RD Sharma Solutions Class 12 Maths Chapter 16 Ex 16.3

Solution 1(i)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

The given equations are

$$y^2 = x \quad \text{---(i)}$$

$$x^2 = y \quad \text{---(ii)}$$

$$m_1 = \frac{dy}{dx} = \frac{1}{2y}$$

$$m_2 = \frac{dy}{dx} = 2x$$

Solving (i) and (ii)

$$x^4 - x = 0 \quad \Rightarrow \quad x\left(x^3 - 1\right) = 0$$

and $y = 0, 1$

$\therefore \quad m_1 = \frac{1}{2}, \infty \quad$ and $\quad m_2 = 0$ or 2

$$\therefore \quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \frac{3}{4}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

and $\quad \tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \infty$

$$\theta = \frac{\pi}{2}$$

Solution 1(ii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad ---(A)$$

Where m_1 and m_2 are slopes of curves.

$$y = x^2 \qquad ---(i)$$

$$x^2 + y^2 = 20 \qquad ---(ii)$$

Solving (i) and (ii)

$$y + y^2 = 20$$

$$\Rightarrow \quad y^2 + y - 20 = 0$$

$$\Rightarrow \quad (y + 5)(y - 4) = 0$$

$$\Rightarrow \quad y = -5, 4$$

$$\therefore \quad x = \sqrt{-5}, \pm 2$$

$\therefore$ Points are $P = (2, 4)$, $Q = (-2, 4)$

Now,

Slope m_1 for (i)

$$m_1 = 2x = 4$$

Slope m_2 for (ii)

$$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{-1}{2}$$

Now,

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{\frac{-1}{2} - 4}{1 - \frac{1}{2} \times 4}\right|$$

$$= \frac{9}{2}$$

$$\therefore \quad \theta = \tan^{-1}\frac{9}{2}$$

Solution 1(iii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \quad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$2y^2 = x^3 \quad \text{---(i)}$$

$$y^2 = 32x \quad \text{---(ii)}$$

Solving (i) and (ii)

$$x^3 = 64x$$

$$\Rightarrow \quad x\left(x^2 - 64\right) = 0$$

$$\Rightarrow \quad x(x+8)(x-8) = 0$$

$$\Rightarrow \quad x = 0, -8, 8$$

$$\therefore \quad y = 0, -, 16$$

$$\therefore \quad P = (0,0),\ Q = (8,16)$$

Now,

$$m_1 = \frac{dy}{dx} = \frac{3x^2}{4y} = 0 \text{ or } 3$$

$$m_2 = \frac{dy}{dx} = \frac{32}{2y} = \infty \text{ or } 1$$

From (A)

$$\tan\theta = \left|\frac{\infty - 0}{10}\right| = \infty \Rightarrow \theta = \frac{\pi}{2}$$

and $$\tan\theta = \left|\frac{3-1}{13} = \frac{2}{4} = \frac{1}{2}\right|$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$$

Thus,

$$\theta = \frac{\pi}{2} \text{ and } \tan^{-1}\left(\frac{1}{2}\right)$$

Solution 1(iv)

We have,

$$x^2 + y^2 - 4x - 1 = 0 \quad \text{---(i)}$$

and $$x^2 + y^2 - 2y - 9 = 0 \quad \text{---(ii)}$$

Equation (i) can be written as

$$(x-2)^2 + y^2 - 5 = 0 \quad \text{---(iii)}$$

Subtracting (ii) from (i), we get

$$-4x + 2y + 8 = 0$$

$$\Rightarrow \quad y = 2x - 4$$

Substituting in (iii), we get

$$(x-2)^2 + (2x-4)^2 - 5 = 0$$

$$\Rightarrow \quad (x-2)^2 + 4(x-2)^2 - 5 = 0$$

$$\Rightarrow \quad (x-2)^2 = 1$$

$$\Rightarrow \quad x - 2 = 1,\ x - 2 = -1$$

$$\Rightarrow \quad x = 3 \text{ or } x = 1$$

$$\therefore \quad y = 2(3) - 4 = 2 \text{ or } y = -2$$

$\therefore$ The points of intersection of the two curves are $(3,2)$ and $(-1,-2)$

