RD Sharma Solutions Class 11 Maths Chapter 28 Ex 28.3

Introduction to 3D Coordinate Geometry 28.3 Q1

We know that angle bisector divides opposite side in ratio of other two sides

$\Rightarrow$ D divides BC in ratio of AB : AC

A(5, 4, 6), B(1, -1, 3) and C(4, 3, 2)

$AB=\sqrt{16+25+9} = \sqrt{50} = 5\sqrt{2}$

$AC=\sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

AB:AC=5:3=m:n

$D(x,y,z)=\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right)$

Substitute values for m:n=5:3,

$(x_1, y_1, z_1)=(1, -1, 3)$

$(x_2, y_2, z_2)=(4, 3, 2)$

$D=\left(\frac{23}{8}, \frac{3}{2}, \frac{19}{8}\right)$

Introduction to 3D Coordinate Geometry 28.3 Q2

z-coordinate 8

A(2, -3, 4) and B(8, 0, 10)

DR's of AB=(6, 3, 6)

DR's of BC= (x-8, y-0, 8-10)

Given A, B, C lie on same line

So values of DR's should be proportional

$\frac{x-8}{6} = \frac{y}{3} = \frac{8-10}{6}$

$So\ x = 6, y = -1$

point is (6, -1, 8)

Introduction to 3D Coordinate Geometry 28.3 Q3

If points are collinear then all points lie on same line

and DR's should be proportional

A(2, 3, 4), B(-1, 2, -3) and C(-4, 1, -10)

DR's of AB=(3, 1, 7)

DR's of BC=(3, 1, 7)

So A, B , C are collinear

Length of $AC=\sqrt{36+4+196} = \sqrt{236}$

Length of $AB=\sqrt{9+1+49} = \sqrt{59}$

Ratio is AC:AB=2:1

So C divides AB in ratio 2:1 externally

Introduction to 3D Coordinate Geometry 28.3 Q4

yz plane means x=0

Given (2, 4, 5) and (3, 5, 4)

assume ratio to be m:n

lets equate x-term

lets equate x term

$$0=\frac{3m+2n}{m+n}$$

$3m=-2n$

$m:n=-2:3$

which means yz plane divides the line in 2:3 ratio externally

Introduction to 3D Coordinate Geometry 28.3 Q5

(2, -1, 3) and (-1, 2, 1)

$x+y+z=5$

Assume plane divides line in ratio λ:1

so point P which is diving line in $\lambda:1$ ratio is

$$P=\left(\frac{-\lambda+2}{\lambda+1},\frac{2\lambda-1}{\lambda+1},\frac{\lambda+3}{\lambda+1}\right)$$

P lies on plane x+y+z=5

$-\lambda+2+2\lambda-1+\lambda+3=5\lambda+5$

$3\lambda=-1\Rightarrow\lambda=-1:3$

So plane diving line in 1:3 ratio externally

Introduction to 3D Coordinate Geometry 28.3 Q6

A(3, 2, -4), B(9, 8, -10) and C(5, 4, -6)

$AC=\sqrt{4+4+4}=2\sqrt{3}$

$AB=\sqrt{36+36+36}=6\sqrt{3}$

$BC=\sqrt{16+16+16}=4\sqrt{3}$

$AC:BC=1:2$

Introduction to 3D Coordinate Geometry 28.3 Q7

Given midpoints D(-2, 3, 5), E(4, -1, 7) and F(6, 5, 3)

Assume D is midpoint of AB, E is midpoint of BC

F is midpoint of CA

$A(x_1, y_1, z_1)$ $B(x_2, y_2, z_2)$ $C(x_3, y_3, z_3)$

From midpoint formula, we get following equations

$x_1+x_2=-4;\ x_2+x_3=8;\ x_3+x_1=12$

$y_1+y_2=6;\ y_2+y_3=-2;\ y_3+y_1=10$

$z_1+z_2=10;\ z_2+z_3=14;\ z_3+z_1=6$

Solving above set of equations we get

A=(0, 9, 1)

B=(-4, -3, 9)

C=(12, 1, 5)

Introduction to 3D Coordinate Geometry 28.3 Q8

A(1, 2, 3), B(0, 4, 1), C(-1, -1, -3)

