RD Sharma Solutions Class 11 Maths Chapter 23 Ex 23.5

Straight Lines Ex 23.5 Q1(i)

Here,

$(x_1 y_1) = (0,0)$

$(x_2 y_2) = (2,-2)$

The equation of the given straight line is:

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - 0 = \frac{-2 - 0}{2 - 0}(x - 0)$$

$$\Rightarrow \quad y = \frac{-2x}{2}$$

$$\Rightarrow \quad y = -x$$

$\therefore$ The equation of the line joining the points $(0,0)$ and $(2,-2)$ is $y = -x$

Straight Lines Ex 23.5 Q1(ii)

Let $A(a,b) = (x_1 y_1)$

$B(a + c\sin\alpha, b + c\cos\alpha) = (x_2 y_2)$

Then equation of line AB is

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - b = \frac{b + c\cos\alpha - b}{a + c\sin\alpha - a}(x - a)$$

$$\Rightarrow \quad y - b = \frac{c\cot\alpha}{c\sin\alpha}(x - a)$$

$$\Rightarrow \quad y - b = \cot\alpha(x - a)$$

$\therefore$ The equation of the line joining the points (a,b) and $(a + c\sin\alpha, b + c\cos\alpha)$ is $y - b = \cot\alpha(x - a)$

Straight Lines Ex 23.5 Q1(iii)

Let $A(a,-a)$ be $(x_1 y_1)$

$B(b,0)$ be $(x_2 y_2)$

Then equation of line AB is

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - (-a) = \frac{0 - (-a)}{b - 0}(x - 0)$$

$$\Rightarrow \quad y + a = \frac{a}{b}(x - 0)$$

$$\Rightarrow \quad ax - by = ab$$

$\therefore$ The equation of the line joining the points $(0,-a)$ and $(b,0)$ is $ax - by = ab$

Straight Lines Ex 23.5 Q1(iv)

Let $A(a,b)$ be (x_1y_1)

$B(a+b,a-b)$ be (x_2y_2)

Then equation of line AB is

$$\Rightarrow \quad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$\Rightarrow \quad y-b=\frac{a-b-b}{a+b-a}(x-a)$$

$$\Rightarrow \quad y-b=\frac{a-2b}{b}(x-a)$$

$$\Rightarrow \quad by-b^2=ax-a^2-2bx+2ba$$

$$\Rightarrow \quad (a-2b)x-by+b^2-a^2+2ab=0$$

$\therefore$ The equation of the line joining the points (a,b) and $(a+b,a-b)$ is $(a-2b)x-by+b^2-a^2+2ab=0$

Straight Lines Ex 23.5 Q1(v)

Let $A(x_1y_1)$ be $\left(at_1,\frac{a}{t_1}\right)$

$B(x_2y_2)$ be $\left(at_2,\frac{a}{t_2}\right)$

Then equation of line AB is

$$\Rightarrow \quad y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$\Rightarrow \quad y-\frac{a}{t_1}=\frac{\frac{a}{t_2}-\frac{a}{t_1}}{at_2-at_1}(x-at_1)$$

$$\Rightarrow \quad y-\frac{a}{t_1}=\frac{a(t_1-t_2)}{at_1t_2(t_2-t_1)}(x-at_1)$$

$$\Rightarrow \quad y-\frac{a}{t_1}=\frac{-1}{t_1t_2}(x-at_1)$$

$$\Rightarrow \quad t_1t_2y+x=a(t_1+t_2)$$

$\therefore$ The equation of the line joining the points $\left(at_1,\frac{a}{t_1}\right)$ and $\left(at_2,\frac{a}{t_2}\right)$ is $t_1t_2y+x=a(t_1+t_2)$

Straight Lines Ex 23.5 Q1(vi)

Let $A(x_1 y_1)$ be $(a\cos\alpha, a\sin\alpha)$

$B(x_2 y_2)$ be $(a\cos\beta, a\sin\beta)$

$$\Rightarrow \quad y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{a\sin\beta - a\sin\alpha}{a\cos\beta - a\cos\alpha}(x - a\cos\alpha)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{a\left(-2\sin\left(\frac{\beta-\alpha}{2}\right)\right)\cos\beta\left(\frac{\beta+\alpha}{2}\right)}{a\left(-2\sin\frac{\beta-\alpha}{2}\right)\sin\left(\frac{\beta+\alpha}{2}\right)}(x - a\cos\alpha)$$

$$\Rightarrow \quad y - a\sin\alpha = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\sin\left(\frac{\alpha+\beta}{2}\right)}(x - a\cos\alpha)$$

$$\Rightarrow \quad x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}$$

$\therefore$ The equation of the line joining the points $(a\cos\alpha, a\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$ is

