RD Sharma Solutions Class 12 Maths Chapter 11 Ex 11.1

Chapter: Differentiation

Exercise: 11.1

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Question 1.
Solution:
Consider $f(x) = e^{-x}$

$$\Rightarrow \quad f(x+h) = e^{-(x+h)}$$

$$\frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{-(x+h)} - e^{-x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{-x} \times e^{-h} - e^{-x}}{h}$$

$$= \lim_{h \to 0} e^{-x} \left\{\left(\frac{e^{-h} - 1}{-h}\right)\right\} \times (-1) \qquad \left[\text{Since, } \lim_{h \to 0} \frac{e^{h} - 1}{h} = 1\right]$$

$$= -e^{-x}$$

So,

$$\frac{d}{dx}\left(e^{-x}\right) = -e^{-x}$$

Question 2.
Solution:
Consider

$f(x) = e^{3x}$

$\Rightarrow f(x+h) = e^{3(x+h)}$

$$\frac{d}{dx}(f(x)) = \frac{\lim}{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \frac{\lim}{h \to 0} \frac{e^{3(x+h)} - e^{3x}}{h}$$

$$= \frac{\lim}{h \to 0} \frac{e^{3x}e^{3h} - e^{3x}}{h}$$

$$= \frac{\lim}{h \to 0} e^{3x} \left\{ \left(e \frac{3h-1}{3h} \right) \right\} \times 3$$

$$3e^{3x}$$

So,

$$\frac{d}{dx}\left(e^{3x}\right) = 3e^{3x}$$

Question 3.
Solution:

Consider

$$f(x) = e^{ax+b}$$

$$\Rightarrow f(x+h) = e^{a(x+h)+b}$$

$$\frac{d}{dx}(f(x)) = \frac{\lim}{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \frac{\lim}{h \to 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h}$$

$$= \frac{\lim}{h \to 0} \frac{e^{ax+b}e^{ax} - e^{ax+b}}{h}$$

$$= \frac{\lim}{h \to 0} e^{ax+b} \left\{ \left(e \frac{ah-1}{ah} \right) \right\} \times a$$

$$ae^{ax+b}$$

So,

$$\frac{d}{dx}\left(e^{ax+b}\right) = ae^{ax+b}$$

Question 4.

Solution:

Consider $f\left(x\right)=e\sqrt{2x}$

$f\left(x+h\right)=e^{\sqrt{2(x+h)}}$

$$\frac{d}{dx}\left(f\left(x\right)\right)=\frac{\lim}{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}$$

$$=\frac{\lim}{h\to 0}\frac{e^{\sqrt{2(x+h)}}-e^{\sqrt{2x}}}{h}$$

$$=e\sqrt{2x}\frac{\lim}{h\to 0}\left(\frac{e^{\sqrt{2(x+h)-\sqrt{2x}}}-1}{\sqrt{2\left(x+h\right)-\sqrt{2x}}}\right)\left(\sqrt{2\frac{\left(x+h\right)-\sqrt{2x}}{h}}\right)$$

$$=e\sqrt{2x}\frac{\lim}{h\to 0}\frac{\sqrt{2\left(x+h\right)-\sqrt{2x}}}{h}\times\frac{\sqrt{2\left(x+h\right)}+\sqrt{2x}}{\sqrt{2\left(x+h\right)}+\sqrt{2x}}$$

$$=de\sqrt{2x\frac{\lim}{h\to 0}\frac{2\left(x+h\right)-2x}{h\left(\sqrt{2\left(x+h\right)}+\sqrt{2x}\right)}}$$

$$=e\sqrt{2x}\frac{\lim}{h\to 0}\frac{2h}{h\left(\sqrt{2\left(x+h\right)+\sqrt{2x}}\right)}$$

$$=e\sqrt{2x\frac{\lim}{h\to 0}\frac{2}{\left(\sqrt{2\left(x+h\right)+\sqrt{2x}}\right)}}$$

So,

$$\frac{d}{dx}\left(e\sqrt{2x}\right)=\frac{e\sqrt{2x}}{\sqrt{2x}}$$

Q.6

Solution:

