RD Sharma Solutions Class 11 Maths Chapter 22 Ex 22.3

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q1

We have,

$$(x-a)^2+(y-b)^2=r^2 \quad \ldots\ldots\text{(i)}$$

Substituting $x = X + (a-c)$, $y = Y + b$ in the equation (i), we get

$$[X+a-c-a]^2+[Y+b-b]^2=r^2$$

$$\Rightarrow \quad [X-c]^2+[Y]^2=r^2$$

$$\Rightarrow \quad X^2+c^2-2Xc+Y^2=r^2$$

$$\Rightarrow \quad X^2+y^2-2cX=r^2-c^2$$

Hence, the required equation is $X^2+y^2-2cX=r^2-c^2$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q2

We have,

$$(a-b)\left(x^2+y^2\right)-2abx=0$$

Substituting $x = X + \dfrac{ab}{a-b}$, $y = Y$

in the given equation, we get

$$(a-b)\left[\left(X+\frac{ab}{a-b}\right)+Y^2\right]-2ab\left[X+\frac{ab}{a-b}\right]=0$$

$$\Rightarrow \quad (a-b)\left[X^2+\left(\frac{ab}{a-b}\right)^2+2\frac{Xab}{a-b}+Y^2\right]-2abX-2\frac{(ab)^2}{a-b}=0$$

$$\Rightarrow \quad (a-b)\left[\frac{X^2(a-b)^2+(ab)h^2+2Xab(a-b)+Y^2(a-b)^2}{(a-b)^2}\right]-\frac{2abX(a-b)+2(ab)^2}{a-b}=0$$

$$\Rightarrow \quad \frac{X^2(a-b)^2+(ab)^2+2ab(a-b)+Y^2(a-b)^2}{a-b}=\frac{2ab(a-b)+2(ab)^2}{a-b}$$

$$\Rightarrow \quad X^2(a-b)^2+Y^2(a-b)^2+(ab)^2+2ab(a-b)=2ab(a-b)+2(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=a^2b^2$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(i)

We have,

$$x^2 + xy - 3x - y + 2 = 0$$

Substituting $x = X + 1,\ Y + 1$ in the equation, we get

$$(X+1)^2 + (X+1)(Y+1) - 3(X+1) - (Y+1) + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3X - 3 - Y - 1 + 2 = 0$$

$$\Rightarrow \quad X^2 + XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(ii)

We have,

$$x^2 - y^2 - 2x + 2y = 0$$

Substituting $x = X + 1, \quad y = Y + 1$ in the equation, we get

$$(X+1)^2 - (Y-1)^2 - 2(X+1) + 2(Y+1) = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2Y = 0$$

$$\Rightarrow \quad X^2 - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(iii)

We have,

$$xy - x - y + 1 = 0$$

Substituting $x = X + 1, \quad y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$

$$\Rightarrow \quad XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(iv)

We have,

$$xy - y^2 - x + y = 0$$

Substituting $x = X + 1, \quad y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (Y+1) - (X+1) + (Y+1) = 0$$

$$\Rightarrow \quad XY + Y + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$$

$$\Rightarrow \quad XY + 2Y - Y^2 - 1 - 2Y + 1 = 0$$

$$\Rightarrow \quad XY - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q4

We have,

$$x^2 + xy - 3x - y + 2 = 0 \text{ .... (i)}$$

Let the origin be shifted to (h, k). Then $x = X + h$ and $y = Y + k$.

Substituting $x = X + h$, $y = Y + k$
in the equation (i), we get

$$(X + h)^2 + (X + h)(Y + k) - 3(X + h) - (Y + k) + 2 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2Xh + XY + Xk + Yh + hk - 3k - 3h - Y - k + 2 = 0$$

$$\Rightarrow \quad X^2 + XY + 2xh + Xk + Yh - Y3 - X + h^2 + hk - 3h - k + 2 = 0$$

$$\Rightarrow \quad X^2 + (2Xh + Xk - 3X) + XY + (Yh - Y) + (h^2 + hk - 3h - k + 2) = 0$$

$$\Rightarrow \quad X^2 + (2h + k - 3)X + XY + (h - 1)Y + (h^2 + hk - 3h - k + 2) = 0$$