$x\cos\left(\frac{\alpha+\beta}{2}\right) + y\sin\frac{\alpha+\beta}{2} = a\cos\frac{\alpha-\beta}{2}$

Straight Lines Ex 23.5 Q2(i)

Let $A(1,4)$, $B(2,-3)$, $C(-1,-2)$

Then equation of AB is

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 4 = \frac{-3-4}{2-1}(x - 1)$

$y - 4 = \frac{-7}{1}(x - 1)$

$7x + y = 11$

Equation of side BC is

$y - y_2 = \frac{y_3 - y_2}{x_3 - x_2}(x - x_2)$

$y - (-3) = \frac{-2-(-3)}{-1-2}(x - 2)$

$y + 3 = \frac{1}{-3}(x - 2)$

$x + 3y + 7 = 0$

Equation of side AC is

$y - y_1 = \frac{y_3 - y_1}{x_3 - x_1}(x - x_1)$

$y - 4 = \frac{-2-4}{-1-1}(x - 1)$

$y - 4 = 3(x - 1)$

$y - 3x = 1$

Straight Lines Ex 23.5 Q2(ii)

Let $A(0,1), B(2,0), C(-1,-2)$

then equation of side AB is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$y - 1 = \frac{0 - 1}{2 - 0}(x - 0)$$
$$y - 1 = \frac{-1}{2}(x)$$
$$x + 2y = 2$$

Equation of side BC is

$$y - y_2 = \frac{y_3 - y_2}{x_3 - x_2}(x - x_1)$$
$$y - 0 = \frac{-2 - 0}{-1 - 2}(x - 2)$$
$$y = \frac{2}{3}(x - 2)$$
$$2x - 3y = 4$$

Equation of side AC is

$$y - y_1 = \frac{y_3 - y_1}{x_3 - x_1}(x - x_1)$$
$$y - 1 = \frac{-2 - 1}{-1 - 0}(x - 0)$$
$$y - 1 = 3(x - 0)$$

$$y - 3x = 1$$

Straight Lines Ex 23.5 Q3

Let $A(-1,6)$ be (x_1y_1)
$B(-3,-9)$ be (x_2y_2)
$C(5,-8)$ be (x_3y_3)

Median is a line segment which joins a vertex to the mid-point of the side opposite to it.
Let D, E and F be the mid points of sides AB, BC, and CA.

Then, using mid point formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ we can find the coordinates of D, E and F as:

$$D = \left(\frac{-3+5}{2}, \frac{-9-8}{2}\right) = \left(-1, \frac{-17}{2}\right)$$
$$E = \left(\frac{-1+5}{2}, \frac{6-8}{2}\right) = (2,-1)$$
$$F = \left(\frac{-1-3}{2}, \frac{6-9}{2}\right) = \left(-2, \frac{-3}{2}\right)$$

Equation of median AD is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-6 = \frac{\frac{-17}{2}-6}{1-(-1)}(x+1) = \frac{-29}{4}(x+1) \qquad \left[A(-1,6), D\left(1, \frac{-17}{2}\right)\right]$$
$$29x+4y+5=0$$

Equation of median BE is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-(-9) = \frac{-1-(-9)}{2-(-3)}(x-(-3)) \qquad \left[B(-3,-9), E(2,-1)\right]$$
$$y+9 = \frac{8}{5}(x+3)$$
$$5y+45 = 8x+24$$
$$8x-5y-21=0$$

Equation of median CF is

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
$$y-(-8) = \frac{\frac{-3}{2}-(-8)}{-2-5}(x-5) \qquad \left[C(5,-8), F\left(-2, \frac{-3}{2}\right)\right]$$
$$y+8 = \frac{-3+16}{2\times(-7)}(x-5)$$
$$y+8 = \frac{-13}{14}(x-5)$$
$$13x+14y+47=0$$

Straight Lines Ex 23.5 Q4

The rectangle ABCD will have diagonals AC and BD

AC passes through $A(a,b)$ and $C(a',b')$.

Thus equation of AC is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-b}{b'-b}=\frac{x-a}{a'-a}$$

$$\Rightarrow (y-b)(a'-a)=(x-a)(b'-b)$$

$$\Rightarrow y(a'-a)-a'b+ab=x(b'-b)-ab'+ab$$

$$\Rightarrow y(a'-a)=x(b'-b)-ab'+a'b$$

$$\Rightarrow y(a'-a)-x(b'-b)=a'b-ab'$$

BD passes through $B(a',b)$ and $D(a,b')$.