Differentiation (i) and (ii), w.r.t x we get

$$2x + 2y\frac{dy}{dx} - 4 = 0 \quad \text{---(iv)}$$

and $$2x + 2y\frac{dy}{dx} - 2\frac{dy}{dx} = 0 \quad \text{---(v)}$$

$\therefore$ At $(3,2)$, from equation (iv) we have,

$$\left(\frac{dy}{dx}\right)_{C_1} = \frac{4 - 2(3)}{2(2)} = \frac{-1}{2}$$

$$\left(\frac{dy}{dx}\right)_{C_2} = \frac{-2(3)}{(2 \times 2 - 3)} = \frac{-6}{2} = -3$$

$\therefore$ If φ is the angle between the curves

Then,

$$\tan\varphi = \frac{\left(\frac{dy}{dx}\right)_{C_1} - \left(\frac{dy}{dx}\right)_{C_2}}{1 + \left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}}$$

$$= \frac{\left(\frac{-1}{2} - (-3)\right)}{1 + \left(\frac{-1}{2}\right)(-3)}$$

$$= \frac{\frac{-1}{2} + 3}{1 + \frac{3}{2}} = \frac{5}{2} \times \frac{2}{5} = 1$$

$$\therefore \quad \varphi = \frac{\pi}{4}$$

Solution 1(v)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad ---(i)$$

$$x^2+y^2=ab \qquad ---(ii)$$

From (ii), we get

$$y^2=ab-x^2$$

$\therefore$ From (i), we get

$$\frac{x^2}{a^2}+\frac{ab-x^2}{b^2}=1$$

$$\Rightarrow \quad b^2x^2+a^3b-a^2x^2=a^2b^2$$

$$\Rightarrow \quad \left(b^2-a^2\right)x^2=a^2b^2-a^3b$$

$$\Rightarrow \quad x^2=\frac{a^2b^2-a^3b}{b^2-a^2}$$

$$=\frac{a^2b\left(b-a\right)}{\left(b-a\right)\left(b+a\right)}$$

$$=\frac{a^2b}{b+a}$$

$$\therefore \quad x=\pm\sqrt{\frac{a^2b}{a+b}}$$

$$\therefore \quad y^2=ab-x^2=ab-\frac{a^2b}{a+b}$$

$$=\frac{a^2b+ab^2-a^2b}{a+b}=\frac{ab^2}{a+b}$$

Differentiating (i) and (ii) w.r.t x we get

$$\frac{2x}{a^2}+\frac{2y}{b^2}\times\left(\frac{dy}{dx}\right)_{C_1}=0$$

and $$2x+2y\left(\frac{dy}{dx}\right)_{C_2}=0$$

$$\therefore \quad \left(\frac{dy}{dx}\right)_{C_1}=\frac{-x}{a^2}\times\frac{b^2}{y}=\frac{-b^2x}{a^2y}$$

$$\left(\frac{dy}{dx}\right)_{C_2}=\frac{-x}{y}$$

At $\left(\pm\sqrt{\frac{a^2b}{a+b}},\pm\sqrt{\frac{ab^2}{a+b}}\right)$ we get

$$\left(\frac{dy}{dx}\right)_{C_1}=\frac{-b^2}{a^2}\sqrt{\frac{a}{b}}=\frac{-b^2\sqrt{a}}{a^2\sqrt{b}}$$

$$\left(\frac{dy}{dx}\right)_{C_2}=-\sqrt{\frac{a}{b}}$$

Let α be the angle between the two curves then,

$$\tan\alpha=\frac{\left(\frac{dy}{dx}\right)_{C_1}-\left(\frac{dy}{dx}\right)_{C_2}}{1+\left(\frac{dy}{dx}\right)_{C_1}\left(\frac{dy}{dx}\right)_{C_2}}$$

$$\frac{-b^2\sqrt{a}}{a^2\sqrt{b}}+\frac{\sqrt{a}}{\sqrt{b}}$$

$$= \frac{a \quad \sqrt{b} \qquad \sqrt{b}}{1 + \left(\frac{-b^2\sqrt{a}}{a^2\sqrt{b}}\right)\left(-\frac{\sqrt{a}}{\sqrt{b}}\right)}$$

$$= \frac{\dfrac{-b^2\sqrt{a} + a^2\sqrt{a}}{a^2\sqrt{b}}}{1 + \dfrac{b^2 a}{a^2 b}}$$

$$= \frac{\sqrt{a}\left(a^2 - b^2\right)}{a^2\sqrt{b}} \times \frac{a}{a+b}$$

$$= \frac{(a-b)}{\sqrt{ab}}$$

$$\therefore \qquad \alpha = \tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$$