Consider $f\left(x\right)=\log\ \cos\ x$

$\Rightarrow f\left(x+h\right)=\log\ \cos\left(x+h\right)$

$$\frac{d}{dx}\left(f(x)\right)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\log\cos(x+h)-\log\cos x}{h}$$

$$=\lim_{h\to 0}\frac{\log\frac{\cos(x+h)}{\cos x}}{h}$$

$$\left[\text{Since}, \log A-\log B=\log\frac{A}{B}\right]$$

$$=\lim_{h\to 0}\frac{\log\left\{1+\frac{\cos(x+h)}{\cos x}-1\right\}}{h}$$

$$=\lim_{h\to 0}\frac{\log\left\{1+\frac{\cos(x+h)}{\cos x}-1\right\}}{\left(\cos\frac{(x+h)}{\cos x}\right)h\times\left(\frac{\cos x}{\cos(x+h)-\cos x}\right)}$$

$$=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{\cos x\times h}$$

$$\left[\text{since}, \lim_{x\to 0}\frac{\log(1+x)}{x}=1\right]$$

$$=\lim_{h\to 0}\frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{\cos x\times h}$$

$$=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{\cos x\times h}$$

$$=\lim_{h\to 0}\frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{\cos x\times h}$$

$$=-2\lim_{h\to 0}\frac{\sin\left(\frac{2x+h}{2}\right)\times\left(\sin\frac{h}{2}\right)}{2\cos x\left(\frac{h}{2}\right)}$$

$$= \frac{-2\sin x}{2\cos x}$$

$$\left[\text{Since, } \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

$$= -\tan x$$

Hence,the solution is $\boxed{\frac{d}{dx}(\log \cos x) = -\tan x}$

Question 7.
Solution:

$$f(x) = e\sqrt{\cos x}$$

$$\Rightarrow f(x+h) = e\sqrt{\cot(x+h)}$$

Consider, $$\frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{e\sqrt{\cot(x+h) - \sqrt{\cot x}} - 1}{h}$$

$$= e\sqrt{\cot x} \lim_{h \to 0} \left(\frac{e\sqrt{\cot(x+h) - \sqrt{\cot x}} - 1}{\sqrt{\cot(x+h)} - \sqrt{\cot x}} \right) \times \left(\frac{\sqrt{\cot(x+h) - \sqrt{\cot x}}}{h} \right)$$

$$= e\sqrt{\cot x \lim_{h \to 0} \left(\frac{\sqrt{\cot(x+h) - \sqrt{\cot x}}}{h} \right) \times \frac{\sqrt{\cot(x+h) + \sqrt{\cot x}}}{\sqrt{\cot(x+h) + \sqrt{\cot x}}}}$$

$$\left[\text{Since, } \lim_{x \to 0} \frac{e^{r} - 1}{x} = 1 \text{ and rationalizing numerator}\right]$$

$$= e\sqrt{\cot x} \lim_{h \to 0} \frac{\cot(x+h) - \cot x}{h\left(\sqrt{\cot(x+h) + \sqrt{\cot x}}\right)}$$

$$= e\sqrt{\cot x} \lim_{h \to 0} \frac{\frac{\cot(x+h)\cot x + 1}{\cot(x+h-x)}}{h\left(\sqrt{\cot(x+h) + \sqrt{\cot x}}\right)}$$

$$\left[\text{Since}, \cot(A-B)=\frac{\cot A\cot B+1}{\cot A-\cot B}\right]$$

$$=e\sqrt{\cot x}\ \frac{\lim}{h\to 0}\frac{\cot(x+h)\cot x+1)}{\coth xh\left(\sqrt{\cot(x+h)}+\sqrt{\cot x}\right)}$$

$$=e\sqrt{\cot x}\ \frac{\lim}{h\to 0}\frac{\left(\cot(x+h)\cot x+1\right)}{\left(\frac{h}{\tanh}\right)\left(\sqrt{\cot(x+h)}+\sqrt{\cot x}\right)}$$

$$=\frac{e\sqrt{\cot x}\times\left(\cot^2 x+1\right)}{2\sqrt{\cot x}}$$

$$\left[\text{Since}, \frac{\lim}{h\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{e\sqrt{\cot x}\times\cos ec^2 x}{2\sqrt{\cot x}}$$

$$\left[\text{Since}, \left(1+\cot^2 x\right)=\cos ec^2 x\right]$$

Hence, the solution is $\boxed{\frac{d}{edx}\left(e\sqrt{\cot x}\right)=\frac{e\sqrt{\cot x}\times\cos ex^2 x}{2\sqrt{\cot x}}}$

