RD Sharma Solutions Class 11 Maths Chapter 28 Ex 28.2

Introduction to 3D Coordinate Geometry Ex 28.2 Q1

(i) Distance between points P and Q

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(1-2)^2 + (-1-1)^2 + (0-2)^2}$$

$$= \sqrt{(-1)^2 + (-2)^2 + (-2)^2}$$

$$= \sqrt{1+4+4}$$

$$PQ = 3 \text{ units}$$

(ii) Distance between points A and B

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 + -z_2)^2}$$

$$= \sqrt{(3+1)^2 + (2+1)^2 + (-1+1)^2}$$

$$= \sqrt{(4)^2 + (3)^2 + (0)^2}$$

$$= \sqrt{16+9+0}$$

$$= \sqrt{25}$$

$$AB = 5 \text{ units}$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q2

Distance between points P and Q

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$= \sqrt{(-2-2)^2 + (3-1)^2 + (1-2)^2}$$

$$= \sqrt{(-4)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{16+4+1}$$

$$PQ = \sqrt{21} \text{ units}$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q3(i)

$A(4,-3,-1)$, $B(5,-7,6)$ and $C(3,1,-8)$

$$AB = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$= \sqrt{(4-5)^2+(-3+7)^2+(-1-6)^2}$$

$$= \sqrt{(-1)^2+(4)^2+(-7)^2}$$

$$= \sqrt{1+16+49}$$

$$= \sqrt{66} \text{ units}$$

$$BC = \sqrt{(5-3)^2+(-7-1)^2+(6+8)^2}$$

$$= \sqrt{(2)^2+(-8)^2+(14)^2}$$

$$= \sqrt{4+64+196}$$

$$= \sqrt{264}$$

$$= 2\sqrt{66} \text{ units}$$

$$AC = \sqrt{(4-3)^2+(-3-1)^2+(-1+8)^2}$$

$$= \sqrt{(1)^2+(-4)^2+(7)^2}$$

$$= \sqrt{1+16+49}$$

$$= \sqrt{66} \text{ units}$$

Since $AC + AB = BC$
so, A, B, C are collinear.

Introduction to 3D Coordinate Geometry Ex 28.2 Q3(ii)

$P(0,7,-7),\ Q(1,4,-5),\ R(-1,10,-9)$

$$PQ = \sqrt{(0-1)^2 + (7-4)^2 + (-7+5)^2}$$

$$= \sqrt{(1)^2 + (3)^2 + (-2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

$$QR = \sqrt{(1+1)^2 + (4-10)^2 + (-5+9)^2}$$

$$= \sqrt{(2)^2 + (-6)^2 + (4)^2}$$

$$= \sqrt{4+36+16}$$

$$= 2\sqrt{14} \text{ units}$$

$$PR = \sqrt{(0+1)^2 + (7-10)^2 + (-7+9)^2}$$

$$= \sqrt{1^2 + (-3)^2 + (2)^2}$$

$$= \sqrt{1+9+4}$$

$$= \sqrt{14} \text{ units}$$

Since $PQ + PR = QR$
so, P, Q, R are collinear

Introduction to 3D Coordinate Geometry Ex 28.2 Q3(iii)

$A(3,-5,1)$, $B(-1,0,8)$, and $C(7,-10,-6)$

$$AB = \sqrt{(3+1)^2 + (-5-0)^2 + (1-8)^2}$$

$$= \sqrt{(4)^2 + (-5)^2 + (-7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

$$BC = \sqrt{(-1-7)^2 + (0+10)^2 + (8+6)^2}$$

$$= \sqrt{(-8)^2 + (10)^2 + (14)^2}$$

$$= \sqrt{64+100+196}$$

$$= \sqrt{360}$$

$$= 6\sqrt{10} \text{ units}$$

$$CA = \sqrt{(3-7)^2 + (-5+10)^2 + (1+6)^2}$$

$$= \sqrt{(-4)^2 + (5)^2 + (7)^2}$$

$$= \sqrt{16+25+49}$$

$$= \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

Since $AB + AC = BC$
so, $A, B,$ and C are collinear

Introduction to 3D Coordinate Geometry Ex 28.2 Q4(i)

Let the point on xy – plane be $P(x, y, 0)$.