For this equation to be free from first degree and the constant term, we must have,

$$2h + k - 3 = 0 \text{ .... (ii)}$$

$$h - 1 = 0$$

$$\Rightarrow h = 1 \text{ .... (iii)}$$

and

$$h^2 + hk - 3h - k + 2 = 0 \text{ .... (iv)}$$

Putting $h = 1$ in equation (ii), we get

$$2 + k - 3 = 0$$

$$\Rightarrow \quad k = 1$$

Putting $h = 1$ and $k = 1$ in equation (iv), we get

$$(1)^2 + 1 - 3 - 1 + 2 = 0$$

Hence, the value of h and k satisfies the equation (iv)

$\therefore$ The origin is shifted at the point $(1,1)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q5

Let the vertices of a triangle be $A(2,3)$, $B(5,7)$ and $C(-3,-1)$.

Then, area of $\triangle ABC$ is given by

$$\triangle = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|2(7+1) + 5(-1-3) - 3(-3-7)\right|$$
$$= \frac{1}{2}\left|2\times 8 + 5\times(-4) - 3\times(-4)\right|$$
$$= \frac{1}{2}\left|16 - 20 + 12\right|$$
$$= \frac{8}{2}$$
$$= 4$$

$\Rightarrow \quad \triangle = 4 sq$ unit

It is given that the origin is shifted at $(-1,3)$. Then new coordinates of the vertices are

$A_1 = (2-3, 3+3) = (-1,6)$

$B_1 = (5-1, 7+3) = (4,10)$

and $\quad C_1 = (-3-1, -1+3) = (-4,2)$

Therefore, the area of the triangle in the new coordinate system is given by

$$\triangle_1 = \frac{1}{2}\left|\left[-1(10-2) + 4(2-6) - 4(6-10)\right]\right|$$
$$= \frac{1}{2}\left|\left[-1\times 8 + 4\times(-4) - 4\times(-4)\right]\right|$$
$$= \frac{1}{2}\left|\left[-8 - 16 + 16\right]\right|$$
$$= \frac{1}{2}\left|-8\right|$$
$$= \frac{8}{2}$$

$\Rightarrow \quad \triangle_1 = 4 \ldots\ldots\ldots\ldots\ldots (2)$

From (i) and (ii), we get

$\triangle = \triangle_1$

Hence, the area of a triangle is invariant under the translation of the axes.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(i)

We have,

$x^2 + xy - 3y^2 - y + 2 = 0 \ldots\ldots\ldots (i)$

Substituting $x = X + 1$, $y = Y + 1$

in equation (i), we get

$$(X+1)^2 + (X+1)(Y+1) - 3(Y+1)^2 - (Y+1) + 2 = 0$$

$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3(Y^2 + 1 + 2Y) - Y - 1 + 2 = 0$

$\Rightarrow \quad X^2 + XY + 3X + 3 - 3Y^2 - 3 - 6Y = 0$

$\Rightarrow \quad X^2 - 3Y^2 + XY + 3X - 6Y = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(ii)

We have,

$xy - y^2 - x + y = 0 \ldots\ldots\ldots (i)$

Substituting $x = X + 1$, $y = Y + 1$

in equation (i), we get

$$(X+1)(Y+1) - (Y+1)^2 - (X+1) + (Y+1) = 0$$

$\Rightarrow \quad XY + X + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$

$\Rightarrow \quad XY + 2Y + 1 - Y^2 - 1 - 2Y = 0$

$\Rightarrow \quad XY - Y^2 = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(iii)

We have,

$$xy - x - y + 1 = 0 \ldots\ldots (i)$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$\Rightarrow$ $XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$

$\Rightarrow$ $XY = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(iv)

We have,

$$x^2 - y^2 - 2x - 2y = 0 \ldots\ldots (i)$$

Substituting $x = X + 1$, $y = Y + 1$
in equation (i), we get

$$(X+1)^2 - (Y+1)^2 - 2(X+1) + 2(Y+1) = 0$$

$\Rightarrow$ $X^2 + 1 + 2X - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$

$\Rightarrow$ $X^2 + 1 - Y^2 - 1 - 2Y + 2X = 0$

$\Rightarrow$ $X^2 + 2X + 1 - (Y^2 + 2Y + 1)$

$\Rightarrow$ $(X+1)^2 - (Y+1)^2$

$\Rightarrow$ $x^2 - y^2 = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(i)

Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$.

