RD Sharma Solutions Class 11 Maths Chapter 32 Ex 32.4

Statistics Ex 32.4 Q1(i)

x	d=(x- Mean)	d^2
2	-5	25
4	-3	9
5	-2	4
6	-1	1
8	1	1
17	10	100
42		140

$\bar{x} = \frac{1}{n}\Sigma x_i = \frac{1}{6}[42] = 7$

$\text{var}(x) = \frac{1}{n}\left\{\Sigma\left(x_i - \bar{x}\right)^2\right\} = \frac{1}{6}\{140\} = 23.33$

$S.D(x) = \sqrt{\text{var}(x)} = \sqrt{23.33} = 4.8$

Statistics Ex 32.4 Q1(ii)

x	d=(x- Mean)	d^2
6	-3	9
7	-2	4
10	1	1
12	3	9
13	4	16
4	-5	25
8	-1	1
12	3	9
72		74

$\bar{x} = \frac{1}{n}\Sigma x_i = \frac{1}{8}[72] = 9$

$\text{var}(x) = \frac{1}{n}\left\{\Sigma\left(x_i - \bar{x}\right)^2\right\} = \frac{1}{8}\{74\} = 9.25$

$S.D(x) = \sqrt{\text{var}(x)} = \sqrt{9.25} = 3.04$

Statistics Ex 32.4 Q1(iii)

x_i	$d_i = x_i - 299$	d_i^2
227	−72	5184
235	−64	4096
255	−44	1936
269	−30	900
292	−7	49
299	0	0
312	13	169
321	22	484
333	34	1156
348	49	2401
	$Total = -99$	$Total = 16375$

$$\overline{X} = 299 + \frac{-99}{10} = 289.1$$

$$Var = \frac{16375}{10} - \left(\frac{-99}{10}\right)^2 = 1637.5 - 98.01 = 1539.49$$

$$S.D = \sqrt{1539.49} = 39.24$$

Statistics Ex 32.4 Q1(iv)

x_i	$d_i = x_i - 15$	d_i^2
15	0	0
22	7	49
27	12	144
11	−4	16
9	−6	36
21	6	36
14	−1	1
9	−6	36
	$Total = 8$	$Total = 318$

$$Mean = 15 + \frac{8}{8} = 16$$

$$Var = \frac{318}{8} - 1 = 38.75$$

$$SD = \sqrt{38.75} = 6.22$$

Statistics Ex 32.4 Q2

We have, $n = 20$, and $\sigma^2 = 5$

Now each observation is multiplied by 2.
Suppose $X = 2x$ be the new data.

$$\therefore \quad \overline{X} = \frac{1}{20}\Sigma 2x_i = \frac{1}{20} \times 2\Sigma x_i = 2\overline{x}$$

$$\Rightarrow \quad \Sigma X_i^2 = 4\Sigma x_i^2$$

Since, $\sigma^2 = 5$

$$\Rightarrow \quad \frac{1}{n}\Sigma x_i^2 - \left(\overline{x}\right)^2 = 5$$

Now, for the new data:

$$\sigma^2 = \frac{1}{n}\Sigma X_i^2 - \left(\overline{X}\right)^2 = 4\Sigma x_i^2 - \left(2\overline{x}\right)^2 = 4\left(\Sigma x_i^2 - \left(\overline{x}\right)^2\right) = 4 \times 5 = 20$$

Statistics Ex 32.4 Q3

We have, $n = 15$, and $\sigma^2 = 4$

Now each observation is increased by 9.
Suppose $X = x + 9$ be the new data.

$$\therefore \quad \overline{X} = \frac{1}{15}\Sigma(x_i + 9) = \left(\frac{1}{15} \times \Sigma x_i\right) + 9 = \overline{x} + 9$$

$$\Rightarrow \quad \Sigma X_i^2 = \Sigma(x_i + 9)^2 = \Sigma x_i^2 + \Sigma 18x_i + \Sigma 9^2$$