Now P is equidistance from $A(1, -1, 0)$, $B(2, 1, 2)$ and $C(3, 2, -1)$.

So, $AP = BP = CP$

Now,

$(AP)^2 = (x-1)^2 + (y+1)^2 + (0-0)^2$
$(BP)^2 = (x-2)^2 + (y-1)^2 + (0-2)^2$
$(CP)^2 = (x-3)^2 + (y-2)^2 + (0+1)^2$

$(AP)^2 = (BP)^2 \Rightarrow (x-1)^2 + (y+1)^2 = (x-2)^2 + (y-1)^2 + 4$

$\Rightarrow x^2 + 1 - 2 + y^2 + 1 + 2y + z^2 = x^2 + 4 - 4x + y^2 + 1 - 2y + 4$

$\Rightarrow 2x + 4y = 7 \quad ...(1)$

$(BP)^2 = (CP)^2 \Rightarrow (x-2)^2 + (y-1)^2 + 4 = (x-3)^2 + (y-2)^2 + 1$
$\Rightarrow x^2 + 4 - 4x + y^2 + 1 - 2y + z^2 + 4 = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$
$\Rightarrow 2x + 2y = 5 \quad ...(2)$

$(AP)^2 = (CP)^2 \Rightarrow (x-1)^2 + (y+1)^2 = (x-3)^2 + (y-2)^2 + 1$
$\Rightarrow x^2 + 1 - 2x + y^2 + 1 + 2y = x^2 + 9 - 6x + y^2 + 4 - 4y + 1$
$\Rightarrow 4x + 6y = 12 \quad ...(3)$

Solving equation (1) and (2) we get

$y = 1, \quad x = 3/2$

Put x and y in equation (3)

$4(3/2) + 6(1) = 12$

$12 = 12$

So, the required point is $(3/2, \ 1, 0)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q4(ii)

Let $Q(0, y, z)$ be the required point.

So

$(AQ)^2 = (BQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-2) + (y-1)^2 + (z-2)^2$

$\Rightarrow \quad 1 + y^2 + 1 + 2y + z^2 = 4 + y^2 + 1 - 2y + z^2 + 4 - 42$

$\Rightarrow \quad 4y + 4z = 7 \ldots (1)$

$(BQ)^2 = (CQ)^2 \Rightarrow (0-z)^2 + (y-1)^2 + (z-2)^2 = (0-3)^2 + (y-2)^2 + (2+1)^2$

$\Rightarrow \quad 4 + y^2 + 1 - 2y + z^2 + 4 - 4z = 9 + y^2 + 4 - 4y + z^2 + 1 + 2z$

$\Rightarrow \quad 2y - 6z = 5 \ \ldots (2)$

$(AQ)^2 = (CQ)^2 \Rightarrow (0-1)^2 + (y+1)^2 + (z-0)^2 = (0-3)^2 + (y-2)^2 + (z+1)^2$

$\Rightarrow \quad 1 + y^2 + 2y + 1 + z^2 = 9 + y^2 - 4y + 4 + z^2 + 1 + 2z$

$\Rightarrow \quad 6y - 2z = 12 \ \ldots (3)$

Solving equation (1) and (2), we get

$$z = \frac{-3}{16} \text{ and } y = \frac{31}{16}$$

Put the value of y and z is equation (3)

$6y - 2z = 12 = 12$

$$6\left(\frac{31}{16}\right) - 2\left(\frac{-3}{16}\right) = 12$$

$$\frac{186}{16} + \frac{6}{16} = 12$$

$$\frac{192}{16} = 12$$

$12 = 12$

LHS = RHS.

so,

$$y = \frac{31}{16}, z = \frac{13}{16}$$

$$\text{Required point} = \left(0, \frac{31}{16}, \frac{-3}{16}\right)$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q4(iii)

Let $R(x, 0, z)$ be the required point.

