+12(9)-48(3)+25$$
$$=243-216+108-144+25=16$$

Hence, we can conclude that the absolute maximum value of f on [0, 3] is 25 occurring at $x = 0$ and the absolute minimum value of f at [0, 3] is − 39 occurring at $x = 2$.

Maxima and Minima 18.4 Q1(iv)

$$f(x) = (x-2)\sqrt{x-1}$$

$$\Rightarrow \quad f'(x) = \sqrt{x-1} + (x-2)\frac{1}{2\sqrt{x-1}}$$

Put $f'(x) = 0$

$$\Rightarrow \quad \sqrt{x-1} + \frac{x-2}{2\sqrt{x-1}} = 0$$

$$\Rightarrow \quad \frac{2(x-1)+(x-2)}{2\sqrt{x-1}} = 0$$

$$\Rightarrow \quad \frac{3x-4}{2\sqrt{x-1}} = 0$$

$$\Rightarrow \quad x = \frac{4}{3}$$

Now,

$$f(1) = 0$$

$$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}-2\right)\sqrt{\frac{4}{3}-1} = \frac{4-6}{3\sqrt{3}} = \frac{-2}{3\sqrt{3}} = \frac{-2\sqrt{3}}{9}$$

$$f(9) = (9-2)\sqrt{9-1} = 7\sqrt{8} = 14\sqrt{2}$$

$\therefore$ The absolute maximum value of $f(x)$ is $14\sqrt{2}$ at $x = 9$ and the absolute minimum value is $\frac{-2\sqrt{3}}{9}$ at $x = \frac{4}{3}$.

Maxima and Minima 18.4 Q2

Let $f(x) = 2x^3 - 24x + 107$.

$$\therefore f'(x) = 6x^2 - 24 = 6(x^2 - 4)$$

Now,

$$f'(x) = 0 \Rightarrow 6(x^2 - 4) = 0 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

We first consider the interval [1, 3].

Then, we evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of the interval [1, 3].

$f(2) = 2(8) - 24(2) + 107 = 16 - 48 + 107 = 75$

$f(1) = 2(1) - 24(1) + 107 = 2 - 24 + 107 = 85$

$f(3) = 2(27) - 24(3) + 107 = 54 - 72 + 107 = 89$

Hence, the absolute maximum value of $f(x)$ in the interval [1, 3] is 89 occurring at $x = 3$.

Next, we consider the interval [−3, −1].

Evaluate the value of f at the critical point $x = -2 \in [-3, -1]$ and at the end points of the interval [1, 3].

$f(-3) = 2(-27) - 24(-3) + 107 = -54 + 72 + 107 = 125$

Maxima and Minima 18.4 Q3

$$f(x) = \cos^2 x + \sin x$$
$$f'(x) = 2\cos x(-\sin x) + \cos x$$
$$= -2\sin x \cos x + \cos x$$

Now, $f'(x) = 0$

$$\Rightarrow 2\sin x \cos x = \cos x \Rightarrow \cos x(2\sin x - 1) = 0$$
$$\Rightarrow \sin x = \frac{1}{2} \text{ or } \cos x = 0$$
$$\Rightarrow x = \frac{\pi}{6}, \text{ or } \frac{\pi}{2} \text{ as } x \in [0, \pi]$$

Now, evaluating the value of f at critical points $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$ and at the end points of the interval $[0,\pi]$ (i.e., at $x = 0$ and $x = \pi$), we have:

$$f\left(\frac{\pi}{6}\right) = \cos^2\frac{\pi}{6} + \sin\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 + \frac{1}{2} = \frac{5}{4}$$
$$f(0) = \cos^2 0 + \sin 0 = 1 + 0 = 1$$
$$f(\pi) = \cos^2 \pi + \sin \pi = (-1)^2 + 0 = 1$$
$$f\left(\frac{\pi}{2}\right) = \cos^2\frac{\pi}{2} + \sin\frac{\pi}{2} = 0 + 1 = 1$$

Hence, the absolute maximum value of f is $\frac{5}{4}$ occurring at $x = \frac{\pi}{6}$ and the absolute minimum value of f is 1 occurring at $x = 0, \frac{\pi}{2}$, and π.

Maxima and Minima 18.4 Q4

We have

$$f(x) = 12x^{\frac{4}{3}} - 6x^{\frac{1}{3}}$$

$$\therefore \quad f'(x) = 16x^{\frac{1}{3}} - \frac{2}{x^{\frac{2}{3}}} = \frac{2(8x-1)}{x^{\frac{2}{3}}}$$

Thus, $f'(x) = 0$

$$\Rightarrow \quad x = \frac{1}{8}$$

Further note that $f'(x)$ isnot defined at $x = 0$.

So, the critical points are $x = 0$ and $x = \frac{1}{8}$.

Evaluating the value of f at critical points $x = 0, \frac{1}{8}$ and at end points of the interval $x = -1$ and $x = 1$

$f(-1) = 12(-1)^{4/3} - 6(-1)^{1/3} = 18$

$f(0) = 12(0) - 6(0) = 0$

$f\left(\frac{1}{8}\right) = 12\left(\frac{1}{8}\right)^{4/3} - 6\left(\frac{1}{8}\right)^{1/3} = \frac{-9}{4}$

$f(1) = 12(1)^{4/3} - 6(1)^{1/3} = 6$

Hence we conclude that absolute maximum value of f is 18 at $x = -1$ and absolute minimum value of f is $\frac{-9}{4}$ at $x = \frac{1}{8}$.

Maxima and Minima 18.4 Q5

Given,

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$\therefore \quad f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6) = 6(x-2)(x-3)$$

Note that $f'(x) = 0$ gives $x = 2$ and $x = 3$

We shall now evaluate the value of f at these points
and at the end points of the interval [1,5],
i.e at x= 1, 2, 3 and 5

At $x = 1$, $f(1) = 2(1^3) - 15(1^2) + 36(1) + 1 = 24$

At $x = 2$, $f(2) = 2(2^3) - 15(2^2) + 36(2) + 1 = 29$

At $x = 3$, $f(3) = 2(3^3) - 15(3^2) + 36(3) + 1 = 28$

At $x = 5$, $f(5) = 2(5^3) - 15(5^2) + 36(5) + 1 = 56$

Thus we conclude that the absolute maximum value of f on [1,5] is 56, occurring at x=5, and absolute minimum value of f on [1,5] is 24 which occurs at x=1.