Q.7
Solution:

Consider

$$f(x)=x^2e^2$$

$$\Rightarrow f(x+h)=(x+h)^2\ e^{(x+h)}$$

$$\frac{d}{dx}(f(x))=\frac{\lim}{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\frac{\lim}{h\to 0}\frac{(x+h)^2\ e^{(x+h)}-x^2e^x}{h}$$

$$=\frac{\lim}{h\to 0}\left(\frac{x^2e^{(x+h)}-x^2e^x}{h}+\frac{2xhe^{(x+h)}}{h}+\frac{h^2e^{(x+h)}}{h}\right)$$

$$= \frac{\lim}{h \to 0}\left(\frac{x^2 e^x \left(e^{(x+h)-x} - 1\right)}{h} + 2xe^{(x+h)} + he^{(x+h)} \right)$$

$$= \frac{\lim}{h \to 0}\left[x^2 e^x \frac{\left(e^h - 1\right)}{h} + 2xe^{(x+h)} + he(x+h) \right]$$

$$= x^2 e^2 + 2xe^x + 0 \times e^x$$

$$\left[\text{Since, } \frac{\lim}{h \to 0} \frac{e^x - 1}{x} = 1 \right]$$

Hence, the solution is $\boxed{\frac{d}{dx}\left(x^2 e^x\right) = e^x\left(x^2 + 2x\right)}$

Q.7
Solution:

Consider $f(x) = \log\ \cos\ ecx$

$\Rightarrow f(x+h) = \log\ \cos ec(x+h)$

$$\frac{d}{dx}(f(x)) = \frac{\lim}{h \to 0} \frac{f(x+h) - (x)}{h}$$

$$= \frac{\lim}{h \to 0} \frac{\log \cos ex(x+h) - \log \cos ecx}{h}$$

$$= \frac{\lim}{h \to 0} \frac{\log\left(\frac{\cos ec(x+h)}{\cos ecx} \right)}{h}$$

$$= \lim_{h \to 0} \frac{\log\left(1+\left(\frac{\sin x}{\sin(x+h)}-1\right)\right)}{h}$$

$$= \lim_{h \to 0} \left\{ \frac{\log\left(1+\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)\right)}{\frac{\sin x - \sin(x+h)}{\sin(x+h)}} \right\} \frac{\left(\frac{\sin x - \sin(x+h)}{\sin(x+h)}\right)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\frac{(x+x+h)}{2}\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)h}$$

$$\left[\text{Since, } \lim_{h \to 0} \frac{\log(1+x)}{x} = 1 \text{ and } \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)(-2)} \left\{ \frac{\sin\left(-\frac{h}{2}\right)}{-\frac{h}{2}} \right\}$$

$$\left[\text{Since, } \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

Hence,the solution is $\boxed{\frac{d}{dx}(\log \operatorname{cosec} x) = -\cot x}$

Q.7

Solution:

Consider

$f(x) = \sin^{-1}(2x+3)$

$\Rightarrow f(x+h) = \sin^{-1}(2(x+h)+3)$

$\Rightarrow f(x+h) = \sin^{-1}(2x+2h+3)$

$$\frac{d}{dx}(f(x)) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin^{-1}(2x+2h+3) - \sin^{-1}(2x+3)}{h}$$

$$= \lim_{h \to 0} \frac{\sin^{-1}\left[(2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}\right]}{h}$$

$$\left[\text{Since}, \sin^{-1} x - \sin^{-1} y = \sin^{-1}\left[x\sqrt{1-y^2} - y\sqrt{1-x^2}\right]\right]$$

$$= \lim_{h \to 0} \frac{\sin^{-1} z}{z} \times \frac{z}{h}$$

$$\text{Where}, z = (2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2} \text{ and} = \lim_{h \to 0} \frac{\sin-1}{h} = 1$$

$$= \lim_{h \to 0} \frac{z}{h}$$

$$= \lim_{h \to 0} \frac{(2x+2h+3)\sqrt{1-(2x+3)^2} - (2x+3)\sqrt{1-(2x+2h+3)^2}}{h}$$

$$= \lim_{h \to 0} \frac{(2x+2h+3)^2 - (2x+3)^2 - (2x+3)^2\left(1-(2x+2h+3)^2\right)}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

