RD Sharma Solutions Class 11 Maths Chapter 22 Ex 22.2

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q1

Let $P(h,k)$ be any point on the locus and let $A(2,4)$ and $B(0,k)$. Then,

$$PA = PB$$

$$\Rightarrow \quad PA^2 = PB^2$$

$$\Rightarrow \quad \left[\sqrt{(2-h)^2+(4-k)^2}\right]^2 = \left[\sqrt{(0-h)^2+(k-k)^2}\right]^2$$

$$\Rightarrow \quad (2-h)^2+(4-k)^2 = (0-h)^2+(0)^2$$

$$\Rightarrow \quad 4+h^2-4h+16+k^2-8k = h^2$$

$$\Rightarrow \quad k^2-8k-4h+20 = 0$$

Hence, locus of (h,k) is $y^2-8y-4x+20=0$

Let $P(h,k)$ be any point on the locus and let $AC(2,4)$ and $B(0,k)$ be the given points.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q2

Let $P(h,k)$ be any point on the locus and let $A(2,0)$ and $B(1,3)$. Then,

$$\frac{PA}{BP} = \frac{5}{4}$$

$$\Rightarrow \quad \frac{PA^2}{BP^2} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{\left[\sqrt{(h-2)^2+(k-0)^2}\right]^2}{\left[\sqrt{(h-1)^2+(k-3)^2}\right]^2} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{(h-2)^2+k^2}{(h-1)^2+(k^2-3)^2} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{h^2+4-4h+k^2}{h^2+1-2h+k^2+9-6k} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{(h^2-4h+k^2+4)}{h^2+k^2-2h-6k+10} = \frac{25}{16}$$

$$\Rightarrow \quad 16(h^2-4h+k^2+4) = 25(h^2+k^2-2h-6k+10)$$

$$\Rightarrow \quad 16h^2-64h+16k^2+64 = 25h^2+25k^2-50h-150k+250$$

$$\Rightarrow \quad 25h^2-16h^2+25k^2-16k^2-50h+64h-150k+250-64 = 0$$

$$\Rightarrow \quad 9h^2+9k^2+14h-150k+186 = 0$$

Hence, locus of (h,k) is $9x^2+9y^2+14x-150y+186=0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q3

Let $P(h,k)$ be any point on the locus and let $A(ae,0)$ and $B(-ae,0)$ be the given points.
By the given condition

$$PA - PB = 2a$$

$$\Rightarrow \quad PA = 2a + PB$$

$$\Rightarrow \quad \sqrt{(ae-h)^2 + (0-k)^2} = 2a + \sqrt{(-ae-h)^2 + (0-k)^2}$$

$$\Rightarrow \quad (ae-h)^2 + k^2 = \left(2a + \sqrt{(ae+h)^2 + k^2}\right)^2 \qquad \text{[Taking square on both sides]}$$

$$\Rightarrow \quad (ae)^2 + h^2 - 2aeh + k^2 = 4a^2 + (ae+h)^2 + k^2 + 2 \times 2a \times \sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad h^2 + k^2 + (ae)^2 - 2aeh = 4a^2 + (ae)^2 + h^2 + 2hae + k^2 + 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4a^2 - 2aeh - 2aeh = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4a^2 - 4aeh = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -4\left[a^2 + aeh\right] = 4a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -\left[a^2 + aeh\right] = a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -a\left[a + eh\right] = a\sqrt{(ae+h)^2 + k^2}$$

$$\Rightarrow \quad -\left[a + eh\right] = \sqrt{(ae+h)^2 + k^2}$$

$\Rightarrow \quad -[a+eh] = \sqrt{(ae+h)^2 + k^2}$

$\Rightarrow \quad (a+eh)^2 = \left(\sqrt{(ae+h)^2 + k^2}\right)^2$ [Taking square on both sides]

$\Rightarrow \quad a^2 + (eh)^2 2hae = (ae+h)^2 + k^2$

$\Rightarrow \quad a^2 + (eh)^2 + 2hae = (ae)^2 + h^2 + 2hae + k^2$

$\Rightarrow \quad a^2 + e^2h^2 = a^2e^2 + h^2 + k^2$

$\Rightarrow \quad e^2h^2 - h^2 - k^2 = a^2e^2 - a^2$

$\Rightarrow \quad h^2(e^2 - 1) - k^2 = a^2(e^2 - 1)$

$\Rightarrow \quad \frac{h^2(e^2-1)}{a^2(e^2-1)} - \frac{k^2}{a^2(e^2-1)} = 1$

$\Rightarrow \quad \frac{h^2}{a^2} - \frac{k^2}{b^2} = 1,$ Where $b^2 = a^2(e^2 - 1)$.