Angle bisctor at A divides BC in ratio of AB:AC

$AB=\sqrt{1+4+4}=3$

$AC=\sqrt{4+9+36}=7$

Assume D divides BC

m:n=3:7

so $D=\left(\frac{-3}{10},\frac{25}{10},\frac{-2}{10}\right)$

Introduction to 3D Coordinate Geometry 28.3 Q9

(12, -4, 8) and (27, -9, 18)

Assume point P is dividing line in $\lambda:1$ ratio, we get

$$P=\left(\frac{27\lambda+12}{\lambda+1},\frac{-9\lambda-4}{\lambda+1},\frac{18\lambda+8}{\lambda+1}\right)$$

P lies on Sphere, so substitute in Sphere equation

$$x^2+y^2+z^2=504$$

$$9(9\lambda+4)^2+(9\lambda+4)^2+4(9\lambda+4)^2=504(\lambda+1)^2$$

$$729\lambda^2+81\lambda^2+324\lambda^2+648\lambda+72\lambda+288\lambda+144+16+64=504\lambda^2+1008\lambda+504$$

$$(1134-504)\lambda^2+(1008-1008)\lambda+224-504=0$$

$$630\lambda^2=280$$

$$\lambda^2=\frac{4}{9}$$

$$\lambda=2:3$$

Introduction to 3D Coordinate Geometry 28.3 Q10

Assume ratio is $\lambda:1$

Plane is $ax+by+cz+d=0$

points (x_1,y_1,z_1) and (x_2,y_2,z_2)

Assume point of intersection of line and plane is D

$$D=\left(\frac{\lambda x_2+x_1}{\lambda+1},\frac{\lambda y_2+y_1}{\lambda+1},\frac{\lambda z_2+z_1}{\lambda+1}\right)$$

As D lies on plane, substitute D in plane equation, we get

$$\lambda(ax_2+by_2+cz_2+d)+ax_1+by_1+cz_1+d=0$$

$$\Rightarrow\lambda=-\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$$

Introduction to 3D Coordinate Geometry 28.3 Q11

(1, 2, -3), (3, 0, 1) and (-1, 1, -4)

Centroid of Traingle is given by

$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

We know that

$x_1+x_2=2$

$x_2+x_3=6$

$x_1+x_3=-2$

Adding all gives $\Rightarrow 2(x_1+x_2+x_3)=6$

so $x_1+x_2+x_3=3$

similarly, $y_1+y_2+y_3=3; z_1+z_2+z_3=-6$

$Centroid=(1,1,-2)$

Introduction to 3D Coordinate Geometry 28.3 Q12

Given Centroid (1, 1, 1)

A(3, -5, 7) and B(-1, 7, -6)

Equating terms, we get

$$1=\frac{3-1+x_3}{3}$$

$$1=\frac{-5+7+y_3}{3}$$

$$1=\frac{7-6+z_3}{3}$$

$(x_3, y_3, z_3)=(1,1,2)$

Introduction to 3D Coordinate Geometry 28.3 Q13

Trisection points are those which divide line in ratio 1:2 or 2:1

P(4, 2, -6) and Q(10, -16, 6)

Consider 1:2 case, we get

$$\left(\frac{10+8}{3}, \frac{-16+4}{3}, \frac{6-12}{3}\right) = (6, -4, -2)$$

Consider 2:1 case, we get

$$\left(\frac{20+4}{3}, \frac{-32+2}{3}, \frac{12-6}{3}\right) = (8, -10, 2)$$

$(6, -4, -2)$ and $(8, -10, 2)$ are trisection points

Introduction to 3D Coordinate Geometry 28.3 Q14

A(2, -3, 4), B(-1, 2, 1) and C(0, 1/3, 2)

DR's of AB are (3, -5, 3)

DR's of BC are $(-1, \frac{5}{3}, -1)$

DR's of AC are $(2, \frac{-10}{3}, 2)$

Its clear that all DR's are proportional

Introduction to 3D Coordinate Geometry 28.3 Q15

P(3, 2, -4), Q(5, 4, -6) and R(9, 8, -10)

$PQ = \sqrt{4+4+4} = 2\sqrt{3}$

$QR = \sqrt{16+16+16} = 4\sqrt{3}$

$PQ : QR = 1 : 2$