So

$(AR)^2 = (BR)^2 \Rightarrow (1-x)^2 + (-1-0)^2 + (0-z)^2 = (2-x) + (1-0)^2 + (2-z)^2$

$\Rightarrow \quad 1 + x^2 - 2x + 1 + z^2 = 4 + x^2 - 4x + 1 + 4 + z^2 - 4z$

$\Rightarrow \quad 2x + 4z = 7 \quad ...(1)$

$(BR)^2 = (CR)^2 \Rightarrow (z-z)^2 + (1-0)^2 + (2-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2$

$\Rightarrow \quad 4 + x^2 - 4x + 4 + z^2 - 4z = 9 + x^2 - 6x + 4 + 1 + z^2 + 2z$

$\Rightarrow \quad 2x - 6z = 5 \quad ...(2)$

$(AR)^2 = (CR)^2 \Rightarrow (1-x)^2 + (1-0)^2 + (0-z)^2 = (3-x)^2 + (2-0)^2 + (-1-z)^2$

$\Rightarrow \quad 1 + x^2 - 2x + 1 + z^2 = 9 + 6x + 4 + 1 + z^2 + 2z$

$\Rightarrow \quad 4x - 2z = 12 \quad ...(3)$

Solving equation (1) and (2), we get

$$z = \frac{1}{5}, \quad x = \frac{31}{10}$$

Put the value of x and z is equation (3)

$4x - 2z = 12$

$$4\left(\frac{31}{10}\right) - 2\left(\frac{1}{5}\right) = 12$$

$$\frac{124}{10} - \frac{2}{10} = 12$$

$$\frac{120}{10} = 12$$

$12 = 12$

LHS = RHS.

so,

$$x = \frac{31}{10}, z = \frac{1}{5}$$

Required point $= \left(\frac{31}{10}, 0, \frac{1}{5}\right)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q5

Let $P(0, 0, z)$ be the point equidistant from $Q(1, 5, 7)$ and $R(5, 1, -4)$.

So,

$(PQ)^2 = (PR)^2 \Rightarrow (0-1)^2 + (0-5)^2 + (z-7)^2 = (0-5)^2 + (0-1)^2 + (z+4)^2$

$\Rightarrow 1 + 25 + (z-7)^2 = 25 + 1 + (z+4)^2$

$\Rightarrow 26 + z^2 + 49 - 14z = 26 + z^2 + 8z + 16$

$\Rightarrow \quad -14z - 8z = 16 - 49$

$\Rightarrow \quad -22z = -33$

$\Rightarrow \quad z = \frac{-33}{-22}$

$\Rightarrow \quad z = \frac{3}{2}$

Required point $= (0, 0, 3/2)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q6

Let $P(0, y, 0)$ be a point on y-axis which is equidistant from $Q(3, 1, 2)$ and $R(5, 5, 2)$.

So,

$(PR)^2 = (PQ)^2 \Rightarrow (0-5)^2 + (y-5)^2 + (0-2)^2 = (0-3)^2 + (y-1)^2 + (0-2)^2$

$\Rightarrow 25 + y^2 + 25 - 10y + 4 = 9 + y^2 + 1 - 2y + 4$

$\Rightarrow \quad -10y + 2y = 14 - 54$

$\Rightarrow \quad -14z - 8z = 16 - 49$

$\Rightarrow \quad -8y = -40$

$\Rightarrow \quad y = 5$

So, the required point is $(0, 5, 0)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q7

Let $P(0, 0, z)$ be at a distance of $\sqrt{21}$ from $Q(1, 2, 3)$.

