RD Sharma Solutions Class 12 Maths Chapter 11 Ex 11.5

Chapter: Differentiation

Exercise: 11.5

Page Number: 11.89

Q1.

Answer :

$$Let\ y = x^{\frac{1}{x}} \qquad ...(\text{i})$$

Taking log on both sides,

$$\log y = \log x^{\frac{1}{x}}$$

$$\Rightarrow \log y = \frac{1}{x}\log x \qquad \left[\because \log a^b = b\log a\right]$$

Differentiating with respect to x,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}(\log\ x) + \log\ x\frac{d}{dx}\left(x^{-1}\right) \qquad [Using\ product\ rule]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\times\frac{1}{x} + (\log\ x)\times\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{\log x}{x^2}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{(1-\log x)}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = x^{\frac{1}{x}}\left[\frac{1-\log x}{x}\right]$$

Q2.

Answer :

$$Let\ y = x^{\sin x} \qquad ...(i)$$

Taking log on both sides,

$$\log y = \log x^{\sin x} \Rightarrow \log y = \sin x\ \log x$$

$$\left[\because \log a^b = b\ \log a\right]$$

Differentiating with respect to x,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x \frac{d}{dx}\log\ x + \log\ x \frac{d}{dx}\sin x \qquad [using\ product\ rule]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x\left(\frac{1}{x}\right) + \log\ x(\cos x)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\sin x}{x} + (\log\ x)(\cos x)\right]$$

$$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + (\log x)(\cos x)\right]$$

Q3.

Answer :

$$Let\ y = (1+\cos x)^x \qquad ...(i)$$

Taking log on both sides,

$$\log y = \log(1+\cos x)^x$$

$$\Rightarrow \log y = x\ \log(1+\cos x)$$

Differentiating with respect to x, we get,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log(1+\cos x) + \log(1+\cos x)\frac{d}{dx}(x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{(1+\cos x)}\frac{d}{dx}(1+\cos x) + \log(1+\cos x)(1)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{(1+\cos x)}(0-\sin\ x) + \log(1+\cos x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log(1+\cos x) - \frac{x\sin x}{(1+\cos x)}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\log(1+\cos x) - \frac{x\sin x}{(1+\cos x)}\right]$$

$$\Rightarrow \frac{dy}{dx} = (1+\cos x)^x\left[\log(1+\cos x) - \frac{x\sin x}{(1+\cos x)}\right] \qquad [using\ equation\ (i)]$$

Q4.

Answer :

$$Let\ y = x^{\cos^{-1}x} \qquad ...(i)$$

Taking log on both sides,

$$\log y = \log x^{\cos^{-1}x}$$

$$\Rightarrow \log y = \cos^{-1}x\ \log x$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \cos^{-1} x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}\cos^{-1} x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos^{-1} x\left(\frac{1}{x}\right) + \log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right]$$

$$\Rightarrow \frac{dy}{dx} = x^{\cos^{-1} x}\left[\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right] \qquad [Using\ equation\ (i)]$$

Q5.

Answer :

$$Let\ y = (\log x)^x \qquad ...(i)$$

Taking log both sides,

$$\log y = \log(\log x)^x \Rightarrow \log y = x\log(\log x)$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log(\log x) + \log(\log x)\frac{d}{dx}(x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\frac{1}{\log x}\frac{d}{dx}(\log x) + \log(\log x)(1)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\log x}\left(\frac{1}{x}\right) + \log(\log x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{\log x} + \log(\log x)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{\log x} + \log(\log x)\right]$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log(\log x)\right] \qquad [using\ equation\ (i)]$$

Q6.

Answer :

$Let\ y=(\log x)^{\cos x} \qquad ...(i)$

Taking log on both sides,

$\log y=\log(\log x)^{\cos x}$

$\Rightarrow \log y=\cos x\ \log(\log x)$

Differentiating with respect to x using chain rule,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\cos x\frac{d}{dx}\log(\log x)+\log(\log x)\frac{d}{dx}(\cos x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{\cos x}{\log x}\frac{d}{dx}(\log x)+\log(\log x)\times(-\sin x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{\cos x}{\log x}\times\left(\frac{1}{x}\right)-\sin x\log(\log x)$$

$$\Rightarrow \frac{dy}{dx}=y\left[\frac{\cos x}{x\log x}-\sin x\log(\log x)\right]$$

$$\Rightarrow \frac{dy}{dx}=(\log x)^{\cos x}\left[\frac{\cos x}{x\log x}-\sin x\log(\log x)\right] \qquad \left[using\ equation\ (i)\right]$$

Q7.

