RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.11

Limits Ex 29.11 Q1

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^{\lim_{n\to\infty}\left(\frac{x}{n}\right)n}$$

$$= e^x$$

Limits Ex 29.11 Q2

$$\lim_{x\to 0^+}\left\{1+\tan^2\sqrt{x}\right\}^{1/2x}$$

$$= e^{\lim_{x\to 0+}\left\{\frac{\tan^2\sqrt{x}}{2x}\right\}}$$

$$= e^{\lim_{x\to 0+}\left\{\frac{\tan^2\sqrt{x}}{2x}\right\}}$$

$$= e^{\lim_{x\to 0+}\left\{\frac{\sin^2\sqrt{x}}{2x\cos^2\sqrt{x}}\right\}}$$

$$= e^{\frac{1}{2}\lim_{x\to 0+}\left\{\left(\frac{\sin\sqrt{x}}{\sqrt{x}}\right)^2\right\}\lim_{x\to 0+}\left\{\frac{1}{\cos^2\sqrt{x}}\right\}}$$

$$= e^{\frac{1}{2}}$$

$$= \sqrt{e}$$

Limits Ex 29.11 Q3

$$\lim_{x\to 0}(\cos x)^{1/\sin x} = \lim_{x\to 0}(1+\cos x-1)^{1/\sin x}$$

$$= \lim_{x\to 0}(1-(1-\cos x))^{1/\sin x}$$

$$= \lim_{x\to 0}\left(1-2\sin^2\left(\frac{x}{2}\right)\right)^{1/\sin x}$$

$$= e^{\lim_{x\to 0}\left(-2\sin^2\left(\frac{x}{2}\right)\right)\times(1/\sin x)}$$

$$= e^{\lim_{x\to 0}\left(\frac{-2\sin^2\left(\frac{x}{2}\right)}{\sin x}\right)}$$

$$= e^{\lim_{x\to 0}\left(\frac{-2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right)}$$

$$= e^{\lim_{x\to 0} -\tan x}$$

$$= e^0$$

$$= 1$$

Limits Ex 29.11 Q4

$$\lim_{x\to 0}(\cos x + \sin x)^{1/x}$$

$$= \lim_{x\to 0}\left(1 + (\cos x + \sin x - 1)\right)^{1/x}$$

$$= e^{\lim_{x\to 0}\frac{(\cos x + \sin x - 1)}{x}}$$

$$= e^{\lim_{x\to 0}\frac{(\sin x - (1-\cos x))}{x}}$$

$$= e^{\lim_{x\to 0}\frac{\left(\sin x - 2\sin^2(x/2)\right)}{x}}$$

$$= e^{\lim_{x\to 0}\frac{\sin x}{x} - \lim_{x\to 0}\frac{2\sin(x/2)\sin(x/2)}{2\left(\frac{x}{2}\right)}}$$

$$= e^{\lim_{x\to 0}\frac{\sin x}{x} - \lim_{x\to 0}\frac{\sin(x/2)\sin(x/2)}{\left(\frac{x}{2}\right)}}$$

$$= e^{1-0}$$

$$= e$$

Limits Ex 29.11 Q5

$$\lim_{x\to 0}\left(\cos x+a\sin bx\right)^{1/x}$$

$$=\lim_{x\to 0}\left(1+(\cos x+a\sin bx-1)\right)^{1/x}$$

$$=e^{\lim_{x\to 0}\frac{(\cos x+a\sin bx-1)}{x}}$$

$$=e^{\lim_{x\to 0}\frac{(a\sin bx-(1-\cos x))}{x}}$$

$$=e^{\lim_{x\to 0}\frac{(a\sin bx-2\sin^2(x/2))}{x}}$$

$$=e^{\lim_{x\to 0}\frac{ab\sin bx}{bx}-\lim_{x\to 0}\frac{2\sin(x/2)\sin(x/2)}{2\left(\frac{x}{2}\right)}}$$

$$=e^{\lim_{x\to 0}\frac{ab\sin bx}{bx}-\lim_{x\to 0}\frac{\sin(x/2)\sin(x/2)}{\left(\frac{x}{2}\right)}}$$

$$=e^{ab-0}$$

$$=e^{ab}$$

Limits Ex 29.11 Q6

$$\lim_{x\to\infty}\left\{\frac{x^2+2x+3}{2x^2+x+5}\right\}^{\frac{3x-2}{3x+2}}$$

$$=e^{\lim_{x\to\infty}\left\{\left(\frac{3x-2}{3x+2}\right)\ln\left(\frac{x^2+2x+3}{2x^2+x+5}\right)\right\}}$$