So

$PQ = \sqrt{(0-1)^2 + (0-2)^2 + (z-3)^2}$

$\sqrt{21} = \sqrt{(1)^2 + (2)^2 + (z-3)^2}$

$21 - 5 = (z-3)^2$

$16 = (z-3)^2$

$z - 3 = \pm 4$

$z = 7$ and $z = -1$

So, the required points are $(0, 0, 7)$ and $(0, 0, -1)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q8

Let the triangle formed be $\triangle ABC$

$$(AB) = \sqrt{(1-2)^2 + (2-3)^2 + (3-1)^2}$$
$$= \sqrt{(-1)^2 + (-1)^2 + (2)^2}$$
$$= \sqrt{6} \text{ units}$$

$$BC = \sqrt{(2-3)^2 + (3-1)^2 + (1-2)^2}$$
$$= \sqrt{(-1)^2 + (2)^2 + (-1)^2}$$
$$= \sqrt{6} \text{ units}$$

$$AC = \sqrt{(1-3)^2 + (2-1)^2 + (3-2)^2}$$
$$= \sqrt{(-2)^2 + (1)^2 + (1)^2}$$
$$= \sqrt{6} \text{ units}$$

since, $AB = BC = CA$

So, $\triangle ABC$ is an equilateral $\triangle$

Introduction to 3D Coordinate Geometry Ex 28.2 Q9

Let $A=(0,7,10)$, $B=(-1,6,6)$ and $C=(-4,9,6)$

$$AB=\sqrt{(0+1)^2+(7-6)^2+(10-6)^2}$$
$$=\sqrt{(1)^2+(1)^2+(4)^2}$$
$$=\sqrt{18}$$
$$=3\sqrt{2} \text{ units}$$

$$BC=\sqrt{(-1+4)^2+(6-9)^2+(6-6)^2}$$
$$=\sqrt{(3)^2+(3)^2+0}$$
$$=\sqrt{18}$$
$$=3\sqrt{2} \text{ units}$$

$$AC=\sqrt{(0+4)^2+(7-9)^2+(10-6)^2}$$
$$=\sqrt{(4)^2+(-2)^2+(4)^2}$$
$$=\sqrt{36}$$
$$=6 \text{ units}$$

$$(AB)^2+(BC)^2$$
$$=(3\sqrt{2})^2+(3\sqrt{2})^2$$
$$=18+18$$
$$=36$$
$$=(AC)^2$$

Also $l(AB)=l(BC)$

Hence $(0,7,10)$, $(-1,6,6)$ and $(-4,9,6)$ are the vertices of an isosceles right-angled triangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q10

Here points are $A(3,3,3)$, $B(0,6,3)$, $C(1,7,7)$ and $D(4,4,7)$.

$$AB = \sqrt{(3-0)^2 + (3-6)^2 + (3-3)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(0-1)^2 + (6-7)^2 + (3-7)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

$$AC = \sqrt{(3-1)^2 + (3-7)^2 + (3-7)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BD = \sqrt{(0-4)^2 + (6-4)^2 + (3-7)^2}$$
$$= \sqrt{16+4+16}$$
$$= 6 \text{ units}$$

$$CD = \sqrt{(1-4)^2 + (7-4)^2 + (7-7)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2} \text{ units}$$

$$AD = \sqrt{(3-4)^2 + (3-4)^2 + (3-7)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

Since,

$AB = BC = CD = DA$

And $AC = BD$

So,

A, B, C, D are vertices of a square.

Introduction to 3D Coordinate Geometry Ex 28.2 Q11

Here,

$$AB = \sqrt{(1+5)^2 + (3-5)^2 + (0-2)^2}$$
$$= \sqrt{36+4+4}$$
$$= \sqrt{44}$$
$$= 2\sqrt{11} \text{ units}$$

$$BC = \sqrt{(-5+9)^2 + (5+1)^2 + (2-2)^2}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13} \text{ units}$$

$$CD = \sqrt{(-9+3)^2 + (-1+3)^2 + (2-0)^2}$$
$$= \sqrt{36+4+4}$$
$$= 2\sqrt{11} \text{ units}$$