Solution 1(vi)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + 4y^2 = 8 \qquad \text{---(i)}$$

$$x^2 - 2y^2 = 2 \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$6y^2 = 6 \Rightarrow \quad y = \pm 1$$

$\therefore \quad x^2 = 2 + 2 \Rightarrow \quad x = \pm 2$

$\therefore$ Point of intersection are

$$P = (2, 1) \text{ and } (-2, -1)$$

Now,

Slope m_1 for (i)

$$8y\frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$$

$\therefore \quad m_1 = \frac{1}{2}$

Slope m_2 for (ii)

$$4y\frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2y}$$

$\therefore \quad m_2 = 1$

From (A)

$$\tan\theta = \left|\frac{1 - \frac{1}{2}}{1 + 1 \times \frac{1}{2}}\right| = \frac{1}{3}$$

$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{3}\right)$

Solution 1(vii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 = 27y \qquad \text{---(i)}$$
$$y^2 = 8x \qquad \text{---(ii)}$$

Solving (i) and (ii) are

$$\frac{y^4}{64} = 27y$$

$$\Rightarrow \quad y\left(y^3 - 27 \times 64\right) = 0$$

$$\Rightarrow \quad y = 0 \text{ or } 12$$

$$\therefore \quad x = 0 \text{ or } 18$$

$\therefore$ Points or intersection is $(0,0)$ and $(18,12)$

Now,

Slope of (i)

$$m_1 = \frac{2x}{27} = \frac{12}{9} = \frac{4}{3}$$

Slope of (ii)

$$m_2 = \frac{8}{2y} = \frac{8}{24} = \frac{1}{3}$$

From (A)

$$\tan\theta = \left|\frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}}\right| = \frac{9}{13}$$

$$\therefore \quad \theta = \tan^{-1}\left(\frac{9}{13}\right)$$

Solution 1(viii)

We know that angle of intersection of two curves is given by

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad \text{---(A)}$$

Where m_1 and m_2 are slopes of curves.

$$x^2 + y^2 = 2x \qquad \text{---(i)}$$

$$y^2 = x \qquad \text{---(ii)}$$

Solving (i) and (ii)

$$x^2 + x = 2x$$

$\Rightarrow \quad x^2 - x = 0$

$\Rightarrow \quad x(x - 1) = 0$

$\Rightarrow \quad x = 0, 1$

$\therefore \quad y = 0 \text{ or } 1$

$\therefore$ The points of intersection is $P = (0,0)$, $Q = (1,1)$

$\therefore$ Slope of (i)

$$2y\frac{dy}{dx} = 2 - 2x$$

$\therefore \quad \frac{dy}{dx} = \frac{2 - 2x}{2y} = \frac{1 - x}{y}$

$\therefore \quad m_1 = 0$

Slope of (ii)

$$m_2 = \frac{1}{2y} = \frac{1}{2}$$

From (A)

$$\tan\theta = \left|\frac{\frac{1}{2} - 0}{1 + \frac{1}{2} \times 0}\right| = \frac{1}{2}$$

$\therefore \quad \theta = \tan^{-1}\left(\frac{1}{2}\right)$

Solution 1 (ix)

$y = 4 - x^2 ......(i)$

$y = x^2 ....(ii)$

Substituting eq (ii) in (i) we get,

$x^2 = 4 - x^2$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

From(i) when $x = \sqrt{2}$,we get $y = 2$ and when $x = -\sqrt{2}$,we get $y =$

Thus the two curves intersect at $\left(\sqrt{2}, 2\right)$ and $\left(-\sqrt{2}, 2\right)$.