$\therefore$ The locus of (h,k) is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Hence proved.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q4

Let $P(h,k)$ be any point on the locus and let $A(0,2)$ and $B(0,-2)$ be the given points.
By the given condition $PA + PB = 6$

$\Rightarrow \quad \sqrt{(h-0)^2 + (k-2)^2} + \sqrt{(h-0)^2 + (k+2)^2} = 6$

$\Rightarrow \quad \sqrt{h^2(k-2)h^2} = 6 - \sqrt{h^2 + (k+2)^2}$

$\Rightarrow \quad h^2 + (k-2)h^2 = 36 - 12\sqrt{h^2 + (k+2)^2} + h^2 + (k+2)^2$

$\Rightarrow \quad -8k - 36 = -12\sqrt{h^2 + (k+2)^2}$

$\Rightarrow \quad (2k+9) = 3\sqrt{h^2 + (k+2)^2}$

$\Rightarrow \quad (2k+9)^2 = 9\left(h^2 + (k+2)^2\right)$

$\Rightarrow \quad 4k^2 + 36k + 81 = 9h^2 + 9k^2 + 36k + 36$

$\Rightarrow \quad 9h^2 + 5k^2 = 45$

Hence, locus of (h,k) is $9x^2 + 5y^2 = 45$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q5

Let $P(h,k)$ be any point on the locus and let $A(1,3)$ and $B(h,0)$. Then,

$PA = PB$

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (1-h)^2 + (3-k)^2 = (h-h)^2 + (0-k)^2$

$\Rightarrow \quad 1 + h^2 - 2h + 9 + k^2 - 6k = 0 + k^2$

$\Rightarrow \quad h^2 - 2h - 6k + 10 = 0$

Hence,locus of (h,k) is $x^2 - 2x - 6y + 10 = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q6

Let $P(h,k)$ be any point on the locus and let $O(0,0)$ be the origin.
By the given condition

$OP = 3k \quad [\because\ k$ is the diffance of point from x-axis$]$

$\Rightarrow \quad OP^2 = 9k^2$

$\Rightarrow \quad \left(\sqrt{(0-h)^2 + (0-k)^2}\right)^2 = 9k^2$

$\Rightarrow \quad h^2 + k^2 = 9k^2$

$\Rightarrow \quad h^2 = 9k^2 - k^2$

$\Rightarrow \quad h^2 = 8k^2$

Hence, locus of (h,k) is $x^2 = 8y^2$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q7

Let $P(h,k)$ be any point on the locus. Then,

Area $(PAB) = 9$ sq units

$$\Rightarrow \quad \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x^3(y_1-y_2)\right| = 9$$

$$\Rightarrow \quad \left|5(-2-k)+3(k-3)+h(3+2)\right| = 18$$

$$\Rightarrow \quad \left|-10-5k+3k-9+5h\right| = 18$$

$$\Rightarrow \quad \left|5h-2k-19\right| = 18$$

$$\Rightarrow \quad 5h-2k-19 = \pm 18$$

$$\Rightarrow \quad 5h-2k-19 \mp 18 = 0$$

$$\Rightarrow \quad 5h-2k-37 = 0 \quad \text{or,} \quad 5h-2k-1 = 0$$

Hence, the locus of (h,k) is

$$5x-2y-37 = 0 \quad \text{or,} \quad 5x-2y-1 = 0.$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q8

Let $P(h,k)$ be the variable point and let $A(2,0)$ and $B(-2,0)$ be the given points.

Then $\angle APB = \pi/2$

$$\Rightarrow \quad AB^2 = PA^2 + PB^2$$

$$\Rightarrow \quad (2+2)^2 + 0 = (2-h)^2 + (0-k^2) + (-2-h)^2 + (0-k)^2$$

$$\Rightarrow \quad 16 = 4+h^2-4h+k^2+4+h^2+4h+k^2$$

$$\Rightarrow \quad 16 = 2h^2+2k^2+8$$

$$\Rightarrow \quad 2h^2+2k^2+8-16 = 0$$

$$\Rightarrow \quad 2h^2+2k^2-8 = 0$$

$$\Rightarrow \quad h^2+k^2-4 = 0$$

Hence, the locus of (h,k) is $x^2+y^2 = 4$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q9