$$DA = \sqrt{(-3-4)^2 + (-3-3)^2 + 0}$$
$$= \sqrt{16+36}$$
$$= \sqrt{52}$$
$$= 2\sqrt{13} \text{ units}$$

$$AC = \sqrt{(1+9)^2 + (3+1)^2 + (0-2)^2}$$
$$= \sqrt{150+16+4}$$
$$= \sqrt{120}$$
$$= 4\sqrt{5} \text{ units}$$

$$BD = \sqrt{(-3+5)^2 + (-3-5)^2 + (0-2)^2}$$
$$= \sqrt{4+64+4}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2} \text{ units}$$

Since,

$AB = CD$ and $BC = DA$

$\Rightarrow$ $ABCD$ is a parallelgram = BD

but, $AC \neq BD$

$\Rightarrow$ $ABCD$ is not a rectangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q12

Here,

$$AB = \sqrt{(1+1)^2 + (3-6)^2 + (4-10)^2}$$
$$= \sqrt{4+9+36}$$
$$= 7 \text{ units}$$

$$BC = \sqrt{(-1+7)^2 + (6-4)^2 + (0-7)^2}$$
$$= \sqrt{36+4+9}$$
$$= 7 \text{ units}$$

$$CD = \sqrt{(-7+5)^2 + (4-1)^2 + (7-1)^2}$$
$$= \sqrt{4+9+36}$$
$$= 7 \text{ units}$$

$$DA = \sqrt{(-5-1)^2 + (1-3)^2 + (1-4)^2}$$
$$= \sqrt{36+4+9}$$
$$= \sqrt{52}$$
$$= 7 \text{ units}$$

Since, $AB = BC = CD = DA$

So, $ABCD$ is a rhombus.

Introduction to 3D Coordinate Geometry Ex 28.2 Q13

Here,

$$AB = \sqrt{(0-1)^2 + (1-0)^2 + (1-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$BC = \sqrt{(1-1)^2 + (0-1)^2 + (1-0)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$CA = \sqrt{(1-0)^2 + (1-1)^2 + (0-1)^2}$$
$$= \sqrt{1+0+1}$$
$$= \sqrt{2} \text{ units}$$

$$DA = \sqrt{(0-0)^2 + (0-1)^2 + (0-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$OB = \sqrt{(0-1)^2 + (0-0)^2 + (0-1)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

$$OC = \sqrt{(0-1)^2 + (0-1)^2 + (0-0)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2} \text{ units}$$

Since, $OA = OB = OC = AB = BC = CA$

So, O, A, B, C represent a regular tetrahedron

Introduction to 3D Coordinate Geometry Ex 28.2 Q14

Here,

$$OA = \sqrt{(1-3)^2 + (3-2)^2 + (4-2)^2}$$
$$= \sqrt{4+1+4}$$
$$= 3\text{units}$$

$$OB = \sqrt{(1+1)^2 + (3-1)^2 + (4-3)^2}$$
$$= \sqrt{4+4+1}$$
$$= 3\text{units}$$

$$OC = \sqrt{(1-0)^2 + (3-5)^2 + (4-6)^2}$$
$$= \sqrt{1+4+4}$$
$$= 3\text{units}$$

$$OD = \sqrt{(1-2)^2 + (3-1)^2 + (4-2)^2}$$
$$= \sqrt{1+4+4}$$
$$= 3\text{units}$$

Since, $OA = OC = OD = OB$, points A, B, C, D lie on a sphere with centre O.