Answer :

$Let\ y=(\sin x)^{\cos x} \qquad ...(i)$

Taking log *on both sides*,

$\log y=\log(\sin x)^{\cos x}$

$\Rightarrow \log y=\cos x\log\ \sin x$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx}=\cos x\frac{d}{dx}(\log\ \sin x)+\log\ \sin x\frac{d}{dx}(\cos x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\cos x\frac{1}{\sin x}\frac{d}{dx}(\sin x)+\log\ \sin x(-\sin x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{\cos x}{\sin x}(\cos x)-\sin x\log\ \sin x$$

$$\Rightarrow \frac{dy}{dx}=y[\cos x\cot x-\sin x\log\ \sin x]$$

$$\Rightarrow \frac{dy}{dx}=(\sin x)^{\cos x}[\cos x\cot x-\sin x\log\ \sin x]$$

Q8.

Answer :

$Let\ y = e^{x \log x}$

$Taking \log\ on\ both\ sides,$

$\log y = x \log x \log e$

$\Rightarrow \log y = x \log x$

$Differentiating\ with\ respect\ to\ x,$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\log x + \log x \frac{d}{dx}x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x(1)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dy}{dx} = y[1 + \log x]$$

$$\Rightarrow \frac{dy}{dx} = e^{x \log x}(1 + \log x) \qquad [using\ equation\ (i)]$$

$$\Rightarrow \frac{dy}{dx} = e^{\log x^x}(1 + \log x)$$

$$\Rightarrow \frac{dy}{dx} = x^x(1 + \log x)$$

Q9.

Answer :

$Let\ y = (\sin x)^{\log x} \qquad ...(i)$

$Taking \log\ on\ both\ sides,$

$\log y = \log(\sin x)^{\log x}$

$\Rightarrow \log y = \log x \log \sin x$

$Differentiating\ with\ respect\ to\ x,$

$$\frac{1}{y}\frac{dy}{dx} = \log x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log x \frac{1}{\sin x}\frac{d}{dx}(\sin x) + \log \sin x\left(\frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\log x}{\sin x}(\cos x) + \frac{\log \sin x}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\log x \cot x + \frac{\log \sin x}{x}\right]$$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^{\log x}\left[\log x \cot x + \frac{\log \sin x}{x}\right]$$

Q10.

Answer :

$$Let\ y = 10^{\log \sin x} \qquad ...(i)$$
$$Taking\ \log\ on\ both\ sides,$$
$$\log y = \log 10^{\log \sin x}$$
$$\Rightarrow \log y = \log\ \sin x \log 10$$
$$Differentiating\ with\ respect\ to\ x,$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log 10 \frac{d}{dx} \log\ \sin x$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log 10 \frac{1}{\sin x}\frac{d}{dx}(\sin x)$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log 10\left(\frac{1}{\sin x}\right)(\cos x)$$
$$\Rightarrow \frac{dy}{dx} = y[\log 10 \times \cot x]$$
$$\Rightarrow \frac{dy}{dx} = 10^{\log \sin x} \times \log 10 \times \cot x \qquad [using\ equation\ (i)]$$

Q11.

Answer :

$$Let\ y = (\log x)^{\log x} \qquad ....(i)$$
$$Taking\ \log\ on\ both\ sides,$$
$$\log y = \log(\log x)^{\log x}$$
$$\Rightarrow \log y = \log x \log\ (\log x)$$
$$Differentiating\ \ both\ side\ with\ respect\ to\ x,$$
$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log(\log x)\frac{d}{dx}\log x + \log x \frac{d}{dx}\log(\log x)$$
$$\Rightarrow \frac{dy}{dx} = y\left[\log(\log x)\frac{1}{x} + \log x \frac{1}{\log x}\frac{d}{dx}(\log x)\right]$$
$$\Rightarrow \frac{dy}{dx} = y\left[\frac{1}{x}\log(\log x) + \frac{1}{x}\right]$$
$$\therefore \frac{dy}{dx} = (\log x)^{\log x}\left[\frac{1+\log(\log x)}{x}\right] \qquad [using\ equation\ (i)]$$

Q12.