Let $P(h,k)$ be any point on the locus. Then,

Area (PAB) = 8 sq units

$$\Rightarrow \quad \frac{1}{2}\left|x_1(y_2-y_3)+(y_3-y_1)+x_3(y_1-y_2)\right| = 8$$

$$\Rightarrow \quad \frac{1}{2}\left|-1(3-k)+2(k-1)+h(1-3)\right| = 8$$

$$\Rightarrow \quad \frac{1}{2}\left|-3+k+2k-2-2h\right| = 8$$

$$\Rightarrow \quad \frac{1}{2}\left|-2h+3k-5\right| = 8$$

$$\Rightarrow \quad \left|-2h+3k-5\right| = 16$$

$$\Rightarrow \quad -2h+3k-5 = \pm 16$$

$$\Rightarrow \quad 2h-3k+5 \pm 16 = 0$$

$$\Rightarrow \quad 2h-3k+21 = 0 \quad \text{or,} \quad 2h-3k-11 = 0$$

Hence, the locus of (h,k) is

$$2x-3y+21 = 0 \quad \text{or,} \quad 2x-3y-11 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q10

Let the two perpendicular lines be the coordinate axes. Let AB be a rod length l. Let the coordinates of A and B be $(a, 0)$ and $(0, b)$ respectively. As the rod slides the value of a and b change. so, a and b are two variables.
Let $P(h, k)$ be the point on the locus. Then,

$$h = \frac{2 \times a + 1 \times 0}{2 + 1}$$

$$\Rightarrow \quad h = \frac{2a}{3}$$

$$\Rightarrow \quad a = \frac{3h}{2}$$

and $k = \dfrac{2 \times 0 + b \times 1}{2 + 1}$

$$\Rightarrow \quad k = \frac{b}{3}$$

$$\Rightarrow \quad b = 3k$$

from $\triangle AOB$, we have

$$AB^2 = OA^2 + OB^2$$

$$\Rightarrow \quad l^2 = \left[(a-0)^2 + (0, 0)\right]^2 + \left[(0-0)^2 + (b-0)^2\right]$$

$$\Rightarrow \quad l^2 = a^2 + b^2$$

$$\Rightarrow \quad a^2 + b^2 = l^2$$

$$\Rightarrow \quad \left(\frac{3h}{2}\right)^2 + (3k)^2 = l^2$$

$$\Rightarrow \quad \frac{9h^2}{4} + 9k^2 = l^2$$

$$\Rightarrow \quad \frac{h^2}{4} + k^2 = \frac{l^2}{9}$$

Hence, the locus of (h, k) is $\dfrac{x^2}{4} + y^2 = \dfrac{l^2}{9}$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q11

Gicen, line is

$$x\cos\alpha + y\sin\alpha = p$$

$$\frac{x}{\frac{p}{\cos\alpha}} + \frac{y}{\frac{p}{\sin\alpha}} = 1$$

Intercepts on x axis is $\frac{p}{\cos\alpha}$ and y - axis is $\frac{p}{\sin\alpha}$

Let $P(x,y)$ be the mid point of AB.

$$(x,y) = \left(\frac{\frac{p}{\cos\alpha}+0}{2}, \frac{\frac{p}{\sin\alpha}+0}{2}\right) = \left(\frac{p}{2\cos\alpha}, \frac{p}{2\sin\alpha}\right)$$

$$\therefore x = \frac{p}{2\cos\alpha}, \quad y = \frac{p}{2\sin\alpha}$$

$$2\cos\alpha = \frac{p}{x}, \quad 2\sin\alpha = \frac{p}{y}$$

Square both sides,

$$4\cos^2\alpha = \frac{p^2}{x^2} \text{ - - - - (1)}$$

and

$$4\sin^2\alpha = \frac{p^2}{y^2} \text{ - - - - - (2)}$$

$[(1)+(2)]$

$$4\cos^2\alpha + 4\sin^2\alpha = \frac{p^2}{x^2} + \frac{p^2}{y^2}$$

$$4 = \frac{p^2\left(x^2+y^2\right)}{x^2y^2}$$

$$4x^2y^2 = p^2\left(x^2+y^2\right)$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.2 Q12

Let $P(h,k)$ be the point on the locus and let the coordinates of a are (a,b). Then,

$$h = \frac{a+0}{2} \text{ and } \frac{b+0}{2} = k \qquad [\because P \text{ is the mid-point of } Q \text{ and the origino}]$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q1