[Since, rationalizing numerator]

$$\left[(2x+3)^2 + 4h^2 + 4h(2x+3)\right]\left(1-(2x+3)^2\right) - (2x+3)^2$$

$$= \lim_{h \to 0} \frac{\left[1-(2x+3)^2 - 4h^2 - 4h(2x+3)\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \lim_{h \to 0} \frac{\left[\begin{array}{r}(2x+3)^2 + 4h^2 + 4h(2x+3) - (2x+3)^4 - 4h^2(2x+3)^2 - 4h(2x+3)^3 - (2x+3)^2 \\ + (2x+3)^4 + 4h^2(2x+3)^2 + 4h(2x+3)^3\end{array}\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2} + (2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \frac{\lim}{h \to 0} \frac{4h\left[h+(2x+3)\right]}{h\left\{(2x+2h+3)\sqrt{1-(2x+3)^2}+(2x+3)\sqrt{1-(2x+2h+3)^2}\right\}}$$

$$= \frac{4(2x+3)}{(2x+3)\sqrt{1-(2x+3)^2}+(2x+3)\sqrt{1-(2x+3)^2}}$$

$$= \frac{4(2x+3)}{2(2x+3)\sqrt{1-2x+3^2}}$$

$$= \frac{2}{\sqrt{1-(2x+3)^2}}$$

Hence,the solution is $\boxed{\frac{d}{dx}(\sin-1(2x+3)) = \frac{2}{\sqrt{1-(2x+3)^2}}}$

Question 8.
Solution:

Consider

$$f(x) = e^{\cos x}$$

$$\Rightarrow f(x+h) = e^{\cos(x+h)}$$

$$\frac{d}{dx}(f(x)) = \frac{\lim}{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \frac{\lim}{h \to 0} \frac{e^{\cos(x+h)} - e^{\cos x}}{h}$$

$$= \frac{\lim}{h \to 0} e^{\cos x}\left[\frac{e^{\cos(x+h)-\cos x} - 1}{h}\right]$$

$$= \frac{\lim}{h \to 0} e^{\cos x} \left[\frac{e^{\cos(x+h)-\cos x} - 1}{\cos(x+h)-\cos x} \right] \times \frac{\cos(x+h)-\cos x}{h}$$

$$= \frac{\lim}{h \to 0} e^{\cos x} \times \left(\frac{\cos(x+h)-\cos x}{h} \right)$$

$$\left[\text{since,} = \frac{\lim}{h \to 0} \frac{e^x - 1}{x} = 1 \right]$$

$$= \frac{\lim}{h \to 0} e^{\cos x} \times \left(\frac{-2\sin\frac{(x+h+x)}{2} \times \sin\frac{x+h-x}{2}}{h} \right)$$

$$\left[\text{since, } \cos A - \cos B = -2\sin\frac{A+B}{2} \quad \sin\frac{A-B}{2} \right]$$

$$= e^{\cos x} = \frac{\lim}{h \to 0} \frac{-2\sin\left(\frac{2x+h}{2}\right)}{2} \times \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= e^{\cos x} = \frac{\lim}{h \to 0} - 2\sin\left(\frac{2x+h}{2}\right) \times \frac{1}{2}$$

$$\left[\text{since, } = \frac{\lim}{h \to 0} \frac{\sin x}{x} = 1 \right]$$

$$= e^{\cos x}(-\sin x)$$

$$= -\sin x e^{\cos x}$$

So the differentiation is

$$\frac{d}{dx}\left(e^{\cos x}\right) = -\sin x e^{\cos x}$$