Answer :

$Let\ y=10^{(10^x)} \qquad ...(i)$

Taking log *on both sides,*

$\log y=\log_e 10^{(10x)}$

$\log y=10^x \log_e 10$

Differentiating with respect to x,

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\log_e 10\times 10^x \log_e 10$

$\Rightarrow \frac{1}{y}\frac{dy}{dx}=10^x\times(\log_e 10)^2$

$\Rightarrow \frac{dy}{dx}=y\left[10^x\times(\log_e 10)^2\right]$

$\therefore \frac{dy}{dx}=10^{(10x)}\times 10^x\times(\log_e 10)^2 \qquad \left[using\ equation\ (i)\right]$

Q13.

Answer :

$Let\ y=\sin\ x^x$

$\Rightarrow \quad \sin^{-1} y=x^x \qquad ...(i)$

Taking log *on both sides,*

$\log\left(\sin^{-1} y\right)=\log x^x$

$\Rightarrow \log\left(\sin^{-1} y\right)=x\log x$

Differentiating with respect to x,

$\Rightarrow \frac{1}{\sin^{-1} y}\frac{dy}{dx}\left(\sin^{-1} y\right)=x\frac{d}{dx}\log x+\log x\frac{d}{dx}x$

$\Rightarrow \frac{1}{\sin^{-1} y}\times\left(\frac{1}{\sqrt{1-y^2}}\right)\frac{dy}{dx}=x\left(\frac{1}{x}\right)+\log x$

$\Rightarrow \frac{dy}{dx}=\sin^{-1} y\sqrt{1-y^2}\,(1+\log x)$

$\Rightarrow \frac{dy}{dx}=\sin^{-1}\left(\sin\ x^x\right)\sqrt{1-\left(\sin\ x^x\right)^2}\,(1+\log x)$

$\Rightarrow \frac{dy}{dx}=\sin^{-1}\left(\sin x^x\right)\sqrt{1-\left(\sin\ x^x\right)^2}\,(1+\log x)$

$\therefore \frac{dy}{dx}=x^x\cos x^x\,(1+\log x) \qquad \left[using\ equation\ (i)\right]$

Q14.

Answer :

$Let\ y = \left(\sin^{-1} x\right)^x \qquad ...(i)$

Taking log *on both sides,*

$\log y = \log\left(\sin^{-1} x\right)^x$

$\Rightarrow \log y = x \log\left(\sin^{-1} x\right)$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}\left(\log \sin^{-1} x\right) + \log \sin^{-1} x \frac{d}{dx} x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\frac{1}{\sin^{-1} x}\frac{d}{dx}\left(\sin^{-1} x\right) + \log \sin^{-1} x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1} x}\left(\frac{1}{\sqrt{1-x^2}}\right) + \log \sin^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = y\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x\left(\sqrt{1-x^2}\right)}\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1} x}\left(\frac{1}{\sqrt{1-x^2}}\right) + \log \sin^{-1} x$$

$$\Rightarrow \frac{dy}{dx} = y\left[\log \sin^{-1} x + \frac{x}{\sin^{-1} x\left(\sqrt{1-x^2}\right)}\right]$$

Q15.

Answer :

$Let\ y = x^{\sin^{-1} x} \qquad ...(i)$

Taking log *on both sides,*

$\log y = \log x^{\sin^{-1} x}$

$\Rightarrow \log y = \sin^{-1} x \log x$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1} x\frac{d}{dx}(\log x) + (\log x)\frac{d}{dx}\left(\sin^{-1} x\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin^{-1} x\left(\frac{1}{x}\right) + (\log x)\left(\frac{1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right]$$

$$\Rightarrow \frac{dy}{dx} = x^{\sin^{-1} x}\left[\frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right] \qquad \left[using\ equation\ (i)\right]$$

Q16.

Answer :

$$Let\ y = (\tan x)^{\frac{1}{x}} \qquad ...(i)$$

Taking log *on both sides,*

$$\log y = \log(\tan x)^{\frac{1}{x}}$$

$$\Rightarrow \log y = \frac{1}{x}\log(\tan x)$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\frac{d}{dx}\{\log(\tan x)\} + \log(\tan x)\frac{d}{dx}\left(\frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\times\frac{1}{\tan x}\frac{d}{dx}(\tan x) + \log(\tan x)\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x\tan x}(\sec^2 x) - \frac{\log(\tan x)}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\sec^2 x}{x\tan x} - \frac{\log(\tan\ x)}{x^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = (\tan x)^{\frac{1}{x}}\left[\frac{\sec^2 x}{x\tan x} - \frac{\log(\tan\ x)}{x^2}\right] \qquad [using\ equation\ (i)]$$

Q17.