Radius = 3 units

Introduction to 3D Coordinate Geometry Ex 28.2 Q15

Let the required point be $P(x_1 y_1 z)$

Here, $0(0,0,0)$, $A(2,0,0)$, $B(0,3,0)$, $C(0,0,8)$

Since, $(OP)^2 = (PA)^2$

$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-2)^2 + (y-0)^2 + (z-0)^2$$
$$x^2 + y^2 + z^2 = x^2 - 4x + 4 + y^2 + z^2$$
$$4x = 4$$
$$x = 1$$

$$(OP)^2 = (PB)^2$$
$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-0)^2 + (y-3)^2 + (z-0)^2$$
$$x^2 + y^2 + z^2 = x^2 + y^2 - 6y + 9 + z^2$$
$$6y = 9$$
$$y = \frac{3}{2}$$

$$(OP)^2 = (PC)^2$$
$$(x-0)^2 + (y-0)^2 + (z-0)^2 = (x-0)^2 + (y-0)^2 + (z-8)^2$$
$$x^2 + y^2 + z^2 = x^2 + y^2 + z^2 - 16z + 64$$
$$16z = 64$$
$$z = 4$$

The required point = $\left(1, \frac{3}{2}, 4\right)$

Introduction to 3D Coordinate Geometry Ex 28.2 Q16

Let P be (x_1y_1z), here, $A(-2,2,3)$ and $B(13,-3,13)$
and $3PA = 2PB$

$$\Rightarrow \quad 3\sqrt{(x+2)^2(y-2)^2+(z-3)^2} = 2\sqrt{(x-13)^2+(y+3)^2+(z-13)^2}$$

squaring both the sides,

$$\Rightarrow \quad 9\left[x^2+4x+4+y^2+4-4y+z^2+9-6z\right]$$
$$= 4\left[x^2+169-26x+y^2+9+6y+z^2+169-26z\right]$$
$$\Rightarrow \quad 9x^2-4x^2+36x+104x+36-676+9y^2-4y^2$$
$$+36-36-36y-24y+9z^2-4z^2+81-676-54z+6yz=0$$
$$\Rightarrow \quad 5x^2+5y^2+5z^2+140x-60y+50z-1235=0$$
$$\Rightarrow \quad 5\left(x^2+y^2+z^2\right)+140x-60y+50z-1235=0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q17

Let $P(x_1y_1z)$, here, $A(3,4,5)$, $B(-1,3,-7)$
$PA^2+PB^2=2k^2$

$$\Rightarrow \quad (x-3)^2+(y-4)^2+(z-5)^2+(x+1)^2+(y-3)^2+(2+7)^2=2k^2$$
$$\Rightarrow \quad x^2+9-6x+y^2+16-8y+z^2+25-10z+x^2+1+2x$$
$$+y^2+9-6y+z^2+49+14z=2k^2$$
$$\Rightarrow \quad 2x^2+2y^2+2z^2-4x-14y+4z+109=2k^2$$
$$\Rightarrow \quad 2\left(x^2+y^2+z^2\right)-4x-14y+4z+109-2k^2=0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q18

Here, $A(a,b,c)$, $B(b,c,a)$, $C(c,a,b)$

$$AB = \sqrt{(a-b)^2+(b-c)^2+(c-a)^2}$$
$$= \sqrt{a^2+b^2-2ab+b^2+c^2-2bc+c^2+a^2-2ac}$$

$$AB = \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ac}$$
$$BC = \sqrt{(b-c)^2+(c-a)^2+(a-b)^2}$$
$$= \sqrt{b^2+c^2-2bc+c^2+a^2-2ca+a^2+b^2-2ab}$$
$$BC = \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$$

$$CA = \sqrt{(a-c)^2+(b-a)^2+(c-b)^2}$$
$$= \sqrt{a^2+c^2-2ac+b^2+a^2-2ab+b^2+c^2-2bc}$$

$$CA = \sqrt{2a^2+2b^2+2c^2-2ab-2bc-2ca}$$

Since, $AB = BC = CA$, so
$\triangle$ABC is an isosceles A

Introduction to 3D Coordinate Geometry Ex 28.2 Q19

Here, $A(3,6,9)$, $B(10.20.30)$, $C(25,41,5)$

$$\begin{aligned}(AB)^2 &= (3-10)^2+(6-20)^2+(9-30)^2\\ &= (-7)^2+(-14)^2+(-21)^2\\ &= 49+196+441\\ &= 586\end{aligned}$$

$$\begin{aligned}(BC)^2 &= (10-25)^2+(20+41)^2+(30-5)^2\\ &= (-15)^2+(61)^2+(25)^2\\ &= 225+3721+625\\ &= 4571\end{aligned}$$

$$\begin{aligned}(CA)^2 &= (3-25)^2+(6+41)^2+(9-5)^2\\ &= (-22)^2+(47)^2+(4)^2\\ &= 484+2209+16\\ &= 2709\end{aligned}$$