Thus equation of BD is:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\Rightarrow \frac{y-b}{b'-b}=\frac{x-a'}{a-a'}$$

$$\Rightarrow (y-b)(a-a')=(x-a')(b'-b)$$

$$\Rightarrow -y(a'-a)-ab+a'b=x(b'-b)-a'b'+a'b$$

$$\Rightarrow a'b'-ab=x(b'-b)+y(a'-a)$$

$$\Rightarrow x(b'-b)+y(a'-a)=a'b'-ab$$

Straight Lines Ex 23.5 Q5

Equation of BC

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-1=\frac{0-1}{2-0}(x-0) \quad [\because B(0,1), C(2,0)]$$

$$2y-2=-x$$

$$x+2y=2$$

D is mid point of BC

So,

$$D=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{0+2}{2},\frac{1+0}{2}\right)=\left(1,\frac{1}{2}\right)$$

$\therefore$ Equation of the median AD:

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-(-2)=\frac{\frac{1}{2}-(-2)}{1-(-1)}\{x-(-1)\}=\frac{\frac{5}{2}}{2}(x+1) \qquad \left[\because A(-1,-2), D\left(1,\frac{1}{2}\right)\right]$$

$$4y+8=5x+5$$

$$5x-4y-3=0$$

Straight Lines Ex 23.5 Q6

The equation of the line passing through points $(-2,-2)$ and $(8,2)$ is

$$y+2=\frac{2+2}{8+2}(x+2)$$

$$2x-5y-6=0$$

Clearly, $(3,0)$ satisfies this equation which means that the line passing through $(-2,-2)$ and $(8,2)$ also passes through $(3,0)$.

Hence three points are collinear.

Straight Lines Ex 23.5 Q7

Let AB be the line segment

Let P be any point which divides the line segment in the ratio 2:3

then using section formula

$$x=\frac{lx_2+mx_1}{l+m},\ y=\frac{ly_2+my_1}{l+m}$$

where $l:m::2:3$

$$\Rightarrow x=\frac{2(8)+3(3)}{2+3}=\frac{16+9}{5}=\frac{25}{5}=5$$

$$y=\frac{2(9)+3(-1)}{2+3}=\frac{18-3}{5}=\frac{15}{5}=3$$

Now P must lie on the line, where P is $(5,3)$

$$y-x+2=0$$

$$\Rightarrow 3-(5)+2=0$$

$$-2+2=0$$

$$0=0$$

Hence, Proved

Straight Lines Ex 23.5 Q8

The line that bisects the distance between the points $A(a,b)$, $B(a'b')$ and between $C(-a,b)$, $D(a'-b')$ means a line passing through the mid-point of AB and CD

mid point of AB is $\left(\frac{a+a'}{2},\frac{b+b'}{2}\right)$

mid point of CD is $\left(\frac{-a+a'}{2},\frac{b-b'}{2}\right)$

Equation is $y-y_1=m(x-x_1)$

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$y-\left(\frac{b+b'}{2}\right)=\frac{\left(\frac{b-b'}{2}\right)-\left(\frac{b+b'}{2}\right)}{\left(\frac{-a+a'}{2}-\frac{a+a'}{2}\right)}\left(x-\left(\frac{a+a'}{2}\right)\right)$$

$$y-\left(\frac{b+b'}{2}\right)=\frac{\frac{b}{2}-\frac{b'}{2}-\frac{b}{2}-\frac{b'}{2}}{\frac{-a}{2}+\frac{a'}{2}-\frac{a}{2}-\frac{a'}{2}}\left(x-\left(\frac{a+a'}{2}\right)\right)$$

$$y-\left(\frac{b+b'}{2}\right)=\frac{+b'}{a}\left(x-\left(\frac{a+a'}{2}\right)\right)$$

$$2ay-2b'x=ab-a'b'$$

Straight Lines Ex 23.5 Q9

In what ratio is the line joining the points $(2,3)$ and $(4,-5)$ divided by the line passing through the points $(6,8)$ and $(-3,-2)$.

