# RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.1

**Limits Ex 29.1 Q1**

$$\lim_{x \to 0} \frac{x}{|x|}$$

We know that $|x| = \begin{cases} x, \text{if } x \ge 0 \\ -x, \text{if } x < 0 \end{cases}$

$\therefore \quad \lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$

Also, $\lim_{x \to 0^-} \frac{x}{|x|} = \lim_{x \to 0^-} \frac{x}{-x} = \lim_{x \to 0^-} -1 = -1$

$\Rightarrow \quad$ LHL of $f(x) \ne$ RHL of $f(x)$

$\Rightarrow \quad \lim_{x \to 0} \frac{x}{|x|}$ does not exist

**Limits Ex 29.1 Q2**

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x + 3)$$
$$= 2(2) + 3$$
$$= 7$$

$\therefore \quad \lim_{x \to 2^-} f(x) = 7$

Also,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (x + k)$$
$$= (2 + k)$$

Since, $\lim_{x \to 2} f(x)$ exists (given)

$\therefore \quad \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\Rightarrow \quad 7 = 2 + k$

$\Rightarrow \quad k = 5$

**Limits Ex 29.1 Q3**

Let $f(x) = \frac{1}{x}$, this function is defined for every value of $x$ except at $x = 0$

As $x \to 0^+, \frac{1}{x} \to \infty.$

As $x \to 0^-, \frac{1}{x} \to -\infty$

$\therefore \quad \lim_{x \to 0} \frac{1}{x}$ does not exist.

**Limits Ex 29.1 Q4**

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{3x}{-x+2x} = \lim_{x\to 0^-} \frac{3x}{x} = 3 \qquad [\because \text{ as } x \to 0^-, |x| = -x]$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{3x}{x+2x} = 1 \qquad [\because \text{ as } x \to 0^+, |x| = x]$$

thus, $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

$\therefore$ $\lim_{x\to 0} f(x)$ does not exist.

**Limits Ex 29.1 Q5**

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x+1$$
$$= \lim_{h\to 0} (0+h)+1 = 1$$

Also, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x-1$

$\lim_{h\to 0} (0-h)-1 = -1$

$\Rightarrow$ $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

*Hence, limit does not exist.*

**Limits Ex 29.1 Q6**

$$\text{LHL} = \lim_{x\to 0^-} f(x)$$
$$= \lim_{h\to 0} f(0-h)$$
$$= \lim_{h\to 0} -h-4$$
$$= 0-4$$
$$= -4$$

$$\text{RHL} = \lim_{x\to 0^+} f(x)$$
$$= \lim_{h\to 0} f(0+h)$$
$$= \lim_{h\to 0} h+5$$
$$= 0+5$$
$$= 5$$

$\therefore \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Hence $\lim_{x\to 0} f(x)$ does not exist.

**Limits Ex 29.1 Q7**

$$\lim_{x\to 3^+} f(x) = 4$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x+1) = \lim_{h\to 0} (3-h+1) = 3+1 = 4$$

*Since,* $\lim_{x\to 3^+} f(x) = 4 = \lim_{x\to 3^-} f(x)$

$\therefore$ $\lim_{x\to 3} f(x)$ is 4

**Limits Ex 29.1 Q8**

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 3(x+1) = 3$$

and

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2(x)+3 = 3$$

$$\therefore \qquad \lim_{x\to 0} f(x) = 3$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} 3(x+1) = 3+3 = 6$$

$$\therefore \qquad \lim_{x\to 1} f(x) = 6$$

**Limits Ex 29.1 Q9**

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 - 1 = \lim_{h\to 0} (-1-h)^2 - 1 = 1-1 = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -x^2 - 1 = \lim_{h\to 0} -(1+h)^2 - 1 = -2$$

Since, $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

$\therefore \lim_{x\to 1} f(x)$ does not exist.

**Limits Ex 29.1 Q10**

$$\text{LHL} = \lim_{x\to 0^-} f(x)$$

$$\Rightarrow \qquad \lim_{x\to 0^-} \frac{|x|}{x}$$

$$\Rightarrow \qquad \lim_{h\to 0} \frac{|0-h|}{0-h}$$

$$\Rightarrow \qquad \lim_{h\to 0} \frac{+h}{-h} = -1 \qquad \text{---(i)}$$

and,

$$\text{RHL} = \lim_{x\to 0^+} f(x)$$

$$\Rightarrow \qquad \lim_{x\to 0^+} \frac{|x|}{x}$$

$$\Rightarrow \qquad \lim_{h\to 0} \frac{|0+h|}{0+h} = \lim_{h\to 0} \frac{h}{h} = 1 \qquad \text{---(ii)}$$

So, LHL $\neq$ RHL

$\therefore \qquad \lim_{x\to 0} f(x)$ does not exist.