Since, $AB^2+BC^2 \neq AC^2$

$AB^2+AC^2 \neq BC^2$

$BC^2+AC^2 \neq AB^2$

So, $\triangle ABC$ is not a right triangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(i)

Here, $A(0,7,-10)$, $B(1,6,-6)$, $C(4,9,-6)$

$$\begin{aligned}AB &= \sqrt{(0-1)^2+(7-6)^2+(-10+6)^2}\\ &= \sqrt{1+1+16}\\ &= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(1-4)^2+(6-9)^2+(-6+6)^2}\\ &= \sqrt{9+9}\\ &= 3\sqrt{2} \text{ units}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(0,-4)^2+(7-9)^2+(-10+6)^2}\\ &= \sqrt{16+4+16}\\ &= 6 \text{ units}\end{aligned}$$

Since, $AB = BC$

So, $\triangle ABC$ is an isosceles $\triangle$

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(ii)

Here, $A(0,7,10)$, $B(-1,6,6)$, $C(-4,9,6)$

$$AB = \sqrt{(0+1)^2 + (7-6)^2 + (10-6)^2}$$
$$= \sqrt{1+1+16}$$
$$= 3\sqrt{2} \text{ units}$$

$$BC = \sqrt{(-1+4)^2 + (6-9)^2 + (6-6)^2}$$
$$= \sqrt{9+9+0}$$
$$= 3\sqrt{2} \text{ units}$$

$$CA = \sqrt{(-4-0)^2 + (9-7)^2 + (6+10)^2}$$
$$= \sqrt{16+4+16}$$
$$= \sqrt{36} \text{ units}$$

Since, $(AB)^2 + (BC)^2 = (AC)^2$

So, $\triangle ABC$ is a right triangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(iii)

Here, $A(-1,2,1)$, $B(1,-2,5)$, $C(4,-7,8)$, $D(2,-3,4)$

$$AB = \sqrt{(-1-1)^2 + (2+2)^2 + (1-5)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BC = \sqrt{(1-4)^2 + (-2+7)^2 + (5-8)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

$$CD = \sqrt{(4-2)^2 + (-7+3)^2 + (8-4)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$DA = \sqrt{(2+1)^2 + (-3-2)^2 + (4-1)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

Since, $AB = CD$ and $BC = DA$

So, $\triangle ABC$ is a parallelogram

Introduction to 3D Coordinate Geometry Ex 28.2 Q20(iv)

Let A(5, -1, 1), B(7, -4, 7), C(1, -6, 10) and D(-1, -3, 4) be the given points.

$$AB = \sqrt{(7-5)^2 + (-4+1)^2 + (7-1)^2} = \sqrt{4+9+36} = 7$$

$$BC = \sqrt{(1-7)^2 + (-6+4)^2 + (10-7)^2} = \sqrt{36+4+9} = 7$$

$$CD = \sqrt{(-1-1)^2 + (-3+6)^2 + (4-10)^2} = \sqrt{4+9+36} = 7$$

$$AD = \sqrt{(-1-5)^2 + (-3+1)^2 + (4-1)^2} = \sqrt{36+4+9} = 7$$

So AB = BC = CD = AD

Hence ABCD is a rhombus.