Answer :

$$Let\ y = x^{\tan^{-1} x} \qquad ...(i)$$

Taking log *on both sides,*

$$\log y = \log\ x^{\tan^{-1} x}$$

$$\Rightarrow \log y = \tan^{-1} x \log x$$

Differentiating with respect to x,

$$\frac{1}{y}\frac{dy}{dx} = \tan^{-1} x\frac{d}{dx}\log x + \log x\frac{d}{dx}(\tan^{-1} x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \tan^{-1} x\left(\frac{1}{x}\right) + \log x\left(\frac{1}{1+x^2}\right)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right]$$

$$\Rightarrow \frac{dy}{dx} = x^{\tan^{-1} x}\left[\frac{\tan^{-1} x}{x} + \frac{\log x}{1+x^2}\right] \qquad [using\ equation\ (i)]$$

Q18.

Answer :

(i)

$$Let\ y = x^x\sqrt{x} \qquad ...(i)$$

Taking log *on both sides,*

$$\log y = \log\left(x^x\sqrt{x}\right)$$

$$\Rightarrow \log y = \log x^x + \log x^{\frac{1}{2}}$$

$$\Rightarrow \log y = x\log x + \frac{1}{2}\log x$$

$$\frac{1}{y}\frac{dy}{dx} = x\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(x) + \frac{1}{2}\frac{d}{dx}(\log x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x(1) + \frac{1}{2}\left(\frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x + \frac{1}{2x}$$

$$\Rightarrow \frac{dy}{dx} = y\left[1 + \log x + \frac{1}{2x}\right]$$

$$\Rightarrow \frac{dy}{dx} = x^x\sqrt{x}\left[1 + \log x + \frac{1}{2x}\right] \qquad \left[using\ equation\ (i)\right]$$

$$\Rightarrow \frac{dy}{dx} = x^{x+\frac{1}{2}}\left[\left(\frac{2x+1}{2x}\right) + \log x\right]$$

$$(ii) \quad Let\ y = x^{(\sin x - \cos x)} + \left(\frac{x^2-1}{x^2+1}\right)$$

$$\Rightarrow y = e^{\log x^{(\sin x-\cos x)}} + \left(\frac{x^2-1}{x^2+1}\right)$$

$$\Rightarrow y = e^{(\sin x-\cos x)\log x} + \left(\frac{x^2-1}{x^2+1}\right)$$

$$\frac{dy}{dx} = \frac{d}{dx}\left[e^{(\sin x-\cos x)\log x}\right] + \frac{d}{dx}\left[\frac{x^2-1}{x^2+1}\right]$$

$$= e^{(\sin x-\cos x)\log x}\frac{d}{dx}\{(\sin x-\cos x)\log x\} + \left[\frac{(x^2+1)\frac{d}{dx}(x^2-1)-(x^2-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}\right]$$

$$= e^{\log x^{(\sin x-\cos x)}}\left[(\sin x-\cos x)\frac{d}{dx}(\log x)+(\log x)\frac{d}{dx}(\sin x-\cos x)\right] + \left[\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}\right]$$

$$= x^{(\sin x-\cos x)}\left[(\sin x-\cos x)\left(\frac{1}{x}\right)+\log x(\sin x+\cos x)\right] + \left[\frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}\right]$$

$$= x^{(\sin x-\cos x)}\left[\frac{(\sin x-\cos x)}{x}+(\sin x+\cos x)\log x\right] + \frac{4x}{(x^2+1)^2}$$

Q19.

Answer :

$We\,have, y = e^x + 10^x + x^x$

$\Rightarrow y = e^x + 10^x + e^{\log x^x}$

$\Rightarrow y = e^x + 10^x + e^{x\log x}$

Differentiating with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(e^x\right) + \frac{d}{dx}\left(10^x\right) + \frac{d}{dx}\left(e^{x\log x}\right)$$

$$= e^x + 10^x \log 10 + e^{x\log x}\frac{d}{dx}\left(x\log x\right)$$

$$= e^x + 10^x \log 10 + e^{x\log x}\left[x\frac{d}{dx}\left(\log x\right) + \log x\frac{d}{dx}\left(x\right)\right]$$

$$= e^x + 10^x \log 10 + e^{\log x x}\left[x\left(\frac{1}{x}\right) + \log x\right]$$

$$= e^x + 10^x \log 10 + x^x\left[1 + \log x\right]$$

$$= e^x + 10^x \log 10 + x^x\left[\log e + \log x\right] \qquad \left[\because \log_e e = 1\right]$$

$$= e^x + 10^x \log 10 + x^x\left(\log ex\right)$$

Q20.