We have,

$$(x-a)^2+(y-b)^2=r^2 \quad \ldots\ldots.(i)$$

Substituting $x=X+(a-c)$, $y=Y+b$ in the equation (i), we get

$$[X+a-c-a]^2+[Y+b-b]^2=r^2$$

$$\Rightarrow \quad [X-c]^2+[Y]^2=r^2$$

$$\Rightarrow \quad X^2+c^2-2Xc+Y^2=r^2$$

$$\Rightarrow \quad X^2+y^2-2cX=r^2-c^2$$

Hence, the required equation is $X^2+y^2-2cX=r^2-c^2$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q2

We have,

$$(a-b)\left(x^2+y^2\right)-2abx=0$$

Substituting $x=X+\frac{ab}{a-b}$, $y=Y$
in the given equation, we get

$$(a-b)\left[\left(X+\frac{ab}{a-b}\right)+Y^2\right]-2ab\left[X+\frac{ab}{a-b}\right]=0$$

$$\Rightarrow \quad (a-b)\left[X^2+\left(\frac{ab}{a-b}\right)^2+2\frac{Xab}{a-b}+Y^2\right]-2abX-2\frac{(ab)^2}{a-b}=0$$

$$\Rightarrow \quad (a-b)\left[\frac{X^2(a-b)^2+(ab)h^2+2Xab(a-b)+Y^2(a-b)^2}{(a-b)^2}\right]-\frac{2abX(a-b)+2(ab)^2}{a-b}=0$$

$$\Rightarrow \quad \frac{X^2(a-b)^2+(ab)^2+2ab(a-b)+Y^2(a-b)^2}{a-b}=\frac{2ab(a-b)+2(ab)^2}{a-b}$$

$$\Rightarrow \quad X^2(a-b)^2+Y^2(a-b)^2+(ab)^2+2ab(a-b)=2ab(a-b)+2(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=(ab)^2$$

$$\Rightarrow \quad (a-b)^2\left(X^2+Y^2\right)=a^2b^2$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(i)

We have,

$$x^2 + xy - 3x - y + 2 = 0$$

Substituting $x = X + 1$, $Y + 1$ in the equation, we get

$$(X+1)^2 + (X+1)(Y+1) - 3(X+1) - (Y+1) + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3X - 3 - Y - 1 + 2 = 0$$

$$\Rightarrow \quad X^2 + XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(ii)

We have,

$$x^2 - y^2 - 2x + 2y = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X+1)^2 - (Y-1)^2 - 2(X+1) + 2(Y+1) = 0$$

$$\Rightarrow \quad X^2 + 1 + 2X - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$$

$$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2Y = 0$$

$$\Rightarrow \quad X^2 - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(iii)

We have,

$$xy - x - y + 1 = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$$

$$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$$

$$\Rightarrow \quad XY = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q3(iv)

We have,

$$xy - y^2 - x + y = 0$$

Substituting $x = X + 1$, $y = Y + 1$ in the equation, we get

$$(X+1)(Y+1) - (Y+1) - (X+1) + (Y+1) = 0$$

$$\Rightarrow \quad XY + Y + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$$

$$\Rightarrow \quad XY + 2Y - Y^2 - 1 - 2Y + 1 = 0$$

$$\Rightarrow \quad XY - Y^2 = 0$$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q4

We have,

$$x^2 + xy - 3x - y + 2 = 0 \text{ .... (i)}$$

Let the origin be shifted to (h,k). Then $x = X + h$ and $y = Y + k$.

Substituting $x = X + h$, $y = Y + k$
in the equation (i), we get

$$(X+h)^2 + (X+h)(Y+k) - 3(X+h) - (Y+k) + 2 = 0$$

$$\Rightarrow \quad X^2 + h^2 + 2Xh + XY + Xk + Yh + hk - 3k - 3h - Y - k + 2 = 0$$

$$\Rightarrow \quad X^2 + XY + 2xh + Xk + Yh - Y3 - X + h^2 + hk - 3h - k + 2 = 0$$

$$\Rightarrow \quad X^2 + (2Xh + Xk - 3X) + XY + (Yh - Y) + (h^2 + hk - 3h - k + 2) = 0$$

$$\Rightarrow \quad X^2 + (2h + k - 3)X + XY + (h-1)Y + (h^2 + hk - 3h - k + 2) = 0$$