**Limits Ex 29.1 Q11**

$$\lim_{x\to a_1} f(x)$$

$$\Rightarrow \lim_{x\to a_1} (x-a_1)(x-a_2)\ldots(x-a_n) \qquad [\text{Putting limit } x\to a_1]$$

$$\Rightarrow (a_1-a_1)(a_1-a_2)\ldots(a_1-a_n)$$

$$\Rightarrow 0$$

And, $\lim_{x\to a} f(x)$

$$\Rightarrow \lim_{x\to a} (x-a_1)(x-a_2)\ldots(x-a_n) \qquad [\text{Putting limit } x\to a]$$

$$\Rightarrow (a-a_1)(a-a_2)\ldots(a-a_n).$$

**Limits Ex 29.1 Q12**

$$\lim_{x\to 1^+}\frac{1}{(x-1)} = \lim_{h\to 0}\frac{1}{(1+h-1)} = \lim_{h\to 0}\frac{1}{h} = \infty$$

**Limits Ex 29.1 Q13(i)**

$$\lim_{x\to 2^+}\frac{x-3}{x^2-4}$$

$$= \lim_{h\to 0}\frac{(2+h)-3}{(2+h)^2-2^2} \qquad \left[\because \lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h)\right]$$

$$= \lim_{h\to 0}\frac{(2-3+h)}{(2+h-2)(2+h+2)}$$

$$= \lim_{h\to 0}\frac{(h-1)}{(h)(4+h)}$$

$$= \lim_{h\to 0}\frac{1-\frac{1}{h}}{4+h}$$

$$= \frac{1-\frac{1}{0}}{4} = -\infty$$

**Limits Ex 29.1 Q13(ii)**

$$\lim_{x\to 2^-}\frac{x-3}{(x^2-4)} = \lim_{h\to 0}\frac{(2-h)-3}{(2-h)^2-4}$$

$$= \lim_{h\to 0}\frac{(2-h-3)}{(2-h+2)(2-h-2)} \qquad \left[\because \lim_{x\to 2^-} f(x) = \lim_{h\to 0} f(2-h)\right]$$