$$=e^{\lim_{x\to\infty}\left\{\left(\frac{3-\frac{2}{x}}{3+\frac{2}{x}}\right)\left(\ln\left(\frac{x^2+2x+3}{2x^2+x+5}\right)\right)\right\}}$$

$$= e^{\lim_{x\to\infty}\left\{\left(\frac{3-\frac{2}{x}}{3+\frac{2}{x}}\right)\left(\ln\left(\frac{1+\frac{2}{x}+\frac{3}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}}\right)\right)\right\}}$$

$$= e^{1.\ln\left(\frac{1}{2}\right)}$$

$$= \frac{1}{2}$$

Limits Ex 29.11 Q7

$$\lim_{x\to 1}\left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{1-\cos(x-1)}{(x-1)^2}}$$

$$= e^{\lim_{x\to 1}\left\{\frac{1-\cos(x-1)}{(x-1)^2}\ln\left(\frac{x^3+2x^2+x+1}{x^2+2x+3}\right)\right\}}$$

$$= e^{\lim_{x\to 1}\left\{\frac{2\sin^2(x-1)}{4\left(\frac{x-1}{2}\right)^2}\ln\left(\frac{x^3+2x^2+x+1}{x^2+2x+3}\right)\right\}}$$

$$= e^{\frac{2}{4}\ln\left(\frac{5}{6}\right)}$$

$$= e^{\ln\left(\frac{5}{6}\right)^{1/2}}$$

$$= \left(\frac{5}{6}\right)^{1/2} = \sqrt{\frac{5}{6}}$$

Limits Ex 29.11 Q8

$$\lim_{x\to 0}\left\{\frac{e^x+e^{-x}-2}{x^2}\right\}^{1/x^2}$$

$$= e^{\lim_{x\to 0}\frac{\left\{\frac{e^x+e^{-x}-2}{x^2}\right\}}{x^2}}$$

Applying L'Hospital's Rule

$$= e^{\lim_{x\to 0}\left\{\frac{2+e^x(-2+x)+x}{2(-1+e^x)x^2}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{1+e^x(-1+x)}{x(-2+e^x(2+x))}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{e^x x}{-2+e^x(2+4x+x^2))}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{1+x}{6+6x+x^2}\right\}}$$

$$= e^{\frac{1}{2}\left\{\frac{\lim_{x\to 0}(1+x)}{\lim_{x\to 0}(6+6x+x^2)}\right\}}$$

$$= e^{\frac{1}{12}}$$

$$= \sqrt[12]{e}$$

Limits Ex 29.11 Q9

$$\lim_{x\to a}\left\{\frac{\sin x}{\sin a}\right\}^{\frac{1}{x-a}}$$

$$=\lim_{x\to a}\left\{1+\left(\frac{\sin x}{\sin a}-1\right)\right\}^{\frac{1}{x-a}}$$

$$=e^{\lim_{x\to a}\left\{\frac{\left(\frac{\sin x}{\sin a}-1\right)}{x-a}\right\}}$$

$$=e^{\lim_{x\to a}\left\{\frac{\left(\frac{\sin x-\sin a}{\sin a}\right)}{x-a}\right\}}$$

$$=e^{\lim_{x\to a}\left\{\left(\frac{\sin x-\sin a}{\sin a(x-a)}\right)\right\}}$$

$$=e^{\lim_{x\to a}\left\{\left(\frac{2\cos\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{\sin a(x-a)}\right)\right\}}$$

$$=e^{\lim_{x\to a}\left\{\left(\frac{2\cos\left(\frac{x+a}{2}\right)}{\sin a}\right)\right\}\lim_{x\to a}\left\{\left(\frac{\sin\left(\frac{x-a}{2}\right)}{2\left(\frac{x-a}{2}\right)}\right)\right\}}$$

$$=e^{\frac{2\cos a}{2\sin a}}$$

$$=e^{\cot a}$$

Limits Ex 29.11 Q10

$$\lim_{x\to\infty}\left\{\frac{3x^2+1}{4x^2-1}\right\}^{\frac{x^3}{1+x}}$$

$$=\lim_{x\to\infty}\left\{1+\frac{-x^2+2}{4x^2-1}\right\}^{\frac{x^3}{1+x}}$$

$$=e^{\lim_{x\to\infty}\left\{\left(\frac{-x^2+2}{4x^2-1}\right)\left(\frac{x^3}{1+x}\right)\right\}}$$

$$=e^{\lim_{x\to\infty}\left\{\left(\frac{-x^5+2x^3}{4x^2-1+4x^3-x}\right)\right\}}$$

$$=e^{-\infty}$$

$$=0$$