For this equation to be free from first degree and the constant term, we must have,

$$2h + k - 3 = 0 \text{ .... (ii)}$$

$$h - 1 = 0$$

$$\Rightarrow h = 1 \text{ .... (iii)}$$

and

$$h^2 + hk - 3h - k + 2 = 0 \text{ .... (iv)}$$

Putting $h = 1$ in equation (ii), we get

$$2 + k - 3 = 0$$

$$\Rightarrow \quad k = 1$$

Putting $h = 1$ and $k = 1$ in equation (iv), we get

$$(1)^2 + 1 - 3 - 1 + 2 = 0$$

Hence, the value of h and k satisfies the equation (iv)

$\therefore$ The origin is shifted at the point $(1,1)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q5

Let the vertices of a triangle be $A(2,3)$, $B(5,7)$ and $C(-3,-1)$.

Then, area of $\triangle ABC$ is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|2(7+1) + 5(-1-3) - 3(-3-7)\right|$$

$$= \frac{1}{2}\left|2 \times 8 + 5 \times (-4) - 3 \times (-4)\right|$$

$$= \frac{1}{2}\left|16 - 20 + 12\right|$$

$$= \frac{8}{2}$$

$$= 4$$

$\Rightarrow \quad \Delta = 4 Sq$ unit

It is given that the origin is shifted at $(-1,3)$. Then new coordinates of the vertices are

$A_1 = (2-3, 3+3) = (-1, 6)$

$B_1 = (5-1, 7+3) = (4, 10)$

and $\quad C_1 = (-3-1, -1+3) = (-4, 2)$

Therefore, the area of the triangle in the new coordinate system is given by

$$\Delta_1 = \frac{1}{2}\left|\left[-1(10-2) + 4(2-6) - 4(6-10)\right]\right|$$

$$= \frac{1}{2}\left|\left[-1 \times 8 + 4 \times (-4) - 4 \times (-4)\right]\right|$$

$$= \frac{1}{2}\left|\left[-8 - 16 + 16\right]\right|$$

$$= \frac{1}{2}\left|-8\right|$$

$$= \frac{8}{2}$$

$\Rightarrow \quad \Delta_1 = 4 \ldots\ldots\ldots\ldots\ldots (2)$

From (i) and (ii), we get

$\Delta = \Delta_1$

Hence, the area of a triangle is invariant under the translation of the axes.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(i)

We have,

$x^2 + xy - 3y^2 - y + 2 = 0 \ldots\ldots\ldots (i)$

Substituting $x = X + 1$, $y = Y + 1$

in equation (i), we get

$$(X+1)^2 + (X+1)(Y+1) - 3(Y+1)^2 - (Y+1) + 2 = 0$$

$\Rightarrow \quad X^2 + 1 + 2X + XY + X + Y + 1 - 3(Y^2 + 1 + 2Y) - Y - 1 + 2 = 0$

$\Rightarrow \quad X^2 + XY + 3X + 3 - 3Y^2 - 3 - 6Y = 0$

$\Rightarrow \quad X^2 - 3Y^2 + XY + 3X - 6Y = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(ii)

We have,

$xy - y^2 - x + y = 0 \ldots\ldots\ldots (i)$

Substituting $x = X + 1$, $y = Y + 1$

in equation (i), we get

$$(X+1)(Y+1) - (Y+1)^2 - (X+1) + (Y+1) = 0$$

$\Rightarrow \quad XY + X + Y + 1 - (Y^2 + 1 + 2Y) - X - 1 + Y + 1 = 0$

$\Rightarrow \quad XY + 2Y + 1 - Y^2 - 1 - 2Y = 0$

$\Rightarrow \quad XY - Y^2 = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(iii)

We have,

$xy - x - y + 1 = 0 \ldots\ldots (i)$

Substituting $x = X + 1, \; y = Y + 1$
in equation (i), we get

$(X+1)(Y+1) - (X+1) - (Y+1) + 1 = 0$

$\Rightarrow \quad XY + X + Y + 1 - X - 1 - Y - 1 + 1 = 0$

$\Rightarrow \quad XY = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q6(iv)

We have,

$x^2 - y^2 - 2x - 2y = 0 \ldots\ldots (i)$

Substituting $x = X + 1, \; y = Y + 1$
in equation (i), we get

$(X+1)^2 - (Y+1)^2 - 2(X+1) + 2(Y+1) = 0$

$\Rightarrow \quad X^2 + 1 + 2X - (Y^2 + 1 + 2Y) - 2X - 2 + 2Y + 2 = 0$

$\Rightarrow \quad X^2 + 1 - Y^2 - 1 - 2Y + 2X = 0$

$\Rightarrow \quad X^2 + 2X + 1 - (Y^2 + 2Y + 1)$

$\Rightarrow \quad (X+1)^2 - (Y+1)^2$

$\Rightarrow \quad x^2 - y^2 = 0$

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(i)

Let the origin be shifted to (h,k). Then, $x = X + h$ and $y = Y + k$.