Since, $\sigma^2 = 5$

$$\Rightarrow \quad \frac{1}{n}\Sigma x_i^2 - \left(\overline{x}\right)^2 = 4$$

Now, for the new data:

$$\sigma^2 = \frac{1}{n}\Sigma X_i^2 - \left(\overline{X}\right)^2 = \frac{1}{15}\left(\Sigma x_i^2 + \Sigma 18x_i + \Sigma 9^2\right) - \left(\overline{x} + 9\right)^2 =$$

$$= \frac{1}{15}\Sigma x_i^2 + \frac{1}{15}\Sigma 18x_i + \frac{1}{15}\Sigma 9^2 - (9)^2 - \left(18\overline{x}\right) - \left(\overline{x}\right)^2$$

$$= \left[\frac{1}{15}\Sigma x_i^2 - \left(\overline{x}\right)^2\right] + \left[\frac{1}{15}\Sigma 18x_i - \left(18\overline{x}\right)\right] + \left[\frac{1}{15}\Sigma 9^2 - (9)^2\right]$$

$$= \left[\frac{1}{15}\Sigma x_i^2 - \left(\overline{x}\right)^2\right] + \left[18 \times \frac{1}{15}\Sigma x_i - \left(18\overline{x}\right)\right] + \left[\frac{1}{15} \times 15 \times (9)^2 - (9)^2\right]$$

$$= \frac{1}{15}\Sigma x_i^2 - \left(\overline{x}\right)^2$$

$$= 4$$

Statistics Ex 32.4 Q4

Let the other two be x and y

$1 + 2 + 6 + x + y = 5 * 4.4$ because of the mean
$x + y = 13$

Variance = $[(1 - 4.4)^2 + (2 - 4.4)^2 + (6 - 4.4)^2 + (x - 4.4)^2 + (y - 4.4)^2]/5$

Hence
$11.56 + 5.76 + 2.56 + (x - 4.4)^2 + (y - 4.4)^2 = 41.2$
$(x - 4.4)^2 + (y - 4.4)^2 = 21.32$

Solve simultaneously
$(x - 4.4)^2 + (13 - x - 4.4)^2 = 21.32$
$(x - 4.4)^2 + (8.6 - x)^2 = 21.32$
$x^2 - 8.8x + 19.36 + 73.96 - 17.2x + x^2 = 21.32$
$2x^2 - 26x + 72 = 0$
$x^2 - 13x + 36 = 0$
$(x - 4)(x - 9) = 0$
$x = 4$ or $x = 9$
If $x = 4$, $y = 9$ and

The other two observations are 4 and 9.

Statistics Ex 32.4 Q5

If mean and SD of observations are $\overline{X}$ and σ respectively,
then mean and SD of observations multiplied by a constant 'k' are
$\text{Mean} = k\overline{X}$
$\text{SD} = |k|\sigma$

In this question, it is given that $k=3$
So New mean $= 8 \times 3 = 24$
New SD $= 4 \times 3 = 12$

Statistics Ex 32.4 Q6

Let x and y be the remaining two observations. Then,

Mean = 9

$$\Rightarrow \quad \frac{6+7+10+12+12+13+x+y}{8} = 9$$

$$\Rightarrow \quad 60 + x + y = 72$$

$$\Rightarrow \quad x + y = 12 \quad \text{-------(i)}$$

Variance = 9.25

$$\Rightarrow \quad \frac{1}{8}\left(6^2+7^2+10^2+12^2+12^2+13^2+x^2+y^2\right) - (\text{Mean})^2 = 9.25$$

$$\Rightarrow \quad \frac{1}{8}\left(36+49+100+144+144+169+x^2+y^2\right) - 81 = 9.25$$

$$\Rightarrow \quad 642 + x^2 + y^2 = 722$$

$$\Rightarrow \quad x^2 + y^2 = 80 \quad \text{------(ii)}$$

Now, $(x+y)^2 + (x-y)^2 = 2\left(x^2+y^2\right)$

$$\Rightarrow \quad 144 + (x-y)^2 = 2 \times 80$$

$$\Rightarrow \quad (x-y)^2 = 16$$

$$\Rightarrow \quad x - y = \pm 4$$

if $x - y = 4$, then $x + y = 12$ and $x - y = 4 \Rightarrow x = 8,\ y = 4$

if $x - y = -4$, then $x + y = 12$ and $x - y = -4 \Rightarrow \quad x = 4,\ y = 8$

Hence, the remaining two observations are 4 and 8.

Statistics Ex 32.4 Q7

We have,

$n = 200, \overline{X} = 40, \sigma = 15.$

$\therefore \quad \overline{X} = \frac{1}{n}\Sigma x_i = \overline{X} = 200 \times 40 = 8000.$

Corrected Σx_i = Incorrect Σx_i − (sum of incorrect values) + (sum of correct values)

$= 8000 - 34 - 53 + 43 + 35 = 7991$

$\therefore \quad$ Corrected mean $= \frac{\text{corrected } \Sigma x_i}{n} = \frac{7991}{200} = 39.955$