Differnentiating (i) wrt x, we get

$$\frac{dy}{dx} = 0 - 2x = -2x$$

Differnentiating (ii) wrt x, we get

$$\frac{dy}{dx} = 2x$$

Angle of intersection at $\left(\sqrt{2}, 2\right)$

$$m_1 = \left(\frac{dy}{dx}\right)_{\left(\sqrt{2},\, 2\right)} = -2\sqrt{2}$$

Angle of intersection at $\left(-\sqrt{2}, 2\right)$

$$m_2 = \left(\frac{dy}{dx}\right)_{\left(-\sqrt{2},\, 2\right)} = 2\sqrt{2}$$

Let θ be the angle of intersection of the two curves.

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{2\sqrt{2} + 2\sqrt{2}}{1 + \left(2\sqrt{2}\right)\left(-2\sqrt{2}\right)}\right| = \left|\frac{4\sqrt{2}}{-7}\right| = \frac{4\sqrt{2}}{7}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$$

Solution 2(i)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where m_1 and m_2 are the slopes of two curves

$$y = x^3 \qquad \text{---(i)}$$

$$6y = 7 - x^2 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{dy}{dx} = 3x^2 = m_1$$

Slope of (ii)

$$\frac{dy}{dx} = -\frac{2}{6}x = m_2$$

Point of intersection of (i) and (ii) is

$$6x^3 = 7 - x^2$$

$$\Rightarrow \quad 6x^3 + x^2 - 7 = 0$$

$$\Rightarrow \quad x = 1$$

$$\therefore \quad y = 1$$

$$\therefore \quad P = (1,1)$$

$$\therefore \quad m_1 = 3 \text{ and } m_2 = -\frac{1}{3}$$

Now,

$$m_1 \times m_2 = 3 \times -\frac{1}{3} = -1$$

$\therefore$ (i) and (ii) cuts orthogonally.

Solution 2(ii)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where m_1 and m_2 are the slopes of two curves

$$x^3 - 3xy^2 = -2 \qquad \text{---(i)}$$

$$3x^2y - y^3 = 2 \qquad \text{---(ii)}$$

Point of intersection of (i) and (ii)

$$(i) + (ii)$$

$$\Rightarrow \quad x^3 - 3xy^2 + 3x^2y - y^3 = 0$$

$$\Rightarrow \quad (x - y)^3 = 0$$

$$\Rightarrow \quad x = y$$

$\therefore$ from (i)

$$x^3 - 3x^2 = -2$$

$$\Rightarrow \quad -2x^3 = -2$$

$$\Rightarrow \quad x = 1$$

$\therefore$ $P = (1,1)$ is the point of intersection

Now,

Slope of (i)

$$3x^2 - 3y^2 - 6xy\frac{dy}{dx} = 0$$

$$\therefore \quad m_1 = \frac{dy}{dx} = \frac{3\left(x^2 - y^2\right)}{6xy}$$

Slope of (ii)

$$6xy + 3x^2\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow \quad m_2 = \frac{dy}{dx} = \frac{-6xy}{3\left(x^2 - y^2\right)}$$

$$\therefore \quad m_1 \times m_2 = \frac{\left(x^2 - y^2\right)}{2xy} \times \frac{-2xy}{\left(x^2 - y^2\right)} = -1$$

Solution 2(iii)

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \qquad \text{---(A)}$$

Where m_1 and m_2 are the slopes of two curves

$$x^2 + 4y^2 = 8 \qquad \text{---(i)}$$
$$x^2 - 2y^2 = 4 \qquad \text{---(ii)}$$

Point of intersection of (i) and (ii) is (i) − (ii), we get

$$6y^2 = 4$$
$$\Rightarrow \quad y = \sqrt{\frac{2}{3}}$$

$$\therefore \quad x^2 = 4 + \frac{8}{6}$$
$$x^2 = \frac{32}{6}$$
$$\Rightarrow \quad x = \frac{4}{\sqrt{3}}$$

Now,

Slope of (i)

$$2x + 8y\frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{4y}$$
$$\Rightarrow \quad m_1 = -\frac{1}{4} \times \frac{4}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \qquad \left[\because \frac{x}{y} = \frac{4}{\sqrt{2}}\right]$$

Slope of (ii)

$$2x - 4y\frac{dy}{dx} = 0$$
$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{2y}$$
$$\Rightarrow \quad m_2 = \frac{1}{2} \times \frac{4}{\sqrt{2}} = \sqrt{2}$$
$$\therefore \quad m_1 \times m_2 = -\frac{1}{\sqrt{2}} \times \sqrt{2} = -1$$

$\therefore$ (i) and (ii) cuts orthogonally.