Substituting $x = X + h$, $y = Y + k$
in the equation $y^2 + x^2 - 4x - 8y + 3 = 0$, we get

$$(Y+k)^2 + (X+h)^2 - 4(X-h) - 8(Y+k) + 3 = 0$$

$\Rightarrow$ $Y^2 + k^2 + 2Yk + X^2 + h^2 + 2Xh - 4X - 4h - 8Y - 8k + 3 = 0$

$\Rightarrow$ $Y^2 + X^2 + 2Yk - 8Y + 2Xh - 4X + k^2 + h^2 - 4h - 8k + 3 = 0$

$\Rightarrow$ $Y^2 + X^2 + (2k - 8)Y + (2h - 4)X + (k^2 + h^2 - 4h - 8k + 3) = 0$

For this equation to be free from the term of first degree, we must have

$2k - 8 = 0$ and $2h - 4 = 0$

$\Rightarrow$ $k = 4$ and $h = 2$

Hence, the origin is shifted at the point $(2, 4)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(ii)

Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$
Substituting $x = X + h$, $y = Y + k$
in the equation $x^2 + y^2 - 5x + 2y - 5 = 0$, we get

$$(X+h)^2 + (Y+k)^2 - 5(X+h) + 2(Y+k) - 5 = 0$$

$\Rightarrow$ $X^2 + h^2 + 2xh + Y^2 + k^2 + 2Yk - 5X - 5h + 2Y + 2k - 5 = 0$

$\Rightarrow$ $X^2 + Y^2 + 2Yk + 2Y + 2Xh - 5x + h^2 + k^2 - 5h + 2k - 5 = 0$

$\Rightarrow$ $X^2 + Y^2 + (2k + 2)Y + (2h - 5)X + h^2 + k^2 - 5h + 2k - 5 = 0$

For this equation to be free from the term of first degree, we must have

$2k + 2 = 0$ and $2h - 5 = 0$

$\Rightarrow$ $k = -1$ and $h = \frac{5}{2}$

Hence, the origin is shifted at the point $\left(\frac{5}{2}, -1\right)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(iii)

Let the origin be shifted to (h, k). Then, $x = X + h$ and $y = Y + k$

Substituting $x = X + h, \quad y = Y + k$

in the equation $x^2 + 12x + 4 = 0$, we get

$$(X + h)^2 - 12(X + h)^2 + 4 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2xh - 12X - 12h + 4 = 0$$

$$\Rightarrow \quad X^2 + (2h - 12)X + h^2 - 12h + 4 = 0$$

For this equation to be free from term of first degree, we must have

$$2h - 12 = 0$$

$$\Rightarrow \quad h = \frac{12}{2}$$

$$\Rightarrow \quad h = 6$$

Hence, the origin is shifted at the point $(6, k) K \varepsilon R$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q8

Let the co-ordinate of the vertex be A(4,6) B(7,10) and C(1,-2)

Now area of the ΔABC is given by

$$\Delta = \frac{1}{2}\left|\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)\right|$$

$$= \frac{1}{2}\left|\left(4(10 + 2) + 7(-2 - 6) + 1(6 - 10)\right)\right|$$

$$= \frac{1}{2}\left|(48 - 56 - 4)\right|$$

$$= 6$$

After transforming the origin to (-2,1), the co-ordinate of the vertex will be

A(2,7),B(5,11) and C(-1,-1). Now the area will be

$$\Delta_1 = \frac{1}{2}\left|\left(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right)\right|$$

$$= \frac{1}{2}\left|\left(2(11 + 1) + 5(-1 - 7) - 1(7 - 11)\right)\right|$$

$$= \frac{1}{2}\left|(24 - 40 + 4)\right|$$

$$= 6$$

Here $\Delta = \Delta_1$

Hence proved.