Answer :

$We\ have,\ y = x^n + n^x + x^x + n^n$

$\Rightarrow y = x^n + n^x + e^{\log x^x} + n^n$

$\Rightarrow y = x^n + n^x + e^{x\log x} + n^n$

Differentiate with respect to x,

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^n\right) + \frac{d}{dx}\left(n^x\right) + \frac{d}{dx}\left(e^{x\log x}\right) + \frac{d}{dx}\left(n^n\right)$$

$$= nx^{n-1} + n^x \log n + e^{\log x^x}\left[x\frac{d}{dx}\log x + \log x\frac{d}{dx}\left(x\right)\right]$$

$$= nx^{n-1} + n^x \log n + x^x\left[x\left(\frac{1}{x}\right) + \log x\right]$$

$$= nx^{n-1} + n^x \log n + x^x\left[1 + \log x\right]$$

$$= nx^{n-1} + n^x \log n + x^x\left[\log e + \log x\right] \qquad \left[\because \log_e e = 1\ and\ \log A + \log B = \log\left(AB\right)\right]$$

$$= nx^{n-1} + n^x \log n + x^x \log\left(ex\right)$$

Q21.

Answer :

$$We\ have,\ y=\frac{\left(x^2-1\right)^3\left(2x-1\right)}{\sqrt{\left(x-3\right)\left(4x-1\right)}} \qquad ...(i)$$

$$\Rightarrow y=\frac{\left(x^2-1\right)^3\left(2x-1\right)}{\left(x-3\right)^{\frac{1}{2}}\left(4x-1\right)^{\frac{1}{2}}}$$

Taking log on both sides,

$$\log\ y=\log\left[\frac{\left(x^2-1\right)^3\left(2x-1\right)}{\left(x-3\right)^{\frac{1}{2}}\left(4x-1\right)^{\frac{1}{2}}}\right]$$

$$\Rightarrow \log y=\log\left(x^2-1\right)^3+\log\left(2x-1\right)-\log\left(x-3\right)^{\frac{1}{2}}-\log\left(4x-1\right)^{\frac{1}{2}}$$

$$\Rightarrow \log y=3\ \log\left(x^2-1\right)+\log\left(2x-1\right)-\frac{1}{2}\log\left(x-3\right)-\frac{1}{2}\log\left(4x-1\right)$$

Differentiating with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx}=3\frac{d}{dx}\left\{\log\left(x^2-1\right)\right\}+\frac{d}{dx}\left\{\log\left(2x-1\right)\right\}-\frac{1}{2}\frac{d}{dx}\left\{\log\left(x-3\right)\right\}-\frac{1}{2}\left\{\log\left(4x-1\right)\right\}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=3\left(\frac{1}{x^2-1}\right)\frac{d}{dx}\left(x^2-1\right)+\frac{1}{\left(2x-1\right)}\frac{d}{dx}\left(2x-1\right)-\frac{1}{2}\left(\frac{1}{x-3}\right)\frac{d}{dx}\left(x-3\right)-\frac{1}{2}\frac{1}{\left(4x-1\right)}\frac{d}{dx}\left(4x-1\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=3\left(\frac{1}{x^2-1}\right)\left(2x\right)+\frac{1}{2x-1}\left(2\right)-\frac{1}{2}\left(\frac{1}{x-3}\right)\left(1\right)-\frac{1}{2}\left(\frac{1}{4x-1}\right)\left(4\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2\left(x-3\right)}-\frac{2}{4x-1}\right]$$

$$\Rightarrow \frac{dy}{dx}=y\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2\left(x-3\right)}-\frac{2}{4x-1}\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{\left(x^2-1\right)^3\left(2x-1\right)}{\sqrt{\left(x-3\right)\left(4x-1\right)}}\left[\frac{6x}{x^2-1}+\frac{2}{2x-1}-\frac{1}{2\left(x-3\right)}-\frac{2}{4x-1}\right] \qquad \left[using\ equation\ \left(i\right)\right]$$