Now, $\sigma = 15$

$$\Rightarrow \quad 15^2 = \frac{1}{200}\left(\Sigma x_i^2\right) - \left(\frac{1}{200}\Sigma x_i\right)^2$$

$$\Rightarrow \quad 255 = \frac{1}{200}\left(\Sigma x_i^2\right) - \left(\frac{8000}{200}\right)^2$$

$$\Rightarrow \quad 255 = \frac{1}{200}\left(\Sigma x_i^2\right) - 1600$$

$$\Rightarrow \quad \Sigma x_i^2 = 200 \times 1825 = 365000$$

$$\Rightarrow \quad \text{Incorrect } \Sigma x_i^2 = 365000.$$

correctedΣx_i^2 = (incorrectΣx_i^2) − (sum of squares of incorrect values)

+ (sum of squares of correct values)

$= 365000 - (34)^2 - 53^2 + (43)^2 + 35^2 = 364109$

891

so, Corrected $\sigma = \sqrt{\frac{1}{n}\Sigma x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2} = \sqrt{\frac{364109}{200} - \left(\frac{7991}{200}\right)^2}$

$= \sqrt{1820.545 - 1596.402} = 14.97$

Statistics Ex 32.4 Q8

We have,

$n = 100, \overline{X} = 40, \sigma = 5.1$

$\therefore \quad \overline{X} = \frac{1}{n}\Sigma x_i = \overline{X} = 100 \times 40 = 4000.$

Corrected Σx_i = Incorrect Σx_i − (sum of incorrect values) + (sum of correct values)

$= 4000 - 50 + 40 = 3990$

$\therefore \quad \text{Corrected mean} = \frac{\text{corrected } \Sigma x_i}{n} = \frac{3990}{100} = 39.9$

Now, $\sigma = 5.1$

$\Rightarrow \quad 5.1^2 = \frac{1}{100}\left(\Sigma x_i^2\right) - \left(\frac{1}{100}\Sigma x_i\right)^2$

$\Rightarrow \quad 26.01 = \frac{1}{100}\left(\Sigma x_i^2\right) - \left(\frac{4000}{100}\right)^2$

$\Rightarrow \quad 26.01 = \frac{1}{100}\left(\Sigma x_i^2\right) - 1600$

$\Rightarrow \quad \Sigma x_i^2 = 100 \times 1626.01 = 162601$

$\Rightarrow \quad \text{Incorrect } \Sigma x_i^2 = 162601.$

corrected Σx_i^2 = (incorrect Σx_i^2) − (sum of squares of incorrect values)

+ (sum of squares of correct values)

$= 162601 - (50)^2 + (40)^2 = 161701$

so, $\quad \text{Corrected } \sigma = \sqrt{\frac{1}{n}\Sigma x_i^2 - \left(\frac{1}{n}\Sigma x_i\right)^2} = \sqrt{\frac{161701}{100} - \left(\frac{3990}{100}\right)^2}$

$= \sqrt{1617.01 - 1592.01} = 5$

Statistics Ex 32.4 Q9

We have, $n = 20$, $\overline{x} = 10$ and $\sigma = 2$

$\therefore \quad \overline{x} = \frac{1}{n}\Sigma x_i$

$\Rightarrow \quad \Sigma x_i = n\overline{x} = 20 \times 10 = 200$

$\Rightarrow \quad \text{Incorrected } \Sigma x_i = 200$

and,

$\sigma = 2$

$\Rightarrow \quad \sigma^2 = 4$

$\Rightarrow \quad \frac{1}{n}\Sigma x_i^2 - (\text{Mean})^2 = 4$

$\Rightarrow \quad \frac{1}{20}\Sigma x_i^2 - 100 = 4$

$\Rightarrow \quad \Sigma x_i^2 = 104 \times 20$

$\Rightarrow \quad \text{Incorrected } \Sigma x_i^2 = 2080.$

(i) When 8 is omitted from the data:
If 8 is omitted from the data, then 19 observations are left.

Now, Incorrected $\Sigma x_i = 200$
$\Rightarrow$ Corrected $\Sigma x_i + 8 = 200$
$\Rightarrow$ Corrected $\Sigma x_i = 192$
and,
Incorrected $\Sigma x_i^2 = 2080$
$\Rightarrow$ Corrected $\Sigma x_i^2 + 8^2 = 2080$
$\Rightarrow$ Corrected $\Sigma x_i^2 = 2080 - 64$
$\Rightarrow$ Corrected $\Sigma x_i^2 = 2016$

$\therefore$ Corrected mean $= \frac{192}{19} = 10.10$

$\Rightarrow$ Corrected variance $= \frac{1}{19}\left(\text{Corrected } \Sigma x_i^2\right) - \left(\text{Corrected mean}\right)^2$

$\Rightarrow$ Corrected variance $= \frac{2016}{19} - \left(\frac{192}{19}\right)^2$