Solution 3(i)

We have,

$$x^2 = 4y \qquad ---(i)$$

$$4y + x^2 = 8 \qquad ---(ii) \qquad P = (2,1)$$

Slope of (i)

$$2x = 4\frac{dy}{dx}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \left(\frac{x}{2}\right)_P = 1$$

Slope of (ii)

$$4\frac{dy}{dx} + 2x = 0$$

$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{x}{2}\right)_P = -1$$

$$\therefore \quad m_1 \times m_2 = 1 \times -1 = -1$$

Hence the result.

Solution 3(ii)

We have,

$$x^2 = y \qquad ---(i)$$

$$x^3 + 6y = 7 \qquad ---(ii) \qquad P = (1,1)$$

Slope of (i)

$$2x = \frac{dy}{dx}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = 2$$

Slope of (ii)

$$3x^2 + 6\frac{dy}{dx} = 0$$

$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{x^2}{2}\right)_P = \frac{-1}{2}$$

$$\therefore \quad m_1 \times m_2 = 2 \times \frac{-1}{2} = -1$$

Solution 3(iii)

We have,

$$y^2 = 8x \qquad \text{---(i)}$$

$$2x^2 + y^2 = 10 \qquad \text{---(ii)} \qquad P\left(1, 2\sqrt{2}\right)$$

Slope of (i)

$$2y\frac{dy}{dx} = 8$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \left(\frac{4}{y}\right)_P = \sqrt{2}$$

Slope of (ii)

$$4x + 2y\frac{dy}{dx} = 0$$

$$\therefore \quad m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{2x}{y}\right)_P = \frac{-1}{\sqrt{2}}$$

$$\therefore \quad m_1 \times m_2 = \sqrt{2} \times \frac{-1}{\sqrt{2}} = -1$$

Solution 4

We have,

$$4x = y^2 \qquad \text{---(i)}$$

$$4xy = k \qquad \text{---(ii)}$$

Slope of (i)

$$4 = 2y\frac{dy}{dx}$$

$$\Rightarrow \quad m_1 = \frac{dy}{dx} = \frac{2}{y}$$

Slope of (ii)

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Solving (i) and (ii)

$$\frac{k}{y} = y^2$$

$$\Rightarrow \quad y^3 = k$$

$$k = \frac{k^{\frac{2}{3}}}{4}$$

$\therefore$ (i) and (ii) cuts orthogonolly

$$\therefore \quad m_1 \times m_2 = -1$$

$$\Rightarrow \quad \frac{2}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \quad \frac{2}{x} = 1$$

$$\Rightarrow \quad x = 2$$

$$\Rightarrow \quad \frac{k^{\frac{2}{3}}}{4} = 2$$

$$\Rightarrow \quad k^{\frac{2}{3}} = 8$$

$$\therefore \quad k^2 = 512$$

Solution 5

We have,

$$2x = y^2 \qquad \text{---(i)}$$

$$2xy = k \qquad \text{---(ii)}$$

Slope of (i)

$$2 = 2y\frac{dy}{dx}$$

$$\Rightarrow \quad m_1 = \frac{dy}{dx} = \frac{1}{y}$$

Slope of (ii)

$$y + x\left(\frac{dy}{dx}\right) = 0$$

$$\therefore \quad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Now,

Solving (i) and (ii)

$$\frac{k}{y} = y^2$$

$$\Rightarrow \quad y^3 = k$$

$$\therefore \quad x = \frac{y^2}{2} = \frac{k^{\frac{2}{3}}}{2}$$

$\therefore$ (i) and (ii) cuts orthogonolly

$$\therefore \quad m_1 \times m_2 = -1$$

$$\frac{1}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \quad \frac{1}{x} = 1$$

$$\Rightarrow \quad x = 1$$

$$\Rightarrow \quad \frac{k^{\frac{2}{3}}}{2} = 1$$

$$\Rightarrow \quad k^{\frac{2}{3}} = 2$$

Closing both side, we get

$$k^2 = 8$$

Solution 6

$xy = 4$

$\Rightarrow x = \frac{4}{y}$......(i)