Let the equation of line AB joining the points $(6,8)$ and $(-3,-2)$ be

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 8 = \frac{-2-8}{-3-6}(x - 6)$$

$$y - 8 = \frac{10}{9}(x - 6)$$

$$9y - 10x = 12 \quad ---(1)$$

Suppose the line joining $(2,3)$ and $(4,-5)$ is divided by the line $9y - 10x = 12$ in the ratio $k:1$ at the point (x, y), then

$$x = \frac{k(4) + 1(2)}{k+1}, y = \frac{k(-5) + 1(3)}{k+1}$$

Substituiting in equation (i), we get:

$$\frac{9(-5k+3)}{k+1} - 10\left(\frac{4k+2}{k+1}\right) = 12$$

$$\Rightarrow -45k + 27 - 40k - 20 = 12k + 12$$

$$\Rightarrow 97k = 5$$

$$\Rightarrow k = \frac{5}{97}$$

Straight Lines Ex 23.5 Q10

The quadrilateral $ABCD$ has diagonals AC and BD.
The required equation is

Since, $A(-2,6), C(10,4)$, the equation for AC is:

$$y - 6 = \frac{4-6}{10-(-2)}(x - (-2)) \qquad \left[y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$y - 6 = \frac{-12}{6}(x + 2)$$

$$y - 6 = \frac{-(x+2)}{6}$$

$$6y - 36 = -x - 2$$

$$x + 6y - 34 = 0$$

Since, $B(1,2), D(7,8)$, the equation for BD is:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 2 = \frac{8-2}{7-1}(x - 1)$$

$$y - 2 = \frac{6}{6}(x - 1)$$

$$y - 2 = x - 1$$

$$x - y + 1 = 0$$

Straight Lines Ex 23.5 Q11

$L_1 = 124.942,\ C_1 = 20$

$L_2 = 125.134,\ C_2 = 110$

Equation of line passing through

(L_1, C_1) and (L_2, C_2)

$$L - L_1 = \left(\frac{L_2 - L_1}{C_2 - C_1}\right)(C - C_1)$$

$$L - 124.942 = \left(\frac{125.134 - 124.942}{110 - 20}\right)(C - 20)$$

$$L - 124.942 = \frac{0.192}{90}(C - 20)$$

$$L - 124.942 = \frac{192}{90000}(C - 20)$$

$$L - 124.942 = \frac{4}{1875}(C - 20)$$

$$L = \frac{4}{1875}C + 124.942 - 4 \times \frac{20}{1875}$$

$$\Rightarrow L = \frac{4}{1875}C + 124.899$$

Straight Lines Ex 23.5 Q12

Assuming x be the price per litre and y be the quantity of the milk.

sold at this price.

So, the line representing the relationship passes through $(14,980)$ and $(16,1220)$.

So its equation is

$$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$$

$$y - 980 = 120(x - 14)$$

$$120x - y - 700 = 0$$

When $x = 17,\ 120 \times 17 - y - 700 = 0$

$$y = 1340$$

Straight Lines Ex 23.5 Q13

Let AD be the bisector of $\angle A$

Then, $BD : DC = AB : AC$

Now,

$$|AB| = \sqrt{(4-0)^2 + (3-0)^2} = 5$$

$$|AC| = \sqrt{(4-2)^2 + (3-3)^2} = 2$$

$$\Rightarrow \frac{AB}{AC} = \frac{BD}{DC} = \frac{5}{2}$$

$\Rightarrow D$ divides BC in the ratio $5:2$

So, coordinates of D are $\left(\frac{5\times 2+0}{5+2}, \frac{5\times 3+0}{5+2}\right) = \left(\frac{10}{7}, \frac{15}{7}\right)$

$\therefore$ The equation of AD is

$$y - 3 = \left(\frac{\frac{15}{7} - 3}{\frac{10}{7} - 4}\right)(x - 4)$$

$$y - 3 = \left(\frac{15 - 21}{10 - 28}\right)(x - 4)$$

$$\Rightarrow y - 3 = \frac{1}{3}(x - 4)$$

$$\Rightarrow 3(y - 3) = x - 4$$

$$\Rightarrow x - 3y + 9 - 4 = 0$$

$$\Rightarrow x - 3y + 5 = 0$$

Straight Lines Ex 23.5 Q14

The required straight line passes through $(0,0)$ and trisects the part of the line $3x + y = 12$ that lies between the axes of coordinates.

The line $3x + y = 12$ has $A(4,0)$ and $B(0,12)$ as x and y intercepts.

Let P and Q be the points of trisection of AB.