Substituting $x = X + h, \quad y = Y + k$
in the equation $y^2 + x^2 - 4x - 8y + 3 = 0$, we get

$(Y+k)^2 + (X+h)^2 - 4(X-h) - 8(Y+k) + 3 = 0$

$\Rightarrow \quad Y^2 + k^2 + 2Yk + X^2 + h^2 + 2Xh - 4X - 4h - 8Y - 8k + 3 = 0$

$\Rightarrow \quad Y^2 + X^2 + 2Yk - 8Y + 2Xh - 4X + k^2 + h^2 - 4h - 8k + 3 = 0$

$\Rightarrow \quad Y^2 + X^2 + (2k - 8)Y + (2h - 4)X + (k^2 + h^2 - 4h - 8k + 3) = 0$

For this equation to be free from the term of first degree, we must have

$2k - 8 = 0$ and $2h - 4 = 0$

$\Rightarrow \quad k = 4$ and $h = 2$

Hence, the origin is shifted at the point $(2,4)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(ii)

Let the origin be shifted to (h,k). Then, $x = X + h$ and $y = Y + k$
Substituting $x = X + h, \quad y = Y + k$
in the equation $x^2 + y^2 - 5x + 2y - 5 = 0$, we get

$(X+h)^2 + (Y+k)^2 - 5(X+h) + 2(Y+k) - 5 = 0$

$\Rightarrow \quad X^2 + h^2 + 2xh + Y^2 + k^2 + 2Yk - 5X - 5h + 2Y + 2k - 5 = 0$

$\Rightarrow \quad X^2 + Y^2 + 2Yk + 2Y + 2Xh - 5x + h^2 + k^2 - 5h + 2k - 5 = 0$

$\Rightarrow \quad X^2 + Y^2 + (2k + 2)Y + (2h - 5)X + h^2 + k^2 - 5h + 2k - 5 = 0$

For this equation to be free from the term of first degree, we must have

$2k + 2 = 0$ and $2h - 5 = 0$

$\Rightarrow \quad k = -1$ and $h = \frac{5}{2}$

Hence, the origin is shifted at the point $\left(\frac{5}{2}, -1\right)$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q7(iii)

Let the origin be shifted to (h,k). Then, $x = X + h$ and $y = Y + k$

Substituting $x = X + h, \quad y = Y + k$

in the equation $x^2 + 12x + 4 = 0$, we get

$$(X + h)^2 - 12(X + h)^2 + 4 = 0$$

$\Rightarrow \quad X^2 + h^2 + 2xh - 12X - 12h + 4 = 0$

$\Rightarrow \quad X^2 + (2h - 12)X + h^2 - 12h + 4 = 0$

For this equation to be free from term of first degree, we must have

$2h - 12 = 0$

$\Rightarrow \quad h = \frac{12}{2}$

$\Rightarrow \quad h = 6$

Hence, the origin is shifted at the point $(6,k) K \in R$.

Brief Review of Cartesian System of Rectangular co-ordinates Ex 22.3 Q8

Let the co-ordinate of the vertex be A(4,6) B(7,10) and C(1,-2)

Now area of the ΔABC is given by

$\Delta = \frac{1}{2}\left|(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))\right|$

$= \frac{1}{2}\left|(4(10+2) + 7(-2-6) + 1(6-10))\right|$

$= \frac{1}{2}\left|(48 - 56 - 4)\right|$

$= 6$

After transforming the origin to (-2,1), the co-ordinate of the vertex will be

A(2,7),B(5,11) and C(-1,-1). Now the area will be

$\Delta_1 = \frac{1}{2}\left|(x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))\right|$

$= \frac{1}{2}\left|(2(11+1) + 5(-1-7) - 1(7-11))\right|$

$= \frac{1}{2}\left|(24 - 40 + 4)\right|$

$= 6$

Here $\Delta = \Delta_1$

Hence proved.