$x^2 + y^2 = 8$.......(ii)

Substituting eq (i) in (ii) we get,

$x^2 + y^2 = 8$

$\Rightarrow \left(\frac{4}{y}\right)^2 + y^2 = 8$

$\Rightarrow 16 + y^4 = 8y^2$

$\Rightarrow y^4 - 8y^2 + 16 = 0$

$\Rightarrow \left(y^2 - 4\right)^2 = 0$

$\Rightarrow y^2 = 4$

$\Rightarrow y = \pm 2$

From(i) when $y = 2$,we get $x = 2$ and when $y = -2$,we get $x = -2$

Thus the two curves intersect at $(2, 2)$ and $(-2, 2)$.

Differnentiating (i) wrt x, we get

$y + x\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

Differnentiating (ii) wrt x, we get

$2x + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$

At $(2, 2)$

$\left(\frac{dy}{dx}\right)_{C_1} = -1$

$\left(\frac{dy}{dx}\right)_{C_2} = -1$

Clearly $\left(\frac{dy}{dx}\right)_{C_1} = \left(\frac{dy}{dx}\right)_{C_2}$ at $(2, 2)$

So given two curves touch each other at $(2, 2)$.

Simillarly, it can be seen that two curves touch each other at (-2,

Solution 7

$y^2 = 4x......(i)$

$x^2 + y^2 - 6x + 1 = 0.......(ii)$

Differnentiating (i) wrt x, we get

$$2y\frac{dy}{dx} = 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

Differnentiating (ii) wrt x, we get

$$2x + 2y\frac{dy}{dx} - 6 + 0 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3-x}{y}$$

At (1, 2)

$$\left(\frac{dy}{dx}\right)_{c_1} = \frac{2}{2} = 1$$

$$\left(\frac{dy}{dx}\right)_{c_2} = \frac{3-1}{2} = \frac{2}{2} = 1$$

Clearly $\left(\frac{dy}{dx}\right)_{c_1} = \left(\frac{dy}{dx}\right)_{c_2}$ at (1, 2)

So given two curves touch each other at (1, 2).

Solution 8(i)

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{---(i)}$$

$$xy = c^2 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\therefore \qquad m_1 = \frac{dy}{dx} = \frac{x}{y} \times \frac{b^2}{a^2}$$

Slope of (ii)

$$y + x\frac{dy}{dx} = 0$$

$$\therefore \qquad m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$ (i) and (ii) cuts orthogonally

$$\therefore \qquad m_1 \times m_2 = -1$$

$$\Rightarrow \qquad \frac{x}{y} \times \frac{-y}{x} \times \frac{a^2}{b^2} = -1$$

$$\Rightarrow \qquad a^2 = b^2$$

Solution 8(ii)

We have,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad \text{---(i)}$$

$$\frac{x^2}{A^2}-\frac{y^2}{B^2}=1 \qquad \text{---(ii)}$$

Slope of (i)

$$\frac{2x}{a^2}+\frac{2y}{b^2}\times\frac{dy}{dx}=0$$

$$\therefore \qquad m_1=\frac{dy}{dx}=-\frac{x}{y}\frac{b^2}{a^2}$$

Slope of (ii)

$$\frac{2x}{A^2}-\frac{2y}{B^2}\times\frac{dy}{dx}=0$$

$$\therefore \qquad m_2=\frac{dy}{dx}=\frac{x}{y}\times\frac{B^2}{A^2}$$

$\therefore$ (i) and (ii) cuts orthogonally

$$\therefore \qquad m_1\times m_2=-1$$

$$\therefore \qquad \frac{-x}{y}\frac{b^2}{a^2}\times\frac{x}{y}\times\frac{B^2}{A^2}=-1$$

$$\Rightarrow \qquad \frac{x^2}{y^2}\times\frac{b^2B^2}{a^2A^2}=1$$

$$\Rightarrow \qquad \frac{x^2}{y^2}=\frac{a^2A^2}{b^2B^2} \qquad \text{---(iii)}$$