$$= \lim_{h\to 0}\frac{-1-h}{(4-h)(-h)}$$

$$= \lim_{h\to 0}\frac{\frac{1}{h}+1}{(4-h)}$$

$$= \frac{\frac{1}{0}+1}{4} = \infty$$

**Limits Ex 29.1 Q13(iii)**

$$\lim_{x\to 0^+} \frac{1}{3x}$$

$$= \lim_{h\to 0} \frac{1}{3(0+h)}$$

$$= \lim_{h\to 0} \frac{1}{0+3h}$$

$$= \frac{1}{0} = \infty$$

**Limits Ex 29.1 Q13(iv)**

$$\lim_{x\to -8^+} \frac{2x}{x+8}$$

$$= \lim_{h\to 0} \frac{2(-8+h)}{(-8+h)+8}$$

$$= \lim_{h\to 0} \frac{-16+2h}{h}$$

$$= \lim_{h\to 0} \frac{-16}{h} + 2$$

$$\Rightarrow \quad \frac{-16}{0} + 2 = \infty$$

**Limits Ex 29.1 Q13(v)**

$$\lim_{x\to 0^+} \frac{2}{x^{\frac{1}{5}}}$$

$$= \lim_{h\to 0} \frac{2}{(0+h)^{\frac{1}{5}}}$$

$$\Rightarrow \frac{2}{0} = \infty$$

**Limits Ex 29.1 Q13(vi)**

$$\lim_{x\to \frac{\pi}{2}^-} \tan x$$

$$= \lim_{h\to 0} \tan\left(\frac{\pi}{2} - h\right)$$

$$= \tan\left(\frac{\pi}{2} - 0\right)$$

$$\Rightarrow \quad \tan\frac{\pi}{2} = \infty$$

**Limits Ex 29.1 Q13(vii)**

$$\lim_{x\to -\frac{\pi}{2}^+} \sec x$$

$$= \lim_{h\to 0} \sec\left(-\frac{\pi}{2}+h\right)$$

$$= \sec\left(-\frac{\pi}{2}+0\right)$$

$$= \sec\left(-\frac{\pi}{2}\right)$$

$$= \frac{1}{\cos\left(-\frac{\pi}{2}\right)}$$

$$= \frac{-1}{\left(\cos\frac{\pi}{2}\right)}$$

$$= \frac{-1}{0} = -\infty$$

**Limits Ex 29.1 Q13(viii)**

$$\lim_{x\to 0^-} \frac{x^2-3x+2}{x^3-2x^2}$$

$$= \lim_{x\to 0^-} \frac{x^2-x-2x+2}{x^2(x-2)}$$

$$= \lim_{x\to 0^-} \frac{x(x-1)-2(x-1)}{x^2(x-2)}$$

$$= \lim_{x\to 0^-} \frac{(x-1)(x-2)}{x^2(x-2)}$$

$$= \lim_{x\to 0^-} \frac{(x-1)}{x^2}$$

$$= \lim_{h\to 0} \frac{(0-h-1)}{(0-h)^2}$$

$$= \frac{-h}{h^2} = \frac{-1}{h} = \frac{-1}{0} = -\infty$$

**Limits Ex 29.1 Q13(ix)**

$$\lim_{x\to -2^+} \frac{x^2-1}{2x+4}$$

$$= \lim_{x\to 2^+} \frac{(x-1)(x+1)}{2(x+2)}$$

$$= \lim_{h\to 0} \frac{(-2+h-1)(-2+h+1)}{2(-2+h+2)}$$

$$= \lim_{h\to 0} \frac{(-3+h)(h-1)}{2h}$$

$$\Rightarrow \quad \frac{-3\times -1}{2\times 0} = \frac{1}{0} = \infty$$

**Limits Ex 29.1 Q13(x)**

$$\lim_{x\to 0^-} 2-\cot x$$

$$= \lim_{h\to 0} 2-\cot(0-h)$$

$$= \lim_{h\to 0} 2-(-1)\coth$$

$$= \lim_{h\to 0} 2+\coth$$

$$= \lim_{h\to 0} 2+\frac{1}{\tanh}$$

$$\Rightarrow \quad 2+\frac{1}{0} \Leftarrow \infty$$

**Limits Ex 29.1 Q13(xi)**

$$\lim_{x\to 0^-} 1+\operatorname{cosec}x$$

$$= \lim_{x\to 0^-} 1+\operatorname{cosec}(0-h)$$

$$= \lim_{h\to 0} 1-\operatorname{cosec}h$$

$$= \lim_{h\to 0} 1-\frac{1}{\sinh}$$

$$\Rightarrow \quad 1-\frac{1}{0} = -\infty$$

**Limits Ex 29.1 Q14**

$$\lim_{x\to 0} e^{\frac{-1}{x}}$$

$$\lim_{x\to 0^+} e^{\frac{-1}{x}} = \lim_{x\to 0^+} \frac{1}{e^{\frac{1}{x}}} = \lim_{h\to 0} \frac{1}{e^{\frac{1}{0+h}}} = \frac{1}{e^{\frac{1}{0}}} = \frac{1}{e^{\infty}} \quad = \frac{1}{\infty} = 0$$

$$And, \lim_{x\to 0^-} e^{\frac{-1}{x}} = \lim_{x\to 0^-} \frac{1}{e^{\frac{1}{x}}} = \lim_{h\to 0} \frac{1}{e^{\frac{1}{0-h}}} = \lim_{h\to 0} \frac{1}{e^{-\frac{1}{h}}} = \frac{1}{e^{-\frac{1}{0}}} = \frac{1}{e^{-\infty}} = e^{\infty} = \infty$$

$$\Rightarrow \quad \lim_{x\to 0^+} e^{\frac{-1}{x}} \neq \lim_{x\to 0^-} e^{\frac{-1}{x}}$$

$$\therefore \quad \lim_{x\to 0} e^{\frac{-1}{x}} \text{ does not exist .}$$

**Limits Ex 29.1 Q15**

(i) $\lim_{x\to 2}[x]$

$$\lim_{x\to 2^-}[x]=1$$
$$\lim_{x\to 2^+}[x]=2$$

Thus, $\lim_{x\to 2}[x]$ does not exist.

(ii) $\lim_{x\to \frac{5}{2}}[x]$

$$\lim_{x\to \frac{5}{2}^+}[x]=2$$
$$\lim_{x\to \frac{5}{2}^-}[x]=2$$

$$\Rightarrow \quad \lim_{x\to \frac{5}{2}}[x]=2$$

(iii) $\lim_{x\to 1}[x]$

$$\lim_{x\to 1^-}[x]=0$$
$$\lim_{x\to 1^+}[x]=1$$

$$\Rightarrow \quad \lim_{x\to 1^-}[x]\neq \lim_{x\to 1^+}[x]$$

Thus, $\lim_{x\to 1}[x]$ does not exist

**Limits Ex 29.1 Q16**

$$\lim_{x\to a^+}[x]$$
$$\Rightarrow \quad \lim_{h\to 0^-}[a+h]=[a]$$
$$\Rightarrow \quad \lim_{h\to 0^-}[x]=[a]\forall a\in R$$
Also, $\lim_{x\to 1^-}[x]$
$$=\lim_{h\to 0}[1-h]$$
$$=0$$
$$\Rightarrow \quad \lim_{x\to 1^-}[x]=0$$