Introduction to 3D Coordinate Geometry Ex 28.2 Q21

Let the point $P(x_1 y_1 z)$ which is equidistance from $A(1,2,3)$ and $B(3,2,-1)$, so

$$AP = BP$$

$$(AP)^2 = (BP)^2$$

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$$x^2 + 1 - 2x + y^2 + 4 - 4y + z^2 + 9 - 6z = x^2 + 9 - 6x + y^2 + 4 - 4y + z^2 + 1 + 2z$$

$$4x - 8z = 14 - 14$$

$$4x - 8z = 0$$

$$x - 2z = 0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q22

Let locus of $P(x_1 y_1 z)$ is the required locus, so

$$PA + P = 10$$

$$\Rightarrow \quad \sqrt{(x-4)^2 + (y-0)^2 + (z-0)^2} + \sqrt{(x+4)^2 + (y-0)^2 + (z-0)^2} = 10$$

$$\Rightarrow \quad \sqrt{x^2 + 16 - 8x + y^2 + z^2} = 10 - \sqrt{x^2 + 8x + 16 + y^2 + z^2}$$

$$\Rightarrow \quad x^2 + y^2 + z^2 - 8x + 16 = (10)^2 + (x^2 + y^2 + z^2 + 8x + 16)$$
$$-20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow \quad -8x + 16 - 100 - 8x - 16 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow \quad -16x - 100 = -20\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

$$\Rightarrow \quad -4(4x + 25) = -20\sqrt{x^2 + yy^2 + z^2 + 8x + 16}$$

$$\Rightarrow \quad (4x + 25) = 5\sqrt{x^2 + y^2 + z^2 + 8x + 16}$$

squiring both the sides,

$$(4x+25)^2 = 25(x^2 + y^2 + z^2 + 8x + 16)$$

$$\Rightarrow \quad 16x^2 + 625 + 200x = 25(x^2 + y^2 + z^2 + 8x + 16)$$

$$\Rightarrow \quad 16x^2 + 625 + 200x = 25x^2 + 25y^2 + 25z^2 + 200x + 400$$

$$\Rightarrow \quad 9x^2 + 25y^2 + 25z^2 - 225 = 0$$

Introduction to 3D Coordinate Geometry Ex 28.2 Q23

$$AB = \sqrt{(1+1)^2 + (2+2)^2 + (3+1)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$BC = \sqrt{(-1-2)^2 + (-2-3)^2 + (-1-2)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

$$CD = \sqrt{(2-4)^2 + (3-7)^2 + (2-6)^2}$$
$$= \sqrt{4+16+16}$$
$$= 6 \text{ units}$$

$$DA = \sqrt{(4-1)^2 + (7-2)^2 + (6-3)^2}$$
$$= \sqrt{9+25+9}$$
$$= \sqrt{43} \text{ units}$$

$$AC = \sqrt{(1-2)^2 + (2-3)^2 + (3-2)^2}$$
$$= \sqrt{1+1+1}$$
$$= \sqrt{3} \text{ units}$$

$$BD = \sqrt{(-1-4)^2 + (-2-7)^2 + (-1-6)^2}$$
$$= \sqrt{25+81+49}$$
$$= \sqrt{155} \text{ units}$$

Since,

$AB = DC$ and $BC = DA$

so,

$ABCD$ is a parallelgram

$$AB^2 + BC^2 = 36 + 43 = 97$$
$$AC = \sqrt{3}$$
$$\therefore AB^2 + BC^2 \neq AC^2$$

i.e. $\angle$B is not a right angle.

ABCD is not a rectangle.

Introduction to 3D Coordinate Geometry Ex 28.2 Q24

Let the point be P (x, y, z)

Given

A=(3, 4, -5)

B=(-2, 1, 4)

$PA=PB \Rightarrow PA^2=PB^2$

$PA^2 = (x-3)^2 + (y-4)^2 + (z+5)^2$

$PB^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$

$PA^2=PB^2 \Rightarrow$

$(x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$

All square terms will be cancelled on both sides, we get

$-6x+9-8y+16+10z+25 = 4x+4-2y+1-8z+16$

$10x+6y-18z-29=0$ is the required equation