Now,

(i) − (ii) gives

$$x^2\left[\frac{1}{a^2}-\frac{1}{A^2}\right]+y^2\left[\frac{1}{b^2}+\frac{1}{B^2}\right]=0$$

$$\Rightarrow \qquad \frac{x^2}{y^2}=\frac{B^2+b^2}{b^2B^2}\times\frac{a^2A^2}{a^2-A^2}$$

Put in (iii), we get

$$\frac{\left(B^2+b^2\right)}{b^2B^2}\times\frac{a^2A^2}{\left(a^2-A^2\right)}=\frac{a^2A^2}{b^2B^2}$$

$$\Rightarrow \qquad B^2+b^2=a^2-A^2$$

$$\Rightarrow \qquad a^2-b^2=A^2+B^2$$

Solution 9

We have,

$$\frac{x^2}{a^2+\lambda_1}+\frac{y^2}{b^2+\lambda_1}=1 \qquad \text{---(i)}$$

$$\frac{x^2}{a^2+\lambda_2}+\frac{y^2}{b^2+\lambda_2}=1 \qquad \text{---(ii)}$$

slope of (i)

$$\frac{2x}{a^2+\lambda_1}+\frac{2y}{b^2+\lambda_1}\times\frac{dy}{dx}=0$$

$$\Rightarrow \quad m_1=\frac{dy}{dx}=\frac{-x}{y}\times\frac{b^2+\lambda_1}{a^2+\lambda_1}$$

Slope of (ii)

$$\frac{2x}{a^2+\lambda_2}+\frac{2y}{b^2+\lambda_2}\times\frac{dy}{dx}=0$$

$$\Rightarrow \quad m_2=\frac{dy}{dx}=\frac{-x}{y}\times\frac{b^2+\lambda_2}{a^2+\lambda_2}$$

Now,

Subtracting (ii) from (i), we get

$$x^2\left[\frac{1}{a^2+\lambda_1}-\frac{1}{a^2+\lambda_2}\right]+y^2\left[\frac{1}{b^2+\lambda_1}-\frac{1}{b^2+\lambda_2}\right]=0$$

$$\Rightarrow \quad \frac{x^2}{y^2}=\frac{\lambda_2-\lambda_1}{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}\times\frac{1}{\frac{\lambda_1-\lambda_2}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}}$$

Now,

$$m_1\times m_2=\frac{x^2}{y^2}\times\frac{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}$$

$$=\frac{\left(\lambda_2-\lambda_1\right)}{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}\times-\frac{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}{\lambda_2-\lambda_1}\times\frac{\left(b^2+\lambda_1\right)\left(b^2+\lambda_2\right)}{\left(a^2+\lambda_1\right)\left(a^2+\lambda_2\right)}$$

$$=-1$$

$\therefore$ (i) and (ii) cuts orthogonolly

Solution 10

Suppose the straight line $x\cos\alpha + y\sin\alpha = p$ touches the curve at $Q(x_1, y$

But equation of tangent to $\frac{x^2}{a^2} + \frac{y}{b^2} = 1$ at $Q(x_1, y_1)$ is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

Thus equation $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ and $x\cos\alpha + y\sin\alpha = p$ represent the same l

$$\therefore \frac{x_1 / a^2}{\cos\alpha} + \frac{y_1 / b^2}{\sin\alpha} = \frac{1}{p}$$

$$\Rightarrow x_1 = \frac{a^2\cos\alpha}{p}, \quad y_1 = \frac{b^2\sin\alpha}{p} \text{..........(i)}$$

The point $Q(x_1, y_1)$ lies on the curve $\frac{x^2}{a^2} + \frac{y}{b^2} = 1$

$$\therefore \frac{a^4\cos^2\alpha}{p^2a^2} + \frac{b^4\sin^2\alpha}{p^2b^2} = 1$$

$$\Rightarrow a^2\cos^2\alpha - b^2\sin^2\alpha = p^2$$