Since P divides AB in the ratio 1:2, coordinates of P are:

$$P = \frac{1(0)+2(4)}{1+2}, \frac{1(12)+2(0)}{1+2} = \left(\frac{8}{3}, 4\right)$$

Since Q divides BA in the ratio 1:2, coordinates of Q are:

$$Q = \frac{2(0)+1(4)}{1+2}, \frac{1(0)+2(12)}{1+2} = \left(\frac{4}{3}, 8\right)$$

Equation of line through $(0,0)$ and $P\left(\frac{8}{3}, 4\right)$ is:

$$y - 0 = \frac{4-0}{\frac{8}{3}-0}(x-0)$$

$$y - 0 = \frac{12}{8}x$$

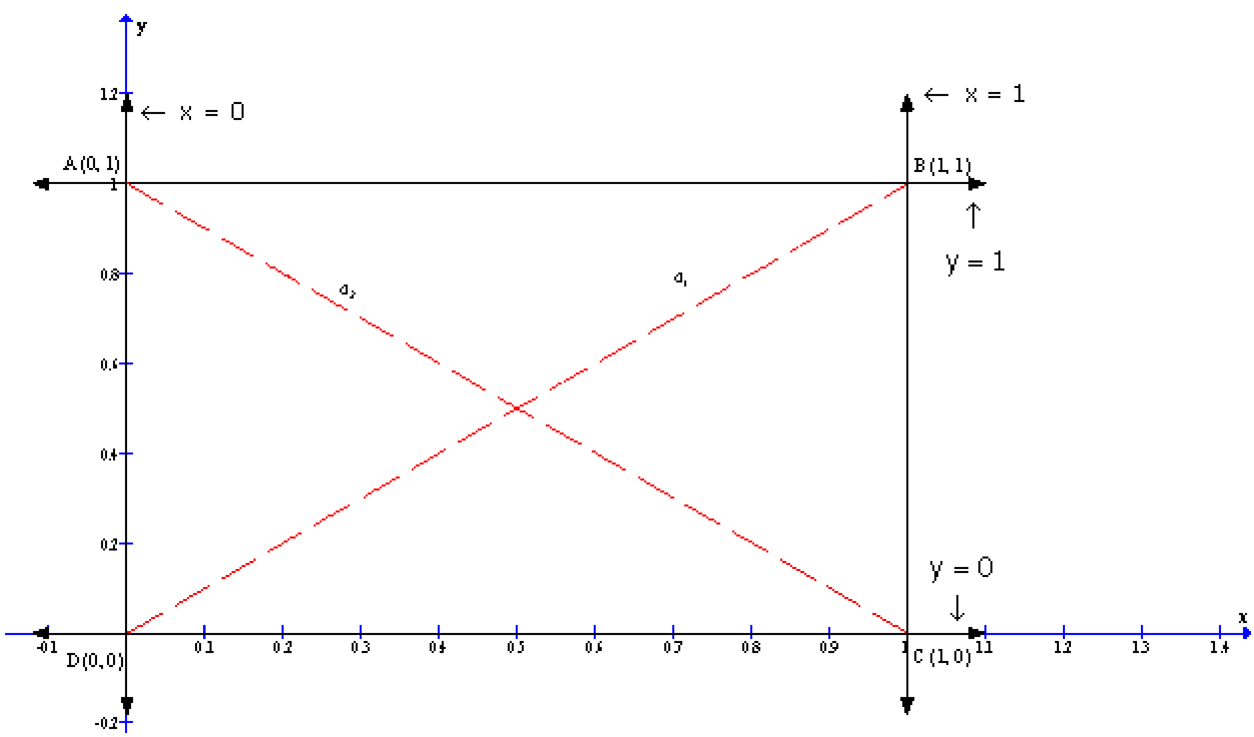
$$2y = 3x$$

Equation of line through $(0,0)$ and $Q\left(\frac{4}{3}, 8\right)$ is:

$$y - 0 = \frac{8-0}{\frac{4}{3}-0}(x-0) = 6x$$

$$y = 6x$$

Straight Lines Ex 23.5 Q15



When we draw all the given equations of lines on the graph we get the points of intersection A(0, 1), B(1,1), C(1,0) and D(0,0).

Let d_1 be the diagonal fomed by joining the points B and D.
Let d_2 be the diagonal fomed by joining the points A and C.

Equation of the diagonal d_1 is given by,

$$(y - 1) = \frac{(0-1)}{(0-1)}(x-1)$$

$$(y-1) = 1(x-1)$$

$$y = x$$

Equation of the diagonal d_2 is given by,

$$(y - 1) = \frac{(0-1)}{(1-0)}(x-0)$$

$$(y-1) = -1(x)$$

$$y + x = 1$$

$\therefore$ The equations of the diagonals are $y = x$ and $y + x = 1$.