Corrected variance $= \frac{38304 - 36864}{361} = \frac{1440}{361}$

$\therefore$ Corrected standard deviation $= \sqrt{\frac{1440}{361}} = \frac{12\sqrt{10}}{19} = 1.997$

(ii) When the incorrect observation 8 is replaced by 12:
We have, Incorrected $\Sigma x_i = 200$

$\therefore$ Corrected $\Sigma x_i = 200 - 8 + 12 = 204$
and,
Incorrected $\Sigma x_i^2 = 2080$
$\therefore$ Corrected $\Sigma x_i^2 = 2080 - 8^2 + 12^2 = 2160$

Now, Corrected mean $= \frac{204}{20} = 10.2$

Corrected variance $= \frac{1}{20}\left(\text{Corrected } \Sigma x_i^2\right) - \left(\text{Corrected mean}\right)^2$

$\Rightarrow$ Corrected variance $= \frac{2160}{20} - \left(\frac{204}{20}\right)^2$

$\Rightarrow$ Corrected variance $= \frac{2160 \times 20 - (204)^2}{(20)^2}$

$\Rightarrow$ Corrected variance $= \frac{43200 - 41616}{400} = \frac{1584}{400}$

$\therefore$ Corrected standard deviation $= \sqrt{\frac{1584}{400}} = \frac{\sqrt{396}}{10} = \frac{19.899}{10} = 1.9899$

Statistics Ex 32.4 Q10

We have, $n = 100$, $\overline{x} = 20$ and $\sigma = 3$

Since $\overline{x} = \frac{1}{n}\Sigma x_i$

$\Rightarrow \quad \Sigma x_i = n\overline{x} = 20 \times 100 = 2000$

$\Rightarrow \quad$ Incorrect $\Sigma x_i = 2000$

and,

$\sigma = 3$

$\Rightarrow \quad \sigma^2 = 9$

$\Rightarrow \quad \frac{1}{n}\Sigma x_i^2 - (\text{Mean})^2 = 9$

$\Rightarrow \quad \frac{1}{100}\Sigma x_i^2 - 400 = 9$

$\Rightarrow \quad \Sigma x_i^2 = 409 \times 100$

$\Rightarrow \quad$ Incorrect $\Sigma x_i^2 = 40900.$

When the incorrect observations 21, 21,18 are omitted from the data:

$n = 97$

Now, Incorrect $\Sigma x_i = 2000$

$\Rightarrow \quad$ Corrected $\Sigma x_i = 2000 - 21 - 21 - 18 = 1940$

and,

Incorrect $\Sigma x_i^2 = 40900$

$\Rightarrow \quad$ Corrected $\Sigma x_i^2 = 40900 - 21^2 - 21^2 - 18^2$

$\Rightarrow \quad$ Corrected $\Sigma x_i^2 = 40900 - 1206$

$\Rightarrow \quad$ Corrected $\Sigma x_i^2 = 39694$

$\therefore \quad$ Corrected mean $= \frac{1940}{97} = 20$

$\Rightarrow \quad$ Corrected variance $= \frac{1}{97}\left(\text{Corrected } \Sigma x_i^2\right) - (\text{Corrected mean})^2$

$\Rightarrow \quad$ Corrected variance $= \frac{39694}{97} - (20)^2 = 409.22 - 400 = 9.22$

$\therefore \quad$ Corrected standard deviation $= \sqrt{9.22} = 3.04$

Statistics Ex 32.4 Q11

We have $\sum(x_i - \overline{X})^2 = \sum\left(x_i^2 - 2x_i\overline{X} + \overline{X}^2\right)$

$$= \sum\left(x_i^2\right) + \sum\left(-2x_i\overline{X}\right) + \sum\left(\overline{X}\right)^2$$

$$= \sum\left(x_i^2\right) - 2\overline{X}\sum\left(x_i\right) + \left(\overline{X}\right)^2\sum 1$$

$$= \sum\left(x_i^2\right) - 2\overline{X}\left(n\overline{X}\right) + n\left(\overline{X}\right)^2$$

$$= \sum\left(x_i^2\right) - n\left(\overline{X}\right)^2$$

Dividing both the sides by n we get,

$$\frac{1}{n}\sum(x_i - \overline{X})^2 = \frac{1}{n}\sum\left(x_i^2\right) - n\left(\overline{X}\right)^2$$

Taking square root on both the sides

$$\sqrt{\frac{1}{n}\sum(x_i - \overline{X})^2} = \sqrt{\frac{1}{n}\sum\left(x_i^2\right) - n\left(\overline{X}\right)^2}$$