**Limits Ex 29.1 Q17**

$$\lim_{x\to 2^-}\frac{x}{[x]} = \lim_{x\to 2^-}\frac{x}{1}=\frac{2}{1}=2 \qquad \left[\because \lim_{x\to k^-}[x]=k-1\right]$$

$$\text{Also, } \lim_{x\to 2^+}\frac{x}{[x]} = \lim_{x\to 2^+}\frac{x}{3}=\frac{2}{3} \qquad \left[\lim_{x\to k^+}[x]=k+1\right]$$

$$\Rightarrow \quad \lim_{x\to 2^-}\frac{x}{[x]}\neq \lim_{x\to 2^+}\frac{x}{[x]}$$

$$\lim_{x\to 3^+}\frac{x}{[x]} = \lim_{x\to 3^+}\frac{x}{3} = \frac{3}{3} = 1$$

$$\lim_{x\to 3^-}\frac{x}{[x]} = \lim_{x\to 3^-}\frac{x}{2} = \frac{3}{2} = 1.5$$

Therefore, $\lim_{x\to 3^+}\frac{x}{[x]} \neq \lim_{x\to 3^-}\frac{x}{[x]}$

**Limits Ex 29.1 Q19**

$$\lim_{x\to \frac{5}{2}}[x]$$

$$\lim_{x\to \frac{5}{2}}[x] = \left[\frac{5}{2}\right],$$

$$= [2.5] = 2 \qquad \text{[By definition of greatest integer function]}$$

$$\Rightarrow \quad \lim_{x\to \frac{5}{2}}[x] = 2$$

**Limits Ex 29.1 Q20**

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(x - [x])$$

$$= \lim_{x\to 2^-} x - \lim_{x\to 2^-}[x]$$

$$= 2 - 1 = 1 \qquad \left[\because \lim_{x\to k^-}[x] = k - 1\right]$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(3x - 5)$$

$$= 3(2) - 5 \qquad [\because x > 2]$$

$$= 6 - 5$$

$$= 1$$

Thus, $\lim_{x\to 2^-} f(x) = 1 = \lim_{x\to 2^+} f(x)$

$$\Rightarrow \quad \lim_{x\to 2} f(x) = 1$$

**Limits Ex 29.1 Q21**

$$\lim_{x\to 0^-}\sin\frac{1}{x} = \lim_{h\to 0}\sin\frac{1}{0-h} = -\lim_{h\to 0}\sin\frac{1}{h}$$

$= -$(An oscillating number which oscillates between $-1$ and $1$).

So, $\lim_{x\to 0^-}\sin\frac{1}{x}$ does not exist.

Similarly, $\lim_{x\to 0^+}\sin\frac{1}{x}$ does not exist.

$\lim_{x\to 0}\sin\frac{1}{x}$ does not exist.

**Limits Ex 29.1 Q22**

Let $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{where } x\neq\dfrac{\pi}{2}\\ 3, & \text{where } x=\dfrac{\pi}{2}\end{cases}$ and if $\lim\limits_{x\to\pi/2} f(x)=f\left(\dfrac{\pi}{2}\right)$, find the value of k.

$$\text{LHL}=\lim_{x\to\frac{\pi}{2}^-} f(x)$$

$$=\lim_{h\to\frac{\pi}{2}} f\left(\frac{\pi}{2}-h\right)$$

$$=\lim_{h\to\frac{\pi}{2}}\frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)}$$

$$=\lim_{h\to\frac{\pi}{2}}\frac{k\cos\left(\frac{\pi}{2}-h\right)}{2h}$$

$$=\frac{k\cos\left(\frac{\pi}{2}-\frac{\pi}{2}\right)}{\pi}$$

$$=\frac{k}{\pi}$$

$$\text{RHL}=\lim_{x\to\frac{\pi}{2}^+} f(x)$$

$$=\lim_{h\to\frac{\pi}{2}} f\left(\frac{\pi}{2}+h\right)$$

$$=\lim_{h\to\frac{\pi}{2}}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}$$

$$=\lim_{h\to\frac{\pi}{2}}\frac{k\cos\left(\frac{\pi}{2}+h\right)}{-2h}$$

$$=\frac{k\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)}{-\pi}$$

$$=\frac{k}{\pi}$$

$$\therefore \lim_{x\to\frac{\pi}{2}^-} f(x)=\lim_{x\to\frac{\pi}{2}^+} f(x)$$

Hence $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ exists.

$$\lim_{x\to\frac{\pi}{2}} f(x)=f\left(\frac{\pi}{2}\right)$$

$$\frac{k}{\pi}=3$$

$$k=3\pi$$